ROTATIONAL DYNAMICS FORMULA SHEET

Formulae

Type I

1. The angular displacement (θ) is

$$\theta = \frac{\text{arc length}}{\text{radius}} = \frac{s}{r}$$

2. The angular speed is

$$\omega = \frac{\theta}{t}$$

3. The angular acceleration is

$$\alpha = \frac{d\omega}{dt}$$

4. The angular acceleration is

$$\alpha = \frac{\omega_2 - \omega_1}{t}$$

Where,

ω_1 = initial angular speed,

ω_2 = final angular speed after time t.

5. The angular acceleration in terms of frequency is

$$\alpha = \frac{2\pi(n_2 - n_1)}{t}$$

where,

n_1 = initial frequency

n_2 = final frequency

6. The relation between linear displacement and angular displacement is

$$s = r\,\theta$$

where r = radius of the circle

7. The relation between linear velocity (tangential velocity) and angular velocity is

$$v = r\,\omega$$

8. The relation between linear acceleration (tangential acceleration) and angular acceleration is

$$a = r\,\alpha$$

Type II

9. The time period in terms of linear velocity is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{v/r} = \frac{2\pi r}{v}$$

10. The time period in terms of angular velocity is

$$T = \frac{2\pi}{\omega}$$

11. The relation between frequency and time period is

$$n = \frac{1}{T}$$

12. The angular speed in terms of frequency is

$$\omega = 2\pi n$$

13. The magnitude of centripetal acceleration is

$$a = r\omega^2 = \frac{v^2}{r} = v\omega$$

14. The magnitude of centripetal force is,

$$F = mr\omega^2 = \frac{mv^2}{r} = mv\omega$$

Where m = mass of the body.

15. The magnitude of centrifugal force is,

$$F = mr\omega^2 = \frac{mv^2}{r} = mv\omega$$

Note : Magnitude of centripetal force and centrifugal force is equal.

- **Time period of hand in a clock :**

Sr. No.	Type of hand in a clock	Time Period (T)
1.	Second hand	60 sec.
2.	Minute hand	60 min. = 3600 s
3.	Hour hand	12 hr. = 12 × 3600 s = 43200 s

- **Time period of Earth :**

Sr. No.	Type of motion of Earth	Time Period (T)
1.	U.C.M.(revolution)	1 year = 365 days
2.	Rotation (spin)	1 day = 24 hours

Note :

i) Tension = C.P.F.

ii) Breaking tension(B.T.) = Maximum C.P.F.

iii) $\begin{pmatrix}\text{Magnitude of} \\ \text{centripetal acc}^{n}\end{pmatrix} = \begin{pmatrix}\text{Magnitude of} \\ \text{centrifugal acc}^{n}\end{pmatrix}$

iv) $\begin{pmatrix}\text{Magnitude of} \\ \text{centripetal force}\end{pmatrix} = \begin{pmatrix}\text{Magnitude of} \\ \text{centrifugal force}\end{pmatrix}$

v) The gravitational unit of force in the S.I. system is kilogram weight (kg.wt)or kilogram force (kg.f)

$$1 \text{ kg.wt} = g \text{ N} = 9.8 \text{ N}$$

vi) The gravitational unit of force in the C.G.S. system gram weight (gm.wt) or gram force (gm.f)

$$1 \text{ gm.wt} = g.\text{dynes} = 980 \text{ dynes}$$

vii) If the force and breaking tension is given in kg.wt or gm.wt then convert it into Newton (N) or dyne.

Type III

On horizontal curved road, we have,

16. Centripetal force ≤ Force of friction

$$\frac{mv^2}{r} \le \mu mg$$

$$\therefore v^2 = \mu rg$$

$$\therefore v_{max} = \sqrt{\mu rg} \quad \text{[without skidding]}$$

Where μ = coefficient of friction.

17. C.P.F. ≤ Force of friction.

$$mr\omega^2 \le \mu mg$$

$$\therefore r\omega^2 = \mu.g.$$

$$\therefore \omega^2 = \frac{\mu.g}{r}$$

$$\therefore \omega_{max} = \sqrt{\frac{\mu.g}{r}} \quad \text{[without skidding]}$$

Note :

i) $\omega = 2\pi n$

ii) $1 km/hr = \frac{5}{18}$ m/s

iii) When a vechicle turns along any curred horizontal road (or track) the horizontal thrust always acts outwards away from the centre in the form of centrifugal force.

Type IV

18. An cyclist, ice skaters or aeroplane negotiating a circular turn must lean inwards at an angle θ made with vertical is given by

$$\theta = \tan^{-1}\left[\frac{v^2}{rg}\right]$$

Note :

i) If angle with horizontal is given, then angle with vertical = 90^0 − angle with horizontal (θ).

ii) Circumference of circle = $2\pi r$

Type V

19. The maximum speed to avoid the slipping of vehicle is

$$v_{max} = \sqrt{rg\left(\frac{\mu + \tan\theta}{1 - \mu\tan\theta}\right)}$$

20. The maximum speed limit for the safety of a vehicle is given by,

$$v = \sqrt{r\,g\tan\theta}$$

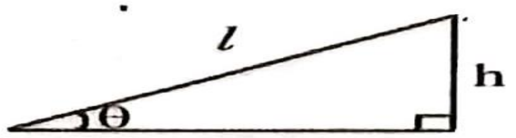
21. Angle of banking is given by,

$$\theta = \tan^{-1}\left[\frac{v^2}{rg}\right]$$

22. If l = distance between two edge of road or width of road.

h = elevation of the outer edge of the road.

Then,



$$\sin\theta = \frac{h}{l}$$

Type VI

23. Tension (T') in the string can be resolved in two components

i) vertical component balances weight of bob
$T' \cos\theta = mg$

$$T' = \frac{mg}{\cos\theta}$$

ii) horizontal component provides C.P.F. $T' \sin\theta = \text{C.P.F.}$

$$T' = \frac{\text{C.P.F.}}{\sin\theta}$$

$$\text{C.P.F} = T'\sin\theta$$

$$= \frac{mg}{\cos\theta}\sin\theta = mg\tan\theta$$

iii) $T' = mL\omega^2$

24. Angular speed of the conical pendulum is

$$\omega = \sqrt{\frac{g\tan\theta}{r}}$$

25. The time period of the conical pendulum

i) in terms of radius

$$T = 2\pi\sqrt{\frac{r}{g\tan\theta}}$$

ii) in terms of length

$$T = 2\pi\sqrt{\frac{L\cos\theta}{g}}$$

iii) in terms of height

$$T = 2\pi\sqrt{\frac{h}{g}}$$

26. $r = L\sin\theta$

27. $h = L\cos\theta$

28. The linear speed of the conical pendulum is

$$v = \sqrt{rg\tan\theta}$$

where $r = L\sin\theta$

29. The angle made by the string with vertical is

$$\theta = \tan^{-1}\left[\frac{v^2}{rg}\right]$$

30. The relation between length of the string (L), radius of the circle (r) and height (h) of the conical pendulum is
$L^2 = r^2 + h^2$

31. The minimum speed at which body remains in contact with wall of rotor (Death well) is

$$v = \sqrt{\frac{rg}{\mu}}$$

32. The minimum angular speed at which body remains in contact with wall of rotor (Death well) is

$$\omega = \sqrt{\frac{g}{\mu r}}$$

Type VIII

On vertical circle at highest point, we have

33. Centripetal force = weight of the body.

$$\frac{mv^2}{r} = mg$$

$$\therefore v = \sqrt{rg}$$

34. C.P.F. = Weight of the body.

$$mr\omega^2 = mg$$

$$\therefore \omega^2 = \frac{g}{r}$$

$$\therefore \omega = \sqrt{\frac{g}{r}}$$

Note : If the centre of gravity of the vehicle is at a height 'h' above the road surface then, $R = r + h$

Where r = radius of vertical circle

$$\therefore v = \sqrt{Rg}$$

Type IX

35. In vertical circular motion the tension in the string at

i) any position is given by

$$T_P = \frac{mv_P^2}{r} + mg\cos\theta$$

OR

$$T_P = 3mg(1 + \cos\theta)$$

ii) highest position is given by

$$T_H = \frac{mv_H^2}{r} - mg$$

iii) lowest position is given by

$$T_L = \frac{mv_L^2}{r} + mg$$

iv) horizontal position is given by

$$T_M = \frac{mv_M^2}{r}$$

36. In vertical circular motion velocity at

i) At any position is given by

$$v_p = \sqrt{v_L^2 - 2gh_p} \qquad \text{OR}$$

$$v_p = \sqrt{rg(3 + 2\cos\theta)}$$

ii) highest position is given by

$$v_H = \sqrt{rg}$$

iii) horizontal position is given by

$$v_M = \sqrt{3rg}$$

iv) lowest position is given by

$$v_L = \sqrt{5rg}$$

37. In vertical circular motion energies

i) At any position is given by

$$(K.E.)_{any} = \frac{1}{2}mrg(3 + 2\cos\theta)$$

$$(P.E.)_{any} = mrg(1 - \cos\theta)$$

$$(T.E.)_{any} = \frac{5}{2}mrg$$

ii) at highest position is given by

$$(K.E.)_H = \frac{1}{2}mrg$$

$$(P.E.)_H = 2mrg$$

$$(T.E.)_H = \frac{5}{2}mrg$$

iii) at lowest position is given by

$$(K.E.)_L = \frac{5}{2}mrg$$

$$(P.E.)_L = 0$$

$$(T.E.)_L = \frac{5}{2}mrg$$

iv) at horizontal position is given by

$$(K.E.)_M = \frac{3}{2}mrg$$

$$(P.E.)_M = mrg$$

$$(T.E.)_M = \frac{5}{2}mrg$$

Note :

$$(T.E.)_H = (T.E.)_L = (T.E.)_M = (T.E.)_{any} = \frac{5}{2} mrg$$

T.E. is in vertical circular motion remains constant i.e. conserved.

Type X

38. The height of the inclined plane from where a body slides down and completes a circular loop of radius r at the bottom

i) in terms of velocity is

$$h = \frac{v^2}{2g}$$

ii) in terms of radius is

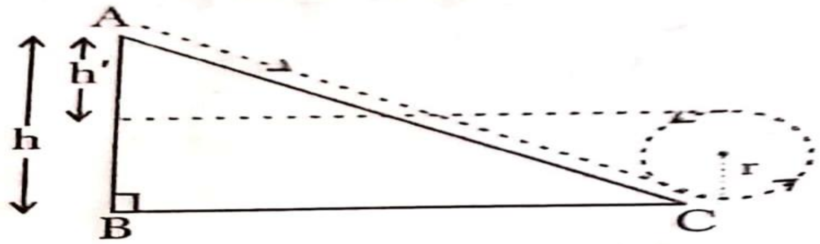
$$h = \frac{5}{2} r$$

iii) In terms of radius is

$$h = \frac{5}{4} d$$

Note : Height of the body above the top of the circular loop (h')will be

$$h' = h - 2r = \frac{5}{2} r - 2r = \frac{r}{2}$$



Type XI

39. Resultant acceleration in non U.C.M. is given by

$$a = \sqrt{a_t^2 + a_r^2}$$

where,

a_t = tangential acceleration of a body performing non U.C.M.

$a_t = r\alpha$

a_r = radial acceleration of a body performing non U.C.M.

$$a_r = \frac{v^2}{r} = r\omega^2$$

Type XII

In circular motion with uniform angular acceleration,

i) $\omega_2 = \omega_1 + \alpha t$

ii) $\theta = \omega_1 t + \frac{1}{2} \alpha t^2$

iii) $\omega_2^2 = \omega_1^2 + 2\alpha\theta$

iv) The number of revolution is

$$N = \frac{\theta}{2\pi}$$

Proof : We know

$$\omega = 2\pi n$$

But $$\omega = \frac{\theta}{t}$$

$$\therefore \quad \frac{\theta}{t} = 2\pi n$$

But the frequency (n) is given by

$$n = \frac{\text{no. of revolution}(N)}{\text{time}(t)}$$

$$\therefore \quad \frac{\theta}{t} = \frac{2\pi N}{t}$$

$$\therefore \quad \theta = 2\pi N$$

$$\therefore \quad N = \frac{\theta}{2\pi}$$

M. I. for a single particle is

$$I = mr^2$$

M. I. for a body of n particle is

$$I = \sum_{i=1}^{n} m_i r_i^2$$

Radius of gyration (K),

$$I = M K^2.$$

$$\therefore K^2 = \frac{I}{M}$$

$$\therefore K = \sqrt{\frac{I}{M}}$$

Principle of parallel axes,

$$I_o = I_c + Mh^2$$

Principle of parallel axes in terms of radius of gyration :

$$K_o^2 = K_c^2 + h^2$$

Principle of perpendicular axes,

$$I_z = I_x + I_y.$$

Principle of perpendicular axes in terms of radius of gyration :

$$K_z^2 = K_x^2 + K_y^2$$

Kinetic energy of rotating body,

$$K.E. = \frac{1}{2} I \omega^2$$

$$\begin{pmatrix} \text{Total} \\ \text{K.E.} \end{pmatrix} = \begin{pmatrix} \text{Rotational} \\ \text{K.E.} \end{pmatrix} + \begin{pmatrix} \text{Translational} \\ \text{K.E.} \end{pmatrix}$$

$$= \frac{1}{2} I\omega^2 + \frac{1}{2} Mv^2$$

Note :

i) $v = R\omega$ $\quad \therefore \omega = \frac{v}{R}$

ii) $L = I.\omega$

Torque acting on a body,

$$\tau = I\,\alpha$$

where,

I = M.I. of the body

α = angular acceleration

The angular acceleration,

$$\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{2\pi n_2 - 2\pi n_1}{t}$$

$$\alpha = 2\pi\left(\frac{n_2 - n_1}{t}\right)$$

where,

$\omega_1 = 2\pi\, n_1$ = Initial angular velocity

$\omega_2 = 2\pi\, n_2$ = Final angular velocity

For a body rotating with a constant angular acceleration α,

i) $\omega_2 = \omega_1 + \alpha t$

ii) $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$

iii) $\omega_2^{\,2} = \omega_1^{\,2} + 2\alpha\theta$

where,

θ = angular displacement.

Work done by a constant torque acting on a body,

$$W = \tau\,\theta$$

The workdone in increasing the angular velocity of a body from ω_1 to ω_2 is given by

$$\omega_2^2 = \omega_1^2 + 2\alpha \cdot \theta$$

$$\therefore \theta = \frac{\omega_2^2 - \omega_1^2}{2\alpha}$$

Now,

$$W = \tau \cdot \theta = \tau \times \left(\frac{\omega_2^2 - \omega_1^2}{2\alpha}\right)$$

$$= I\alpha\left(\frac{\omega_2^2 - \omega_1^2}{2\alpha}\right) \qquad \left(\because I = \frac{\tau}{\alpha}\right)$$

$$W = \frac{1}{2} I(\omega_2^2 - \omega_1^2)$$

$$\theta = 2\pi\left(\begin{matrix}\text{number of rotations}\\ \text{completed}\end{matrix}\right) = 2\pi \times N$$

$$\therefore \; N = \frac{\theta}{2\pi}$$

Power $= \tau\,\omega$

Note : For retardation

$$\omega_2 = \omega_1 - \alpha t$$

$$\theta = \omega_1 t - \frac{1}{2}\alpha t^2$$

$$\omega_2^2 = \omega_1^2 - 2\alpha\theta$$

Angular momentum of a body,

$$L = I\,\omega$$

The torque is

$$\tau = \frac{dL}{dt}$$

The angular momentum of a body is conserved.

$$\therefore I_1 \times \omega_1 = I_2 \times \omega_2$$

$$I_1 \times 2\pi n_1 = I_2 \times 2\pi n_2$$

$$I_1 n_1 = I_2 n_2$$

When two bodies are rotated with different angular speed ω_1 and ω_2 and then these are coupled and rotate with a common angular speed (ω) then

$$I_1\,\omega_1 + I_2\,\omega_2 = (I_1 + I_2)\,\omega$$

Note :

i) $\omega = 2\pi\,n$ ii) $\omega = \frac{2\pi}{T}$

The change in rotational K.E. is

Change in K.E. = final K.E. – initial K.E.

$$= \frac{1}{2} I\omega_2^2 - \frac{1}{2} I\omega_1^2 = \frac{1}{2} I(\omega_2^2 - \omega_1^2)$$

The change in angular momentum is

Change in ang. momentum $= L_2 - L_1$

$$= I\omega_2 - I\omega_1 = I\,(\omega_2 - \omega_1)$$

Relation between rotational K.E. and angular momentum is

i) $$K.E. = \frac{1}{2} I\omega^2 = \frac{1}{2}(I\omega)\omega$$

$$\therefore K.E. = \frac{1}{2} L\omega$$

ii) $$K.E. = \frac{1}{2} I\omega^2 = \frac{1}{2}\frac{I^2\omega^2}{I} = \frac{1}{2}\frac{(I\omega)^2}{I}$$

$$\therefore K.E. = \frac{1}{2}\frac{L^2}{I}$$

iii) $$K.E. = \frac{1}{2}\frac{L^2}{I}$$

$$\therefore L^2 = 2\ K.E. \times I$$

$$\therefore\ L = \sqrt{2K.E. \times I}$$