ASPIRE STUDY

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CTQ 4.0

Trigonometry Ratio and Identity

Think the Aspire Study Way !

CLASS 01

CTQ 4.0

If $u = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$, then the difference between the maximum and minimum values of u^2 is given by :

(a) $(a-b)^2$ (b) $2\sqrt{a^2+b^2}$

(c) $(a+b)^2$ (d) $2(a^2+b^2)$

$a = b = 1$

$u = \sqrt{1} + \sqrt{1}$

$u = 2$

$\boxed{u^2 = 4}$

max = 4

min = 4

Let α, β such that $\pi < \alpha - \beta < 3\pi$. If $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$, then the value of $\cos\frac{\alpha-\beta}{2}$

(a) $-\frac{6}{65}$ (b) $\frac{3}{\sqrt{130}}$

(c) $\frac{6}{65}$ (d) $-\frac{3}{\sqrt{130}}$

$$1+1+2\sin\alpha\sin\beta+2\cos\alpha\cos\beta = \frac{21^2}{65^2}+\frac{27^2}{65^2}$$

$$2+2(\cos(\alpha-\beta)) = \frac{441+729}{4225}$$

$$2(1+\cos(\alpha-\beta)) = \frac{1170}{4225}$$

$$\cancel{2}(\cancel{1}+2\cos^2\left(\frac{\alpha-\beta}{2}\right)-\cancel{1}) = \frac{\overset{117}{\cancel{585}}}{\underset{845}{\cancel{4225}}}$$

$$\cos^2\left(\frac{\alpha-\beta}{2}\right) = \frac{\overset{9}{\cancel{117}}}{\underset{65}{\cancel{845}}\times 2}$$

$$\cos^2\left(\frac{\alpha-\beta}{2}\right) = \frac{9}{2\times 65}$$

$$\cos\left(\frac{\alpha-\beta}{2}\right) = -\frac{3}{\sqrt{130}}$$

If $0 < x < \pi$ and $\cos x + \sin x = \frac{1}{2}$, then $\tan x =$

(a) $\frac{1-\sqrt{7}}{4}$ (b) $\frac{4-\sqrt{7}}{3}$ (c) $-\frac{4+\sqrt{7}}{3}$ (d) $\frac{1+\sqrt{7}}{4}$

$$1+\sin 2x = \frac{1}{4}$$

$$\sin 2x = \frac{1}{4} - 1 = \frac{-3}{4}$$

$$\frac{2\tan x}{1+\tan^2 x} = \frac{-3}{4}$$

$$3\tan^2 x + 8\tan x + 3 = 0$$

$$\tan x = \frac{-8 \pm \sqrt{64 - 4\times 9}}{2\times 3}$$

$$\tan x = \frac{-8 \pm \sqrt{28}}{2\times 3}$$

$$= \frac{-8 \pm 2\sqrt{7}}{2\times 3} = \frac{-4 \pm \sqrt{7}}{3} = \frac{-4-\sqrt{7}}{3}$$

Let $\cos(\alpha+\beta)=\frac{4}{5}$ and $\sin(\alpha-\beta)=\frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then $\tan 2\alpha =$

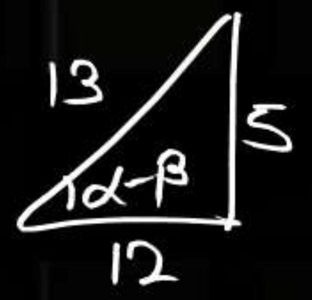
(a) $\frac{56}{33}$ (b) $\frac{19}{12}$ (c) $\frac{20}{7}$ (d) $\frac{25}{16}$

$$\tan(\alpha+\beta+\alpha-\beta) = \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}$$

$$= \frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\times\frac{5}{12}} = \frac{\frac{36+20}{\cancel{48}}}{\frac{33}{\cancel{48}}}$$

$$= \frac{56}{33}$$

$\tan(\alpha+\beta)=\frac{3}{4}$



$\tan(\alpha-\beta)=\frac{5}{12}$

If $A = \sin^2 x + \cos^4 x$, then for all real x :

(a) $\frac{13}{16} \le A \le 1$ (b) $1 \le A \le 2$

(c) $\frac{3}{4} \le A \le \frac{13}{16}$ (d) $\frac{3}{4} \le A \le 1$

The expression $\frac{\tan A}{1-\cos A}+\frac{\cos A}{1-\tan A}$ be written as

(a) $\sin A\cos A+1$ (b) $\sec A \operatorname{cosec} A+1$

(c) $\tan A+\cot A$ (d) $\sec A+\operatorname{cosec} A$

JEE MAIN 2013

$\boxed{A=135^\circ}$

Let $f_k(x) = \frac{1}{k}\left(\sin^k x + \cos^k x\right)$, where $x \in R$ and $k \geq 1$. Then $f_4(x) - f_6(x)$ equals

(a) $\frac{1}{4}$ (b) $\frac{1}{12}$ (c) $\frac{1}{6}$ (d) $\frac{1}{3}$

JEE MAIN 2014

$$f_4 - f_6 = \frac{1}{4}\left(\sin^4 x + \cos^4 x\right) - \frac{1}{6}\left(\sin^6 x + \cos^6 x\right)$$

$\underline{\underline{x = 0}}$

$$\frac{1}{4} - \frac{1}{6} = \underline{\frac{1}{12}}$$

If m and M are the minimum and maximum values of $4+\frac{1}{2}\sin^2 2x-2\cos^4 x, x \in R$, then $M-m=$

(a) $\frac{15}{4}$ ~~(b)~~ $\frac{9}{4}$ (c) $\frac{7}{4}$ (d) $\frac{1}{4}$

JEE MAIN 2016

$4+\frac{1}{2}\times 4\sin^2 x\cos^2 x-2\cos^4 x$

$4+2(1-\cos^2 x)\cos^2 x-2\cos^4 x$

$4+2\cos^2 x-2\cos^4 x-2\cos^4 x$

$y=-4\cos^4 x+2\cos^2 x+4$

$y=-4\left[\cos^4 x-\frac{1}{2}\cos^2 x-1\right]$

$y=-4\left[\cos^4 x-\frac{1}{2}\cos^2 x+\frac{1}{16}-\frac{1}{16}-1\right]$

$y=-4\left[\left(\cos^2 x-\frac{1}{4}\right)^2-\frac{17}{16}\right]$

$0\le \cos^2 x\le 1$

$-\frac{1}{4}\le \cos^2 x-\frac{1}{4}\le \frac{3}{4}$

$0\le \left(\cos^2 x-\frac{1}{4}\right)^2\le \frac{9}{16}$

$-\frac{17}{16}\le \left(\cos^2 x-\frac{1}{4}\right)^2\le -\frac{1}{2}$

$2\le y\le \frac{17}{4}$

$M-m$

$\frac{17}{4}-2$

$=\frac{9}{4}$

If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of $\cos 4x =$

(a) $\frac{1}{3}$ (b) $\frac{2}{9}$ (c) $-\frac{7}{9}$ (d) $-\frac{3}{5}$

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For any $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$, the expression $3(\cos\theta - \sin\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$ equals :

(a) $13 - 4\cos^2\theta + 6\sin^2\theta\cos^2\theta$

(b) $13 - 4\cos^6\theta$

(c) $13 - 4\cos^2\theta + 6\cos^4\theta$

(d) $13 - 4\cos^4\theta + 2\sin^2\theta\cos^2\theta$

JEE MAIN 2019

$3(1-\sin 2\theta)^2 + 6(1+\sin 2\theta) + 4\sin^6\theta$

$3 + 3\times\sin^2 2\theta - \cancel{6\sin 2\theta} + 6 + \cancel{6\sin 2\theta} + 4\sin^6\theta$

$9 + 3\sin^2 2\theta + 4\sin^6\theta$

$9 + 3\times 4\sin^2\theta\cdot\cos^2\theta + 4\sin^6\theta$

$9 + 12(1-\cos^2\theta)\cos^2\theta + 4(1-\cos^2\theta)^3$

$9 + 12\cos^2\theta - 12\cos^4\theta + 4\left(1 - \cos^6\theta - 3\cos^2\theta(1-\cos^2\theta)\right)$

$9 + \cancel{12\cos^2\theta} - \cancel{12\cos^4\theta} + 4 - 4\cos^6\theta - \cancel{12\cos^2\theta} + \cancel{12\cos^4\theta}$

The value of $\cos\frac{\pi}{2^2}.\cos\frac{\pi}{2^3}\ldots\cos\frac{\pi}{2^{10}}.\sin\frac{\pi}{2^{10}}$ is

(a) $\frac{1}{256}$ (b) $\frac{1}{2}$ (c) $\frac{1}{1024}$ (d) $\frac{1}{512}$

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$$\cos\frac{\pi}{2^{10}}\cdot\cos\frac{\pi}{2^9}\cdot\cos\frac{\pi}{2^8}\cdots\cos\frac{\pi}{2^2}\cdot\sin\frac{\pi}{2^{10}}$$

$$= \frac{\sin\left(\cancel{2}^9\times\frac{\pi}{\cancel{2^{10}}}\right)}{2^9\cdot\cancel{\sin\frac{\pi}{2^{10}}}}\times\cancel{\sin\frac{\pi}{2^{10}}}$$

$$= \frac{1}{2^9} = \frac{1}{512}$$

$$\cos\theta\cdot\cos2\theta\cdots\cos2^{n-1}\theta$$

$$= \frac{\sin2^n\theta}{2^n\sin\theta}$$

$\boxed{\text{Angle GP}}$

The maximum value of $3\cos\theta + 5\sin\left(\theta - \frac{\pi}{6}\right)$ for any real value of θ is :

(a) $\sqrt{34}$ (b) $\sqrt{31}$ (c) $\sqrt{19}$ (d) $\frac{\sqrt{79}}{2}$

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$3\cos\theta + 5\sin\theta\cos\frac{\pi}{6} - 5\cos\theta\sin\frac{\pi}{6}$

$3\cos\theta + 5\times\frac{\sqrt{3}}{2}\sin\theta - \frac{5}{2}\cos\theta$

$\frac{1}{2}\cos\theta + \frac{5\sqrt{3}}{2}\sin\theta$

$\max = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{5\sqrt{3}}{2}\right)^2}$

$= \sqrt{\frac{1}{4} + \frac{75}{4}} = \sqrt{\frac{76}{4}} = \sqrt{19}$

\# $a\sin\theta + b\cos\theta$ $\max = \sqrt{a^2+b^2}$

$\min = -\sqrt{a^2+b^2}$

If $\cos(\alpha+\beta)=\frac{3}{5}$, $\sin(\alpha-\beta)=\frac{5}{13}$ and $0<\alpha,\beta<\frac{\pi}{4}$, then $\tan 2\alpha =$

(a) $\frac{21}{16}$ (b) $\frac{63}{52}$ (c) $\frac{33}{52}$ ~~(d)~~ $\frac{63}{16}$

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$\tan(\underline{\alpha+\beta}+\underline{\alpha-\beta})$

The value of

$\cos^2 10° - \cos 10° \cos 50° + \cos^2 50°$ is

(a) $\frac{3}{2} + \cos 20°$ (b) $\frac{3}{4}$

(c) $\frac{3}{2}(1 + \cos 20°)$ (d) $\frac{3}{2}$

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$$\frac{\cos 20° + 1}{2} - \frac{1}{2}(2\cos 10° \cos 50) + \frac{\cos 100° + 1}{2}$$

$$\frac{1}{2}\left[\cos 20° + 1 - \cos 60° - \cos 40° + \cos 100° + 1\right]$$

$$\frac{1}{2}\left[\frac{3}{2} + \cos 20° - \cos 40° + \cos 100°\right]$$

$$\frac{1}{2}\left[\frac{3}{2} + \left(\cancel{2}\sin \cancel{30°} \cdot \cancel{\sin 10°}\right) - \cancel{\sin} 10\right]$$

$$= \frac{3}{4}$$

$\cos 2\theta = 2\cos^2\theta - 1$

$\frac{\cos 2\theta + 1}{2} = \cos^2\theta$

The value of $\sin 10° \sin \cancel{30}° \sin 50° \sin 70°$ is

(a) $\frac{1}{36}$ (b) $\frac{1}{16}$ (c) $\frac{1}{32}$ (d) $\frac{1}{18}$

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$\frac{1}{2} \times \sin 10° \sin(60° - 10°) \sin(60 + \theta)$

$\frac{1}{2} \times \frac{1}{4} \sin(3 \times 10°)$

$\frac{1}{8} \sin 30° = \frac{1}{16}$

$$y = \sin x \sin(x+2) - \sin^2(x+1)$$

$$y = \frac{1}{2} \times 2 \sin \sin(x+2) - \left(\frac{1-\cos 2(x+1)}{2}\right)$$

$$y = \frac{1}{2}\left[\cos(x-x-2) - \cancel{\cos(x+x+2)}\right] - \frac{1}{2} + \cancel{\frac{1}{2}\cos(2x+2)}$$

$$y = \frac{1}{2}\cos 2 - \frac{1}{2} = \frac{1}{2}(\cos 2 - 1) = \frac{1}{2}(\cancel{1} - 2\sin^2 1 - \cancel{1})$$

The equation $y = \sin x \, \sin(x + 2) - \sin^2(x + 1)$ represents a straight line lying in :

(a) first, second and fourth quadrants

(b) first, third and fourth quadrants

(c) second and third quadrants only

(d) third and fourth quadrants only

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The value of $\cos^3\left(\frac{\pi}{8}\right)\cos\left(\frac{3\pi}{8}\right)+\sin^3\left(\frac{\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)$

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2\sqrt{2}}$

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$$\cos^3\frac{\pi}{8}\cdot\cos\left(\frac{\pi}{2}-\frac{\pi}{8}\right)+\sin^3\frac{\pi}{8}\,\sin\left(\frac{\pi}{2}-\frac{\pi}{8}\right)$$

$$\cos^3\frac{\pi}{8}\times\sin\frac{\pi}{8}+\sin^3\frac{\pi}{8}\cos\frac{\pi}{8}$$

$$\sin\frac{\pi}{8}\cos\left(\frac{\pi}{8}\right)\left(\sin^2\frac{\pi}{8}+\cos^2\frac{\pi}{8}\right)$$

$$=\frac{1}{2}\times 2\sin\frac{\pi}{8}\cos\frac{\pi}{8}$$

$$=\frac{1}{2}\sin\left(2\times\frac{\pi}{8}\right)=\frac{1}{2}\times\frac{1}{\sqrt{2}}$$

If $x = \Sigma_{n=0}^{\infty}(-1)^n \tan^{2n}\theta$ and $y = \Sigma_{n=0}^{\infty}\cos^{2n}\theta$ for $0 < \theta < \frac{\pi}{4}$, then

(a) $x(1+y) = 1$ (b) $y(1-x) = 1$

(c) $y(1+x) = 1$ (d) $x(1-y) = 1$

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$x = 1 - \tan^2\theta + \tan^4\theta + \cdots$

$x = \frac{1}{1+\tan^2\theta} \Rightarrow \frac{1}{\sec^2\theta}$

$\boxed{x = \cos^2\theta}$

$y = 1 + \cos^2\theta + \cos^4\theta - \cdots$

$y = \frac{1}{1-\cos^2\theta}$

$y = \frac{1}{\sin^2\theta}$

$y = \frac{1}{1-x}$

$y(1-x) = 1$

If the equation $\cos^4 x + \sin^4 x + \lambda = 0$ has real solutions for θ, then λ lies in the interval :

(a) $\left[-\frac{3}{2}, -\frac{5}{4}\right]$ (b) $\left(-\frac{1}{2}, -\frac{1}{4}\right]$

(c) $\left(-\frac{5}{4}, -1\right]$ (d) $\left[-1, -\frac{1}{2}\right]$

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$\cos^4\theta + \sin^4\theta = -\lambda$

$1 - 2\sin^2\theta\cos^2\theta = -\lambda$

$\lambda = 2\sin^2\theta\cos^2\theta - 1$

$= \frac{1}{2} \times 4\sin^2\theta\cos^2\theta - 1$

$= \frac{1}{2} \times \sin^2 2\theta - 1$

$0 \le \sin^2 2\theta \le 1$

$0 \le \frac{\sin^2 2\theta}{2} \le \frac{1}{2}$

$-1 \le y \le -\frac{1}{2}$

If $L = \sin^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$ and

$M = \cos^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$, then

× (a) $L = -\frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$

~~(b)~~ $M = \frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$

(c) $M = \frac{1}{4\sqrt{2}} + \frac{1}{4}\cos\frac{\pi}{8}$

× (d) $L = \frac{1}{4\sqrt{2}} - \frac{1}{4}\cos\frac{\pi}{8}$

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$$L = \sin^2\frac{\pi}{16} - \sin^2\frac{\pi}{8}$$

$$L = \frac{1-\cos\frac{\pi}{8}}{2} - \frac{1-\cos\frac{\pi}{4}}{2}$$

$$L = \cancel{\frac{1}{2}} - \frac{1}{2}\cos\frac{\pi}{8} - \cancel{\frac{1}{2}} + \frac{1}{2\sqrt{2}}$$

$$M = \frac{1+\cos\frac{\pi}{8}}{2} - \frac{1-\cos\frac{\pi}{4}}{2}$$

$$= \frac{1}{2}\left[\cos\frac{\pi}{8} + \frac{1}{\sqrt{2}}\right]$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

If $e^{(\cos^2 x+\cos^4 x+\cos^6 x+\cdots\infty)\log_e 2}$ satisfied the equation $t^2-9t+8=0$, then the value of $\frac{2\sin x}{\sin x+\sqrt{3}\cos x}\left(0<x<\frac{\pi}{2}\right)$, is

(a) $\sqrt{3}$ (b) $\frac{3}{2}$ (c) $2\sqrt{3}$ (d) $\frac{1}{2}$

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$$e^{\log_e 2^{(\cos^2 x+\cos^4 x+\cdots)}}$$

$$=2^{\cos^2 x\left(\frac{1}{1-\cos^2 x}\right)}=2^{\cot^2 x}$$

$$(t-1)(t-8)=0$$

$$t=1,\quad t=8$$

$$2^{\cot^2 x}=1,\quad 2^3=2^{\cot^2 x}$$

$$x=\frac{\pi}{2}\ \times$$

$$\cot^2 x=3$$

$$\cot x=\pm\sqrt{3}$$

$$\boxed{\cot x=\sqrt{3}}$$

$$\boxed{x=30^\circ}$$

$$\frac{\cancel{2}\times\frac{1}{\cancel{2}}}{\frac{1}{2}+\cancel{\sqrt{3}}\times\frac{\cancel{\sqrt{3}}}{2}}=\frac{1}{\frac{1}{2}+\frac{3}{2}}=\frac{1}{2}$$

$\sin x + \cos y$

$\frac{\sqrt{3}}{2} + \frac{1}{2}$

If $0 < x, y < \pi$ and $\cos x + \cos y - \cos(x+y) = \frac{3}{2}$, then $\sin x + \cos y$ is equal to :

~~(a)~~ $\frac{1+\sqrt{3}}{2}$ (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1-\sqrt{3}}{2}$

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$x = 60^\circ, y = 60^\circ$

If for $x \in \left(0, \frac{\pi}{2}\right)$, $\log_{10} \sin x + \log_{10} \cos x = -1$ and $\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1)$, n > 0, then the value of n is equal to :

(a) 16 (b) 9 (c) 12 (d) 20

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$$\log_{10} \sin x \cdot \cos x = -1$$

$$\boxed{\sin x \cos x = \frac{1}{10}}$$

$$2\log_{10}(\sin x + \cos x) = \log_{10} n - \log_{10} 10$$

$$\log_{10}(1 + 2\sin x \cos x) = \log_{10} \frac{n}{10}$$

$$1 + \frac{2}{10} = \frac{n}{10}$$

$$\frac{12}{10} = \frac{n}{10}$$

$$\boxed{n = 12}$$

If $15\sin^4\alpha + 10\cos^4\alpha = 6$, for some $\alpha \in R$, then the value of $27\sec^6\alpha + 8\,\text{cosec}^6\alpha$ is equal to

(a) 500 (b) 400 (c) 250 (d) 350

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$$15\frac{\sin^4\alpha}{\cos^4\alpha} + 10 = \frac{6}{\cos^4\alpha}$$

$$15\tan^4\alpha + 10 = 6\sec^4\alpha$$

$$15\tan^4\alpha + 10 = 6(1+\tan^2\alpha)^2$$

$$15\tan^4\alpha + 10 = 6 + 6\tan^4\alpha + 12\tan^2\alpha$$

$$9\tan^4\alpha - 12\tan^2\alpha + 4 = 0$$

$$(3\tan^2\alpha - 2)^2 = 0$$

$$3\tan^2\alpha - 2 = 0$$

$$\tan^2\alpha = \frac{2}{3}$$

$$27(1+\tan^2\alpha)^3 + 8\left(1+\frac{1}{\tan^2\alpha}\right)^3$$

$$27\left(1+\frac{2}{3}\right)^3 + 8\left(1+\frac{3}{2}\right)^3$$

$$\cancel{27}\times\frac{125}{\cancel{27}} + \cancel{8}\times\frac{125}{\cancel{8}}$$

$$= 250$$

The value of $\cot\frac{\pi}{24}$ is

(a) $\sqrt{2}+\sqrt{3}+2-\sqrt{6}$ (b) $\sqrt{2}+\sqrt{3}+2+\sqrt{6}$

(c) $\sqrt{2}-\sqrt{3}-2+\sqrt{6}$ (d) $3\sqrt{2}-\sqrt{3}-\sqrt{6}$

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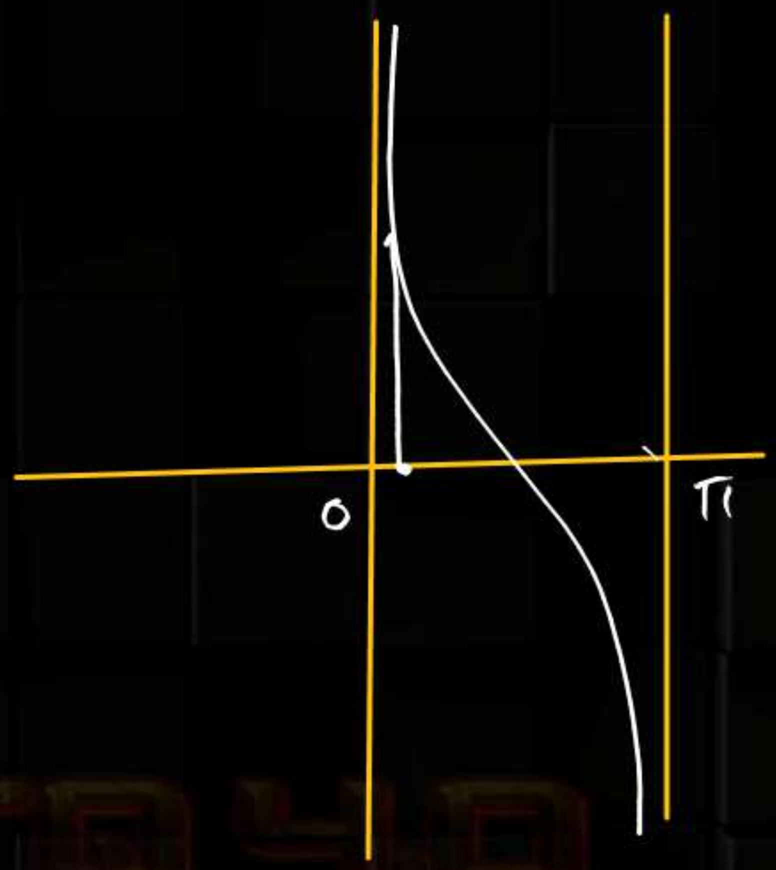

$\sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$

If $\sin\theta + \cos\theta = \frac{1}{2}$, then

$16(\sin 2\theta + \cos 4\theta + \sin 6\theta)$ is equal to :

(a) 23 (b) -27 (c) -23 (d) 27

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$$1 + \sin 2\theta = \frac{1}{4}$$

$$\sin 2\theta = -\frac{3}{4}$$

$$\cos 4\theta = 1 - 2\sin^2 2\theta$$

$$= 1 - \cancel{2} \times \frac{9}{\cancel{16}\,8}$$

$$= -\frac{1}{8}$$

$$16\left(-\frac{3}{4} - \frac{1}{8} + \sin(3\times(2\theta))\right]$$

$$16\left(-\frac{7}{8} + 3\sin 2\theta - 4\sin^3 2\theta\right]$$

$$16\left[-\frac{7}{8} + 3\left(-\frac{3}{4}\right) - \cancel{4}\left(-\frac{27}{\cancel{64}\,16}\right)\right] = -23$$

$$16\left[-\frac{14}{16} - \frac{36}{16} + \frac{27}{16}\right]$$

If $\tan\left(\frac{\pi}{9}\right), x, \tan\left(\frac{7\pi}{18}\right)$ are in arithmetic progression and $\tan\left(\frac{\pi}{9}\right), y, \tan\left(\frac{5\pi}{18}\right)$ are also in arithmetic progression, then $|x-2y|$ is equal to :

(a) 4 (b) 3 (c) 0 (d) 1

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$$\tan\frac{\pi}{9}+\tan\frac{7\pi}{18}=2x$$

$$\tan\frac{\pi}{9}+\tan\frac{5\pi}{18}=2y$$

$$\left|\frac{\tan 20^\circ+\tan 70}{2}-\frac{2(\tan 20+\tan 50)}{2}\right|$$

$$\left|\frac{\tan 70^\circ-\tan 20^\circ-2\tan 50}{2}\right|$$

$$\left|\frac{\cot 20-\tan 20-2\tan 50}{2}\right|$$

$$\frac{2\cot 40-2\cot 40}{2}$$

$$=\underline{0}$$

$$\cot\theta-\tan\theta$$
$$=2\cot 2\theta$$

The value of

$\cancel{2}\sin\left(\frac{\pi}{8}\right)\sin\cancel{\left(\frac{2\pi}{8}\right)}\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\cancel{\left(\frac{6\pi}{8}\right)}\sin\left(\frac{7\pi}{8}\right)$ is

(a) $\frac{1}{4\sqrt{2}}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $\frac{1}{8\sqrt{2}}$

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$\sin\frac{\pi}{8}$ $\quad \sin\frac{3\pi}{8}\ \sin\left(\pi-\frac{3\pi}{8}\right)\ \sin\left(\pi-\frac{\pi}{8}\right)$

$\sin\frac{\pi}{8}\sin\frac{\pi}{8}\cdot\sin\frac{3\pi}{8}\cdot\sin\frac{3\pi}{8}$

$\sin^2\frac{\pi}{8}\cdot\sin^2\frac{3\pi}{8}$

$\frac{1}{4}\left(2\sin 22\frac{1}{2}^\circ\cos 22\frac{1}{2}^\circ\right)^2$

$\frac{1}{4}(\sin 45^\circ)^2 = \frac{1}{8}$

The value of $2\sin 12° - \sin 72°$ is

(a) $\frac{\sqrt{5}(1-\sqrt{3})}{4}$ (b) $\frac{1-\sqrt{5}}{8}$

(c) $\frac{\sqrt{3}(1-\sqrt{5})}{2}$ (d) $\frac{\sqrt{3}(1-\sqrt{5})}{4}$

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$\sin 12° + \sin 12° - \sin 72°$

$\sin 12° + 2\cos\left(\frac{72+12}{2}\right)\sin\left(\frac{12-72}{2}\right)$

$\sin 12° - \cancel{2}\cos 42° \times \frac{1}{\cancel{2}}$

$\sin 12° - \cos 42°$

$\cos 78 - \cos 42°$

$= 2\sin\frac{120}{2}\cdot\sin\left(\frac{42-78}{2}\right) = -2\times\frac{\sqrt{3}}{2}\left(\sin 18°\right)$

$-\sqrt{3}\times\left(\frac{\sqrt{5}-1}{4}\right)$

$\frac{\sqrt{3}(1-\sqrt{5})}{4}$

$16 \sin 20° \sin 40° \sin 80° =$

(a) $\sqrt{3}$ (b) $2\sqrt{3}$ (c) 3 (d) $4\sqrt{3}$

JEE MAIN 2022

$16 \times \frac{1}{4} \sin(3 \times 20°)$

$4 \sin 60°$

$4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$

The value of $\cos\left(\frac{2\pi}{7}\right)+\cos\left(\frac{4\pi}{7}\right)+\cos\left(\frac{6\pi}{7}\right)$ is equal to

(a) -1 (b) $-\frac{1}{2}$ (c) $-\frac{1}{3}$ (d) $-\frac{1}{4}$

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$$\cos\left(\frac{2\pi}{7}\right)+\cos\left(\frac{2\pi}{7}+\frac{2\pi}{7}\right)+\cos\left(\frac{2\pi}{7}+2\cdot\frac{2\pi}{7}\right)$$

$$=\frac{\cos\left(\frac{2\pi}{7}+\frac{2\pi}{7}\right)\cdot\sin\left(\frac{3}{2}\cdot\frac{2\pi}{7}\right)}{\sin\left(\frac{2\pi}{14}\right)}$$

$$\sin\alpha+\sin(\alpha+\beta)+\cdots\sin(\alpha+(n-1)\beta)$$

$$=\frac{\sin\left(\alpha+\frac{(n-1)\beta}{2}\right)\sin\left(\frac{n\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)}$$

$$\frac{\cos\frac{4\pi}{7}\cdot\sin\frac{3\pi}{7}}{\sin\frac{\pi}{7}}=\frac{\cos\left(\pi-\frac{3\pi}{7}\right)\sin\frac{3\pi}{7}}{\sin\frac{\pi}{7}}$$

$$=-\frac{1}{2}\times\frac{2\cos\frac{3\pi}{7}\sin\frac{3\pi}{7}}{\sin\pi/7}$$

$$-\frac{1}{2}\times\frac{\cancel{\sin\frac{6\pi}{7}}}{\cancel{\sin}\frac{\pi}{7}}$$

$\alpha = \sin 36°$ is a root of which of the following equation?

(a) $16x^4 - 10x^2 - 5 = 0$

(b) $16x^4 + 20x^2 - 5 = 0$

(c) $16x^4 - 20x^2 + 5 = 0$

(d) $4x^4 - 10x^2 + 5 = 0$

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$$\alpha = \frac{\sqrt{10-2\sqrt{5}}}{4}$$

$$4\alpha = \sqrt{10-2\sqrt{5}}$$

$$16\alpha^2 = 10-2\sqrt{5}$$

$$8\alpha^2 = 5-\sqrt{5}$$

$$\sqrt{5} = 5-8\alpha^2$$

$$5 = 25+64\alpha^4-80\alpha^2$$

$$64\alpha^4-80\alpha^2+20=0$$

$$\boxed{16\alpha^4-20\alpha^2+5=0}$$

If $\cot\alpha = 1$ and $\sec\beta = -\frac{5}{3}$, where $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$, then the value of $\tan(\alpha+\beta)$ and the quadrant in which $\alpha+\beta$ lies, respectively are :

(a) $-\frac{1}{7}$ and IV^{th} quadrant

(b) 7 and I^{st} quadrant

(c) -7 and IV^{th} quadrant

(d) $\frac{1}{7}$ and I^{st} Quadrant

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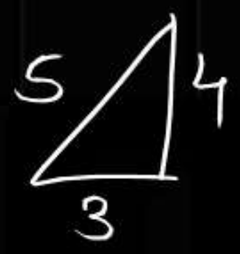
$\tan\alpha = 1$ $\quad \cos\beta = -\frac{3}{5}$, $\tan\beta = \frac{-4}{3}$



$\frac{3\pi}{2} < \alpha+\beta < \frac{5\pi}{2}$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$= \frac{1-\frac{4}{3}}{1+\frac{4}{3}} = \frac{-\frac{1}{3}}{\frac{7}{3}} = \frac{-1}{7}$$

$2\sin\left(\frac{\pi}{22}\right)\sin\left(\frac{3\pi}{22}\right)\sin\left(\frac{5\pi}{22}\right)\sin\left(\frac{7\pi}{22}\right)\sin\left(\frac{9\pi}{22}\right)$ is equal to

(a) $\frac{3}{16}$ (b) $\frac{1}{16}$ (c) $\frac{1}{32}$ (d) $\frac{9}{32}$

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$$2\sin\left(\frac{11\cdot\pi}{11\cdot 2}-\frac{10\pi}{22}\right)\cdot\sin\left(\frac{11\pi}{22}-\frac{8\pi}{22}\right)\cdot\sin\left(\frac{11\pi}{22}-\frac{6\pi}{22}\right)\cdot\sin\left(\frac{11\pi}{22}-\frac{4\pi}{22}\right)\cdot\sin\left(\frac{11\pi}{22}-\frac{2\pi}{22}\right)$$

$$2\cos\frac{10\pi}{22}\cdot\cos\frac{8\pi}{22}\cdot\cos\frac{6\pi}{2}\cdot\cos\frac{4\pi}{22}\cdot\cos\frac{2\pi}{22}$$

$$2\cdot\cos\frac{\pi}{11}\cdot\cos\frac{2\pi}{11}\cdot\cos\frac{3\pi}{11}\cdot\cos\frac{4\pi}{11}\cdot\cos\frac{5\pi}{11}$$

$$= -2\cos\frac{\pi}{11}\cos\frac{2\pi}{11}\cos\frac{8\pi}{11}\cos\frac{4\pi}{11}\cos\frac{5\pi}{11}$$

$$-2\times\frac{\sin 2^4\cdot\frac{\pi}{11}}{2^4\cdot\sin\frac{\pi}{11}}\times\cos\frac{5\pi}{11}$$

$$=\frac{+1}{\cancel{8}\,2}\times 2\,\frac{\sin\left(\pi+\frac{5\pi}{11}\right)\times\cos\frac{5\pi}{11}}{\sin\frac{\pi}{11}}=\frac{1}{16}\times\frac{\cancel{\sin\frac{10\pi}{11}}}{\cancel{\sin\frac{\pi}{11}}}$$

Let $f(\theta) = 3\left(\sin^4\left(\frac{3\pi}{2} - \theta\right) + \sin^4(3\pi + \theta)\right) - 2(1 - \sin^2 2\theta)$ and $S = \left\{\theta \in [0, \pi] : f'(\theta) = -\frac{\sqrt{3}}{2}\right\}$. If $4\beta = \Sigma_{\theta \in S}\theta$, then $f(\beta) =$

(a) $\frac{9}{8}$ (b) $\frac{3}{2}$ (c) $\frac{5}{4}$ (d) $\frac{11}{8}$

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$$\boxed{f(\theta) = \frac{5}{4} - \frac{\cos 4\theta}{4}} \Rightarrow$$

$$\boxed{f\left(\frac{5\pi}{8}\right) = \frac{5}{4} - \frac{\cos\frac{5\pi}{2}}{4}}$$

$$f'(\theta) = +\frac{1}{\cancel{4}} \times \sin 4\theta \times \cancel{4}$$

$$f'(\theta) = \sin 4\theta$$

$$\sin 4\theta = -\frac{\sqrt{3}}{2}$$

$$\sin 4\theta = \sin\left(-\frac{\pi}{3}\right)$$

$$4\theta = n\pi + (-1)^n\left(-\frac{\pi}{3}\right)$$

$$4\theta = \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3}, \frac{16\pi}{3}$$

$$\theta = \frac{4\pi}{12}, \frac{5\pi}{12}, \frac{10\pi}{12}, \frac{11\pi}{12}, \cancel{\frac{16\pi}{12}}$$

$$S = \frac{30}{12}\pi$$

$$4\beta = \frac{\cancel{30}\pi}{\cancel{12}}$$

$$\beta = \frac{5\pi}{8}$$

$$f(\theta) = \frac{9}{2}\sin^2 2\theta - 3$$

$$f'(\theta) = \frac{9}{\cancel{2}} \times 2\sin 2\theta \cdot \cos 2\theta \times \cancel{2}$$

$$f'(\theta) = 9 \times \sin 4\theta$$

$$9\sin 4\theta = -\frac{\sqrt{3}}{2}$$

The set of all values of λ for which the equation $\cos^2 2x - 2\sin^4 x - 2\cos^2 x = \lambda$ has a real solution x, is :

(a) $[-2, -1]$ (b) $\left[-\frac{3}{2}, -1\right]$

(c) $\left[-2, -\frac{3}{2}\right]$ (d) $\left[-1, -\frac{1}{2}\right]$

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$(1-2\sin^2 x)^2 - 2\sin^4 x - 2(1-\sin^2 x)$

$1 + 4\sin^4 x - 4\sin^2 x - 2\sin^4 x - 2 + 2\sin^2 x$

$f(x) = 2\sin^4 x - 2\sin^2 x - 1$

$f(x) = 2[\sin^4 x - \sin^2 x] - 1$

$= 2[\sin^4 x - \sin^2 x + \frac{1}{4} - \frac{1}{4}] - 1$

$= 2\left(\sin^2 x - \frac{1}{2}\right)^2 - \frac{3}{2}$

$0 \le \left(\sin^2 x - \frac{1}{2}\right)^2 \le \frac{1}{4}$

$0 \le 2(\quad) \le \frac{1}{2}$

$-\frac{3}{2} \le y \le -1$

$x = 0 \quad 1 - 2 = -1$

$x = 45^\circ \Rightarrow 0 - \frac{2}{4} - 1 = -\frac{1}{2} - 1 = -\frac{3}{2}$

$x = 90^\circ \Rightarrow 1 - 2 = -1$

If $\tan 15° + \frac{1}{\tan 75°} + \frac{1}{\tan 105°} + \tan 195° = 2a$, then value od $\left(a + \frac{1}{a}\right) =$

(a) $5 - \frac{3}{2}\sqrt{3}$ (b) $4 - 2\sqrt{3}$

(c) 2 (d) 4

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$\tan 15° + \tan 15° - \tan 15° + \tan 15°$

$2 \tan 15°$

$2(2-\sqrt{2}) = 2a$

$a = 2 - \sqrt{3}$

$\frac{1}{a} = 2 + \sqrt{3}$

The value of

$36\,(4\cos^2 9^\circ - 1)(4\cos^2 27^\circ - 1)(4\cos^2 81^\circ - 1)(4\cos^2 243^\circ - 1)$ is :

(a) 18 (b) 36 (c) 54 (d) 27

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$36(4\cos^2 9-1)(4\sin^2 9^\circ-1)(4\cos^2 27^\circ-1)(4\sin^2 27^\circ-1)$

$36(16\sin^2 9\cos^2 9-4\cos^2 9-4\sin^2 9+1)(16\cos^2 27\sin^2 27-4\sin^2 27-4\cos^2 27+1)$

$36(4\times\sin^2 18^\circ-4+1)(4\times\sin^2 54^\circ-4+1)$

$36\left(4\times\left(\frac{\sqrt{5}-1}{4}\right)^2-3\right)\left(4\times\left(\frac{\sqrt{5}+1}{4}\right)^2-3\right)$

$36\left(\frac{6-2\sqrt{5}}{4}-3\right)\left(\frac{6+2\sqrt{5}}{4}-3\right)$

$\frac{9}{4}(-2\sqrt{5}-6)(2\sqrt{5}-6)$

$96 \cos\frac{\pi}{33} \cos\frac{2\pi}{33} \cos\frac{4\pi}{33} \cos\frac{8\pi}{33} \cos\frac{16\pi}{33}$ is equal to

(a) 4 (b) 2 (c) 1 (d) 3

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$$96 \times \frac{\sin\left(2^5 \times \frac{\pi}{33}\right)}{2^5 \sin\left(\frac{\pi}{33}\right)}$$

$$\frac{96}{32} \times \frac{\cancel{\sin\left(\frac{32\pi}{33}\right)}}{\cancel{\sin\frac{\pi}{33}}}$$

$$= \underline{\underline{3}}$$

For $\alpha, \beta \in \left(0, \frac{\pi}{2}\right)$. Let $3\sin(\alpha+\beta) = 2\sin(\alpha-\beta)$ and a real number k be such that $\tan\alpha = k\tan\beta$. *Then, the value of* k=

(a) 5 (b) $-\frac{2}{3}$ (c) -5 (d) $\frac{2}{3}$

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$$3\sin(\alpha+\beta) = 2\sin(\alpha-\beta)$$

$$\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)} = \frac{2}{3}$$

$$\frac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{\sin(\alpha+\beta)-\sin(\alpha-\beta)} = \frac{2+3}{2-3}$$

$$\Rightarrow \frac{\cancel{2}\sin\alpha\cos\beta}{\cancel{2}\cos\alpha\sin\beta} = \frac{5}{-1}$$

$$\Rightarrow \frac{\tan\alpha}{\tan\beta} = \frac{5}{-1}$$

$$-\tan\alpha = 5\tan\beta$$

$$\tan\alpha = -5\tan\beta$$

The number of solutions, of the equation $e^{\sin x} - 2e^{-\sin x} = 2$, is

(a) 0 (b) 1 (c) 2 (d) more than 2

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If $\tan A = \frac{1}{\sqrt{x(x^2+x+1)}}$, $\tan B = \frac{\sqrt{x}}{\sqrt{x^2+x+1}}$ and $\tan C = (x^{-3}+x^{-2}+x^{-1})^{1/2}$, $0 < A, B, C < \frac{\pi}{2}$, then $A + B =$

(a) C (b) $\pi - C$ (c) $2\pi - C$ (d) $\frac{\pi}{2} - C$

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$x = 1$

$\tan A = \frac{1}{\sqrt{3}}$, $\tan B = \frac{1}{\sqrt{3}}$

$\tan C = (1+1+1)^{1/2}$

$= \sqrt{3}$

$A = 30^\circ$ $B = 30^\circ$ $C = 60^\circ$

$\boxed{A+B=C}$

Suppose $\theta \in \left[0, \frac{\pi}{4}\right]$ is a solution of $4\cos\theta - 3\sin\theta = 1$. Then $\cos\theta$ is equal to :

(a) $\frac{6-\sqrt{6}}{(3\sqrt{6}-2)}$ (b) $\frac{4}{(3\sqrt{6}+2)}$

(c) $\frac{6+\sqrt{6}}{(3\sqrt{6}+2)}$ (d) $\frac{4}{(3\sqrt{6}-2)}$

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$$4\cos\theta - 1 = 3\sin\theta$$

$$16\cos^2\theta + 1 - 8\cos\theta = 9(1-\cos^2\theta)$$

$$25\cos^2\theta - 8\cos\theta - 8 = 0$$

$$\cos\theta = \frac{8 \pm \sqrt{64+4\times8\times25}}{25\times2}$$

$$\cos\theta = \frac{\cancel{8} \pm 2\times\cancel{2}\sqrt{4+50}}{25\times\cancel{2}} = \frac{4\pm2\sqrt{54}}{25} = \frac{4\pm2\,3\sqrt{6}}{25}$$

$$= \frac{4+2\,3\sqrt{6}}{25}\times\frac{4-6\sqrt{6}}{4-6\sqrt{6}}$$

$$= \frac{16-216}{25(4-6\sqrt{6})} = \frac{-8}{4-6\sqrt{6}} = \frac{-4}{2-3\sqrt{6}}$$

If $\sin x = -\frac{3}{5}$ where $\pi < x < \frac{3\pi}{2}$, then $80(\tan^2 x - \cos x)$ is equal to

(a) 109 (b) 108 (c) 19 (d) 18

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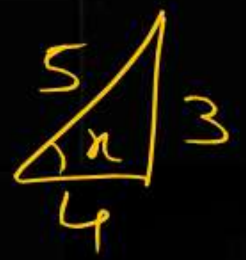
$\sin x = -\frac{3}{5}$



$\tan x = \frac{3}{4}$, $\cos x = -\frac{4}{5}$

$80\left(\frac{9}{16} + \frac{4}{5}\right)$

$\cancel{80}\left(\frac{45+64}{\cancel{80}}\right)$

$= \underline{109}$

If the value of $\frac{3\cos 36°+5\sin 18°}{5\cos 36°-3\sin 18°}$ is $\frac{a\sqrt{5}-b}{c}$, where a, b, c are natural numbers and $\gcd(a, c)=1$, then $a+b+c$ is equal to :

(a) 54 (b) 52 (c) 50 (d) 40

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$$\frac{3\left(\frac{\sqrt{5}+1}{\cancel{4}}\right)+5\left(\frac{\sqrt{5}-1}{\cancel{4}}\right)}{5\left(\frac{\sqrt{5}+1}{\cancel{4}}\right)-3\left(\frac{\sqrt{5}-1}{\cancel{4}}\right)}$$

$$\frac{3\sqrt{5}+3+5\sqrt{5}-5}{5\sqrt{5}+5-3\sqrt{5}+3}$$

$$\frac{8\sqrt{5}-2}{2\sqrt{5}+8}=\frac{4\sqrt{5}-1}{\sqrt{5}+4}$$

$$\frac{4\sqrt{5}-1}{\sqrt{5}+4}\times\frac{\sqrt{5}-4}{\sqrt{5}-4}=\frac{(4\sqrt{5}-1)(\sqrt{5}-4)}{-11}$$

$$=\frac{20-16\sqrt{5}-\sqrt{5}+4}{-11}$$

$$=\frac{24-17\sqrt{5}}{-11}=\frac{17\sqrt{5}-24}{11}$$

$a=17$, $b=24$

$c=11$

The value of $(\sin 70°)(\cot 10° \cot 70° - 1)$ *is*

(a) 0 (b) $\frac{2}{3}$ (c) 1 (d) $\frac{3}{2}$

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$$\sin 70°\left(\frac{\cos 10°}{\sin 10°} \times \frac{\cos 70°}{\sin 70°} - 1\right)$$

$$\cancel{\sin 70°}\left(\frac{\cos 10° \cos 70° - \sin 10 \sin 70°}{\sin 10° \cancel{\sin 70°}}\right)$$

$$= \left(\frac{\cos(10+70)}{\sin 10}\right)$$

$$= \frac{\cos 80}{\sin 10} = \frac{\sin 10}{\sin 10} = 1$$