Exercise -1.1

1. Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$?

Sol:

Yes, zero is a rotational number. It can be written in the form of $\frac{p}{q}$ where q to as such as

$\frac{0}{3}, \frac{0}{5}, \frac{0}{11}, etc.........$

2. Find five rational numbers between 1 and 2.

Sol:

Given to find five rotational numbers between 1 and 2

A rotational number lying between 1 and 2 is

$(1+2) \div 2 = 3 \div 2 = \frac{3}{2}$ $i.e., 1 < \frac{3}{2} < 2$

Now, a rotational number lying between 1 and $\frac{3}{2}$ is

$$\left(1+\frac{3}{2}\right) \div 2 = \left(\frac{2+3}{2}\right) \div 2 = \frac{5}{2} \div 2 = \frac{5}{2} \times \frac{1}{2} = \frac{5}{4}$$

i.e., $1 < \frac{5}{4} < \frac{3}{2}$

Similarly, a rotational number lying between 1 and $\frac{5}{4}$ is

$$\left(1+\frac{5}{4}\right) \div 2 = \left(\frac{4+5}{2}\right) \div 2 = \frac{9}{4} \div 2 = \frac{9}{4} \times \frac{1}{2} = \frac{9}{8}$$

i.e., $1 < \frac{9}{8} < \frac{5}{4}$

Now, a rotational number lying between $\frac{3}{2}$ and 2 is

$$\left(1+\frac{5}{4}\right) \div 2 = \left(\frac{4+5}{4}\right) \div 2 = \frac{9}{4} \div 2 = \frac{9}{4} \times \frac{1}{2} = \frac{9}{8}$$

i.e., $1 < \frac{9}{8} < \frac{5}{4}$

Now, a rotational number lying between $\frac{3}{2}$ and 2 is

$$\left(\frac{3}{2}+2\right) \div 2 = \left(\frac{3+4}{2}\right) \div 2 = \frac{7}{2} \times \frac{1}{2} = \frac{7}{4}$$

i.e., $\frac{3}{2}<\frac{7}{4}<2$

Similarly, a rotational number lying between $\frac{7}{4}$ and 2 is

$\left(\frac{7}{4}+2\right)\div 2=\left(\frac{7+8}{4}\right)\div 2=\frac{15}{4}\times\frac{1}{2}=\frac{15}{8}$

i.e., $\frac{7}{4}<\frac{15}{8}<2$

$\therefore 1<\frac{9}{8}<\frac{5}{4}<\frac{3}{2}<\frac{7}{4}<\frac{15}{8}<2$

Recall that to find a rational number between r and s, you can add r and s and divide the sum by 2, that is $\frac{r+s}{2}$ lies between r and s So, $\frac{3}{2}$ is a number between 1 and 2. you can proceed in this manner to find four more rational numbers between 1 and 2, These four numbers are, $\frac{5}{4},\frac{11}{8},\frac{13}{8}\ and\ \frac{7}{4}$

3. Find six rational numbers between 3 and 4.

Sol:

Given to find six rotational number between 3 and 4

We have,

$3\times\frac{7}{7}=\frac{21}{7}\ and\ 4\times\frac{7}{7}=\frac{28}{7}$

We know that

$21<22<23<24<25<26<27<28$

$\Rightarrow\frac{21}{7}<\frac{22}{7}<\frac{23}{7}<\frac{24}{7}<\frac{25}{7}<\frac{26}{7}<\frac{27}{7}<\frac{28}{7}$

$\Rightarrow 3<\frac{22}{7}<\frac{23}{7}<\frac{24}{7}<\frac{25}{7}<\frac{26}{7}<\frac{27}{7}<4$

Hence, 6 rotational number between 3 and 4 are

$\frac{22}{7},\frac{23}{7},\frac{24}{7},\frac{25}{7},\frac{26}{7},\frac{27}{7}$

4. Find five rational numbers between $\frac{3}{4}\ and\ \frac{4}{5}$

Sol:

Given to find 5 rotational numbers lying between $\frac{3}{5}\ and\ \frac{4}{5}$.

We have,

$\frac{3}{5}\times\frac{6}{6}=\frac{18}{100}$ and $\frac{4}{5}\times\frac{6}{6}=\frac{24}{30}$

We know that

$18<19<20<21<22<23<24$

$\Rightarrow\frac{18}{30}<\frac{19}{30}<\frac{20}{30}<\frac{21}{30}<\frac{22}{30}<\frac{23}{30}<\frac{24}{30}$

$\Rightarrow\frac{3}{5}<\frac{19}{30}<\frac{20}{30}<\frac{21}{30}<\frac{22}{30},\frac{23}{30},\frac{4}{5}$

$\Rightarrow\frac{3}{5}<\frac{19}{30}<\frac{2}{3}<\frac{7}{10}<\frac{11}{15}<\frac{23}{30}<\frac{4}{5}$

Hence, 5 rotational number between $\frac{3}{5}$ and $\frac{4}{5}$ are

$\frac{19}{30},\frac{2}{3},\frac{7}{10},\frac{11}{15},\frac{23}{30}$.

5. Are the following statements true or false? Give reasons for your answer.

(i) Every whole number is a rational number.
(ii) Every integer is a rational number.
(iii) Every rational number is a integer.
(iv) Every natural number is a whole number.
(v) Every integer is a whole number.
(vi) Evert rational number is a whole number.

Sol:

(i) False. As whole numbers include zero, whereas natural number does not include zero
(ii) True. As integers are a part of rotational numbers.
(iii) False. As integers are a part of rotational numbers.
(iv) True. As whole numbers include all the natural numbers.
(v) False. As whole numbers are a part of integers
(vi) False. As rotational numbers includes all the whole numbers.

Exercise – 1.2

Express the following rational numbers as decimals:

1. (i) $\frac{42}{100}$ (ii) $\frac{327}{500}$ (iii) $\frac{15}{4}$

Sol:

(i) By long division, we have

$$100\overline{)42.00}\ 0\cdot 42$$

$$\begin{array}{r} 400 \\ \hline 200 \\ 200 \\ \hline 0 \end{array}$$

$$\therefore \boxed{\frac{42}{100}=0\cdot 42}$$

(ii) By long division, we have

$$500\overline{)327\cdot 000}\ 0.654$$

$$\begin{array}{r} 3000 \\ \hline 2700 \\ 2500 \\ \hline 2000 \\ 2000 \\ \hline 0 \end{array}$$

$$\therefore \boxed{\frac{327}{500}=0\cdot 654}$$

(iii) By long division, we have

$$4\overline{)15\cdot 00}\quad 3\cdot 75$$

$$\begin{array}{r} 12 \\ \hline 30 \\ 28 \\ \hline 20 \\ 20 \\ \hline 0 \end{array}$$

$$\therefore \boxed{\frac{15}{4}=3\cdot 75}$$

2. (i) $\frac{2}{3}$ (ii) $-\frac{4}{9}$ (iii) $\frac{-2}{15}$ (iv) $-\frac{22}{13}$ (v) $\frac{437}{999}$

Sol:

(i) By long division, we have

$$3\overline{)2\cdot0000}(0.6666$$

$$\begin{array}{r} 18 \\ \hline 20 \\ 18 \\ \hline 20 \\ 18 \\ \hline 20 \\ 18 \\ \hline 2 \end{array}$$

$$\therefore \boxed{\frac{2}{3} = 0\cdot6666..... = 0.\overline{6}}$$

(ii) By long division, we have

$$9\overline{)4\cdot0000}(0\cdot4444$$

$$\begin{array}{r} 36 \\ \hline 40 \\ 36 \\ \hline 40 \\ 36 \\ \hline 40 \\ 36 \\ \hline 4 \end{array}$$

$$\therefore \boxed{\frac{4}{9} = 0\cdot4444..... = 0.\overline{4}}$$

Hence, $\boxed{-\frac{4}{9} = -0.\overline{4}}$

(iii) By long division, we have

$$5\overline{)2\cdot0000}(0.13333$$

$$\begin{array}{r} 15 \\ \hline 50 \\ 45 \\ \hline 50 \\ 45 \\ \hline \end{array}$$

50
45
5

$\therefore \frac{2}{15} = 0.1333\ ... \ = 0.1\bar{3}$

Hence, $\boxed{\frac{-2}{15} = -0.1\bar{3}}$

(iv) By long division, we have

$13\overline{)22.0000}\ (1.692307692307$

−13
90
−78
120
−117
30
−26
40
−39
100
−91
90
−78
120
−117
30
26

$\therefore \frac{22}{13} = 1\cdot 692307692307....... = 1.\overline{692307} \Rightarrow \boxed{-\frac{22}{13} = 1.\overline{692307}}$

(v) By long division, we have

$999\overline{)437.000000}\ (0.437437$

3996
3740
2997
7430
6993

$$\begin{array}{r} 4370 \\ \underline{3996} \\ 3740 \\ \underline{2997} \\ 7430 \\ \underline{6993} \\ 4370 \end{array}$$

$$\therefore \boxed{\frac{437}{999} = 0.437437......... = 0.\overline{437}}$$

(vi) By long division, we have

$$26\overline{)33.0000000000000}(1.2692307692307$$

$$\begin{array}{r} \underline{26} \\ 70 \\ \underline{\_52} \\ 180 \\ \underline{\_156} \\ 240 \\ \underline{\_234} \\ 60 \\ \underline{\_52} \\ 80 \\ \underline{\_78} \\ 200 \\ \underline{\_182} \\ 180 \\ \underline{\_156} \\ 240 \\ \underline{\_234} \\ 60 \\ \underline{\_52} \\ 80 \\ \underline{\_78} \\ 200 \\ \underline{\_182} \\ 18 \end{array}$$

$$\therefore \frac{33}{26} = 1.2692307698307............ = 1.2\overline{692307}$$

3. Look at several examples of rational numbers in the form $\frac{p}{q}$ $(q \neq 0)$, where p and q are integers with no common factors other than 1 and having terminating decimal representations. Can you guess what property q must satisfy?

Sol:

A rational number $\frac{p}{q}$ is a terminating decimal only, when prime factors of q are q and 5 only. Therefore, $\frac{p}{q}$ is a terminating decimal only, when prime factorization of q must have only powers of 2 or 5 or both.

Exercise -1.3

1. Express each of the following decimals in the form $\frac{p}{q}$:

(i) 0.39
(ii) 0.750
(iii) 2.15
(iv) 7.010
(v) 9.90
(vi) 1.0001

Sol:

(i) We have,

$$0\cdot39=\frac{39}{100}$$

$$\Rightarrow \boxed{0\cdot39=\frac{39}{100}}$$

(ii) We have,

$$0\cdot750=\frac{750}{1000}=\frac{750\div250}{1000\div250}=\frac{3}{4}$$

$$\therefore \boxed{0.750=\frac{3}{4}}$$

(iii) We have

$$2\cdot15=\frac{215}{100}=\frac{215\div5}{100\div5}=\frac{43}{20}$$

$$\therefore \boxed{2\cdot15=\frac{43}{20}}$$

(iv) We have,

$$7\cdot010=\frac{7010}{1000}=\frac{7010\div10}{1000\div10}=\frac{701}{100}$$

$$\therefore \boxed{7010=\frac{701}{100}}$$

(v) We have,

$$9\cdot90=\frac{990}{100}=\frac{990\div10}{100\div10}=\frac{99}{10}$$

$$\therefore \boxed{9\cdot90=\frac{99}{10}}$$

(vi) We have,

$$1\cdot0001=\frac{10001}{10000}$$

$$\therefore \boxed{1\cdot0001=\frac{10001}{10000}}$$

2. Express each of the following decimals in the form $\frac{p}{q}$:

(i) $0.\overline{4}$

(ii) $0.\overline{37}$

Sol:

(i) Let $x=0\cdot\overline{4}$

Now, $x=0\cdot\overline{4}=0.444....$ $---(1)$

Multiplying both sides of equation (1) by 10, we get,

$10x=4.444....$ $---(2)$

Subtracting equation (1) by (2)

$\therefore 10x-x=4.444...-0.444...$

$\Rightarrow 9x=4$

$$\Rightarrow x=\frac{4}{9}$$

Hence, $0\cdot\overline{4}=\frac{4}{9}$

(ii) Let $x=0\cdot\overline{37}$

Now, $x=0.3737...$ $.....(1)$

Multiplying equation (1) by l0.

$\therefore 10x=3.737....$ $---(2)$

$100x = 37.3737...$ $\quad ---(3)$

Subtracting equation (1) by equation (3)

$\therefore 100x - x = 37$

$\Rightarrow 99x = 37$

$\Rightarrow x = \frac{37}{99}$

Hence, $0\cdot\overline{37} = \frac{37}{99}$

Exercise -1.4

1. Define an irrational number.

Sol:

A number which can neither be expressed as a terminating decimal nor as a repeating decimal is called an irrational number. For example,

1.01001000100001…

2. Explain, how irrational numbers differ from rational numbers?

Sol:

A number which can neither be expressed as a terminating decimal nor as a repeating decimal is called an irrational number, For example,

0.33033003300033...

On the other hand, every rational number is expressible either as a terminating decimal or as a repeating decimal. For examples, $3.2\overline{4}$ and 6.2876 are rational numbers

3. Examine, whether the following numbers are rational or irrational:

(i) $\sqrt{7}$

(ii) $\sqrt{4}$

(iii) $2+\sqrt{3}$

(iv) $\sqrt{3}+\sqrt{2}$

(v) $\sqrt{3}+\sqrt{5}$

(vi) $\left(\sqrt{2}-2\right)^2$

(vii) $\left(2-\sqrt{2}\right)\left(2+\sqrt{2}\right)$

(viii) $\left(\sqrt{2}+\sqrt{3}\right)^2$

(ix) $\sqrt{5}-2$

(x) $\sqrt{23}$
(xi) $\sqrt{225}$
(xii) 0.3796
(xiii) 7.478478……
(xiv) 1.101001000100001……

Sol:

$\sqrt{7}$ is not a perfect square root, so it is an irrational number.

We have,

$\sqrt{4}=2=\frac{2}{1}$

$\therefore$ $\sqrt{4}$ can be expressed in the form of $\frac{p}{q}$, so it is a rational number.

The decimal representation of $\sqrt{4}$ is $2.0.$

2 is a rational number, whereas $\sqrt{3}$ is an irrational number.

Because, sum of a rational number and an irrational number is an irrational number, so $2+\sqrt{3}$ is an irrational number.

$\sqrt{2}$ is an irrational number. Also $\sqrt{3}$ is an irrational number.

The sum of two irrational numbers is irrational.

$\therefore \sqrt{3}+\sqrt{2}$ is an irrational number.

$\sqrt{5}$ is an irrational number. Also $\sqrt{3}$ is an irrational number.

The sum of two irrational numbers is irrational.

$\therefore \sqrt{3}+\sqrt{5}$ is an irrational number.

We have,

$\left(\sqrt{2}-2\right)^2=\left(\sqrt{2}\right)^2-2\times\sqrt{2}\times 2+\left(2\right)^2$

$=2-4\sqrt{2}+4$

$=6-4\sqrt{2}$

Now, 6 is a rational number, whereas $4\sqrt{2}$ is an irrational number.

The difference of a rational number and an irrational number is an irrational number.

So, $6-4\sqrt{2}$ is an irrational number.

$\therefore \left(\sqrt{2}-2\right)^2$ is an irrational number.

We have,

$\left(2-\sqrt{2}\right)\left(2+\sqrt{2}\right)=\left(2\right)^2-\left(\sqrt{2}\right)^2$ $\left[\because \left(a-b\right)\left(a+b\right)=a^2-b^2\right]$

$=4-2$

$=2=\frac{2}{1}$

Since, 2 is a rational number.

$\therefore \left(2-\sqrt{2}\right)\left(2+\sqrt{2}\right)$ is a rational number.

We have,

$\left(\sqrt{2}+\sqrt{3}\right)^2 = \left(\sqrt{2}\right)^2 + 2\times\sqrt{2}\times\sqrt{3}+\left(\sqrt{3}\right)^2$

$= 2+2\sqrt{6}+3$

$= 5+2\sqrt{6}$

The sum of a rational number and an irrational number is an irrational number, so $5+2\sqrt{6}$ is an irrational number.

$\therefore \left(\sqrt{2}+\sqrt{3}\right)^2$ is an irrational number.

The difference of a rational number and an irrational number is an irrational number

$\therefore \sqrt{5}-2$ is an irrational number.

$\sqrt{23} = 4.79583152331........$

$\sqrt{225} = 15 = \frac{15}{1}$

Rational number as it can be represented in $\frac{p}{q}$ form.

0.3796

As decimal expansion of this number is terminating, so it is a rational number.

$7.478478............ = 7.\overline{478}$

As decimal expansion of this number is non-terminating recurring so it is a rational number.

4. Identify the following as rational numbers. Give the decimal representation of rational numbers:

(i) $\sqrt{4}$

(ii) $3\sqrt{18}$

(iii) $\sqrt{1.44}$

(iv) $\sqrt{\frac{9}{27}}$

(v) $-\sqrt{64}$

(vi) $\sqrt{100}$

Sol:

We have

$\sqrt{4} = 2 = \frac{2}{1}$

$\sqrt{4}$ can be written in the form of $\frac{p}{q}$, so it is a rational number.

Its decimal representation is 2.0.
We have,

$3\sqrt{18} = 3\sqrt{2\times3\times3}$

$= 3\times3\sqrt{2}$

$= 9\sqrt{2}$

Since, the product of a rations and an irrational is an irrational number.

$\therefore 9\sqrt{2}$ is an irrational

$\Rightarrow 3\sqrt{18}$ is an irrational number.

We have,

$\sqrt{1\cdot44} = \sqrt{\frac{144}{100}}$

$= \frac{12}{10}$

$= 1.2$

Every terminating decimal is a rational number, so 1.2 is a rational number.
Its decimal representation is 1.2.
We have,

$\sqrt{\frac{9}{27}} = \frac{3}{\sqrt{27}} = \frac{3}{\sqrt{3\times3\times3}}$

$= \frac{3}{3\sqrt{3}}$

$= \frac{1}{\sqrt{3}}$

Quotient of a rational and an irrational number is irrational numbers so $\frac{1}{\sqrt{3}}$ is an irrational number.

$\Rightarrow \sqrt{\frac{9}{27}}$ is an irrational number.

We have,

$-\sqrt{64} = -\sqrt{8\times8}$

$= -8$

$= -\frac{8}{1}$

$-\sqrt{64}$ can be expressed in the form of $\frac{p}{q}$, so $-\sqrt{64}$ is a rotational number.

Its decimal representation is -8.0.
We have,

$\sqrt{100}=10$

$=\frac{10}{1}$

$\sqrt{100}$ can be expressed in the form of $\frac{p}{q}$. so $\sqrt{100}$ is a rational number.

The decimal representation of $\sqrt{100}$ is 10.0.

5. In the following equations, find which variables x, y, z etc. represent rational or irrational numbers:

(i) $x^2=5$

(ii) $y^2=9$

(iii) $z^2=0.04$

(iv) $u^2=\frac{17}{4}$

(v) $v^2=3$

(vi) $w^2=27$

(vii) $t^2=0.4$

Sol:

(i) We have

$x^2=5$

Taking square root on both sides.

$\Rightarrow \sqrt{x^2}=\sqrt{5}$

$\Rightarrow x=\sqrt{5}$

$\sqrt{5}$ is not a perfect square root, so it is an irrational number.

(ii) We have

$y^2=9$

$\Rightarrow y=\sqrt{9}$

$=3$

$=\frac{3}{1}$

$\sqrt{9}$ can be expressed in the form of $\frac{p}{q}$, so it a rational number.

(iii) We have

$z^2=0.04$

Taking square root on the both sides, we get,

$\sqrt{z^2}=\sqrt{0.04}$

$\Rightarrow z = \sqrt{0.04}$
$= 0.2$
$= \frac{2}{10}$
$= \frac{1}{5}$

z can be expressed in the form of $\frac{p}{q}$, so it is a rational number.

(iv) We have

$u^2 = \frac{17}{4}$

Taking square root on both sides, we get,

$\sqrt{u^2} = \sqrt{\frac{17}{4}}$

$\Rightarrow u = \sqrt{\frac{17}{2}}$

Quotient of an irrational and a rational number is irrational, so u is an irrational number.

(v) We have

$v^2 = 3$

Taking square root on both sides, we get,

$\sqrt{v^2} = \sqrt{13}$

$\Rightarrow v = \sqrt{3}$

$\sqrt{3}$ is not a perfect square root, so y is an irrational number.

(vi) We have

$w^2 = 27$

Taking square root on both des, we get,

$\sqrt{w^2} = \sqrt{27}$

$\Rightarrow w == \sqrt{3 \times 3 \times 3}$
$= 3\sqrt{3}$

Product of a rational and an irrational is irrational number, so w is an irrational number.

(vii) We have

$t^2 = 0.4$

Taking square root on both sides, we get

$\sqrt{t^2} = \sqrt{0.4}$

$\Rightarrow t = \sqrt{\frac{4}{10}}$

$= \frac{2}{\sqrt{10}}$

Since, quotient of a rational and an irrational number is irrational number, so *t* is an irrational number.

6. Give an example of each, of two irrational numbers whose:
(i) difference is a rational number.
(ii) difference is an irrational number.
(iii) sum is a rational number.
(iv) sum is an irrational number.
(v) product is a rational number.
(vi) product is an irrational number.
(vii) quotient is a rational number.
(viii) quotient is an irrational number.

Sol:

(i) $\sqrt{3}$ is an irrational number.

Now, $\left(\sqrt{3}\right)-\left(\sqrt{3}\right)=0$

0 is the rational number.

(ii) Let two irrational numbers are $5\sqrt{2}$ and $\sqrt{2}$

Now, $\left(5\sqrt{2}\right)-\left(\sqrt{2}\right)=4\sqrt{2}$

$4\sqrt{2}$ is the rational number.

(iii) Let two irrational numbers are $\sqrt{11}$ and $-\sqrt{11}$

Now, $\left(\sqrt{11}\right)+\left(-\sqrt{11}\right)=0$

0 is the rational number.

(iv) Let two irrational numbers are $4\sqrt{6}$ and $\sqrt{6}$

Now, $\left(4\sqrt{6}\right)+\left(\sqrt{6}\right)=5\sqrt{6}$

$5\sqrt{6}$ is the rational number.

(v) Let two irrational numbers are $2\sqrt{3}$ and $\sqrt{3}$

Now, $2\sqrt{3}\times\sqrt{3}=2\times3$

$=6$

6 is the rational number.

(vi) Let two irrational numbers are $\sqrt{2}$ and $\sqrt{5}$

Now, $\sqrt{2}\times\sqrt{5}=\sqrt{10}$

$\sqrt{10}$ is the rational number.

(vii) Let two irrational numbers are $3\sqrt{6}$ and $\sqrt{6}$

Now, $\frac{3\sqrt{6}}{\sqrt{6}} = 3$

3 is the rational number.

(viii) Let two irrational numbers are $\sqrt{6}$ and $\sqrt{2}$

Now, $\frac{\sqrt{6}}{\sqrt{2}} = \frac{\sqrt{3+2}}{\sqrt{2}}$

$= \frac{\sqrt{3} \times \sqrt{2}}{\sqrt{2}}$

$= \sqrt{3}$

$\sqrt{3}$ is an irrational number.

7. Give two rational numbers lying between 0.232332333233332 . . . and 0.212112111211112.

Sol:

Let, a = 0.212112111211112

And, b = 0.232332333233332...

Clearly, $a < b$ because in the second decimal place a has digit 1 and b has digit 3

If we consider rational numbers in which the second decimal place has the digit 2, then they will lie between a and b.

Let,

x = 0.22

y = 0.22112211...

Then,

$a < x < y < b$

Hence, x, and y are required rational numbers.

8. Give two rational numbers lying between 0.515115111511115 ... and 0.5353353335 ...

Sol:

Let, a = 0.515115111511115...

And, b = 0.5353353335...

We observe that in the second decimal place a has digit 1 and b has digit 3, therefore, $a < b$. So if we consider rational numbers

$x = 0.52$

y = 0.52052052...

We find that,

$a < x < y < b$

Hence x, and y are required rational numbers.

9. Find one irrational number between 0.2101 and 0.2222 . . . = $0.\overline{2}$

Sol:

Let, a = 0.2101

And, b = 0.2222...

We observe that in the second decimal place a has digit 1 and b has digit 2, therefore $a < b.$ in the third decimal place a has digit 0. So, if we consider irrational numbers

$x = 0.211011001100011....$

We find that

$a < x < b$

Hence, *x* is required irrational number.

10. Find a rational number and also an irrational number lying between the numbers 0.3030030003 ... and 0.3010010001 ...

Sol:

Let, $a = 0.3010010001$

And, $b = 0.3030030003...$

We observe that in the third decimal place a has digit 1 and b has digit 3, therefore $a < b.$ in the third decimal place a has digit 1. so, if we consider rational and irrational numbers

$x = 0.302$

$y = 0.302002000200002.....$

We find that

$a < x < b$

And, $a < y < b$

Hence, x and y are required rational and irrational numbers respectively.

11. Find two irrational numbers between 0.5 and 0.55.

Sol:

Let $a = 0.5 = 0.50$

And, $b = 0.55$

We observe that in the second decimal place a has digit 0 and b has digit 5, therefore $a < b.$ so, if we consider irrational numbers

$x = 0.51051005100051...$

$y = 0.530535305353530...$

We find that

$a < x < y < b$

Hence, *x* and *y* are required irrational numbers.

12. Find two irrational numbers lying between 0.1 and 0.12.

Sol:

Let, $a = 0.1 = 0.10$

And, $b = 0.12$

We observe that in the second decimal place a has digit 0 and b has digit 2, Therefore $a < b.$ So, if we consider irrational numbers

$x = 0.11011001100011...$

$y = 0.111011110111110...$

We find that,

$a < x < y < b$

Hence, x and y are required irrational numbers.

13. Prove that $\sqrt{3} + \sqrt{5}$ is an irrational number.

Sol:

If possible, let $\sqrt{3}+\sqrt{5}$ be a rational number equal to x. Then,

$x = \sqrt{3}+\sqrt{5}$

$\Rightarrow x^2 = \left(\sqrt{3}+\sqrt{5}\right)^2$

$\Rightarrow x^2 = \left(\sqrt{3}\right)^2 + \left(\sqrt{5}\right)^2 + 2\times\sqrt{3}\times\sqrt{5}$

$= 3+5+2\sqrt{15}$

$= 8+2\sqrt{15}$

$\Rightarrow x^2 - 8 = 2\sqrt{15}$

$\Rightarrow \dfrac{x^2-8}{2} = \sqrt{15}$

Now, x is rational

$\Rightarrow x^2$ is rational

$\Rightarrow \dfrac{x^2-8}{2}$ is rational

$\Rightarrow \sqrt{15}$ is rational

But, $\sqrt{15}$ is rational

Thus, we arrive at a contradiction. So, our supposition that $\sqrt{3}+\sqrt{5}$ is rational is wrong.

Hence, $\sqrt{3}+\sqrt{5}$ is an irrational number.

14. Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$

Sol:

$\frac{5}{7} = 0.\overline{714285}$

$\frac{9}{11} = 0.\overline{81}$

3 irrational numbers are

0.73073007300073000073.......

0.75075007500075000075.......

0.79079007900079000079.......

Exercise -1.5

1. Complete the following sentences:

(i) Every point on the number line corresponds to a ______ number which many be either ______ or ______

(ii) The decimal form of an irrational number is neither ______ nor ______

(iii) The decimal representation of a rational number is either ______ or ______

(iv) Every real number is either ______ number or ______ number.

Sol:

(i) Every point on the number line corresponds to a **Real** number which may be either **rational** or **irrational**.

(ii) The decimal form of an irrational number is neither **terminating** nor **repeating**

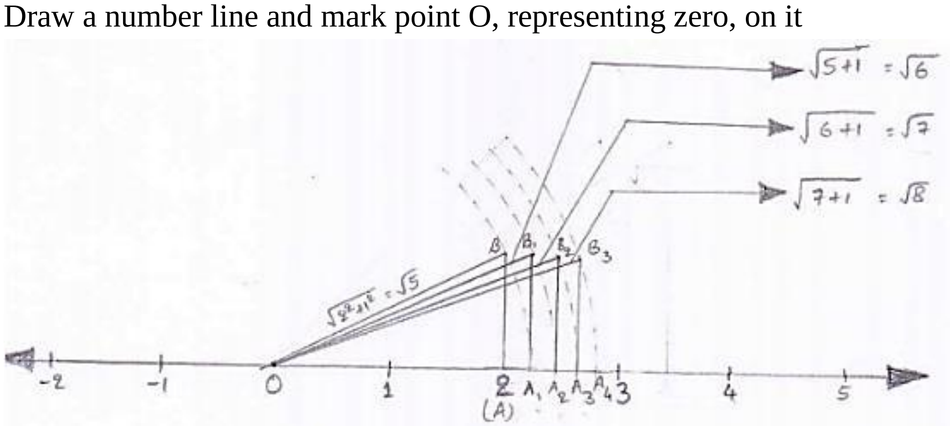
(iii) The decimal representation of a rational number is either **terminating, non-terminating** or **recurring**.

(iv) Every real number is either **a rational** number or **an irrational** number.

2. Represent $\sqrt{6}, \sqrt{7}, \sqrt{8}$ on the number line.

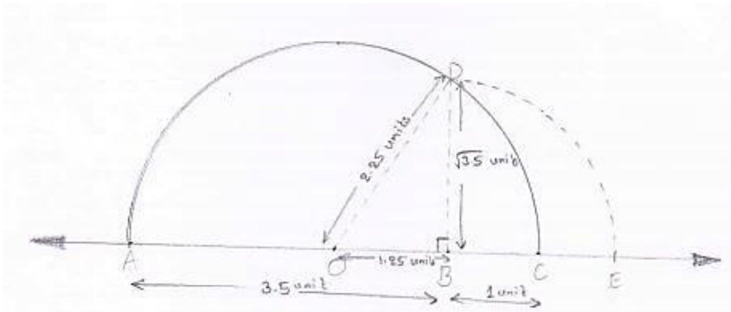
Sol:

Draw a number line and mark point O, representing zero, on it



Suppose point A represents 2 as shown in the figure
Then $OA = 2.$ Now, draw a right triangle OAB such that $AB = 1.$
By Pythagoras theorem, we have
$OB^2 = OA^2 + AB^2$
$\Rightarrow OB^2 = 2^2 + 1^2$
$\Rightarrow OB^2 = 4 + 1 = 5 \Rightarrow OB = \sqrt{5}$
Now, draw a circle with center O and radius OB.
We fine that the circle cuts the number line at A
Clearly, $OA_1 = OB =$ radius of circle $= \sqrt{5}$
Thus, A_1 represents $\sqrt{5}$ on the number line.
But, we have seen that $\sqrt{5}$ is not a rational number. Thus we find that there is a point on the number which is not a rational number.
Now, draw a right triangle OA_1B_1, Such that $A_1B_1 = AB = 1$
Again, by Pythagoras theorem, we have
$(OB_1)^2 = (OA_1)^2 + (A_1B_1)^2$
$\Rightarrow (OB_1)^2 = (\sqrt{5})^2 + (1)^2$
$\Rightarrow (OB_1^2) = 5 + 1 = 6 \Rightarrow OB_1 = \sqrt{6}$
Draw a circle with center O and radius $OB_1 = \sqrt{6}.$ This circle cuts the number line at A_2 as shown in figure
Clearly $OA_2 = OB_1 = \sqrt{6}$
Thus, A_2 represents $\sqrt{6}$ on the number line.
Also, we know that $\sqrt{6}$ is not a rational number.
Thus, A_2 is a point on the number line not representing a rational number
Continuing in this manner, we can represent $\sqrt{7}$ and $\sqrt{8}$ also on the number lines as shown in the figure
Thus, $OA_3 = OB_2 = \sqrt{7}$ and $OA_4 = OB_3 = \sqrt{8}$

3. Represent $\sqrt{3.5}, \sqrt{9.4}, \sqrt{10.5}$ on the real number line.
Sol:
Given to represent $\sqrt{3\cdot 5}, \sqrt{9\cdot 4}, \sqrt{10\cdot 5}$ on the real number line
Representation of $\sqrt{3\cdot 5}$ on real number line:
Steps involved:
(i) Draw a line and mark A on it.



(ii) Mark a point B on the line drawn in step - (i) such that $AB = 3\cdot5$ units

(iii) Mark a point C on AB produced such that BC = 1unit

(iv) Find mid-point of AC. Let the midpoint be O

$\Rightarrow AC = AB + BC = 3\cdot5 + 1 = 4\cdot5$

$\Rightarrow AO = OC = \dfrac{AC}{2} = \dfrac{4\cdot5}{2} = 2\cdot25$

(v) Taking O as the center and $OC = OA$ as radius drawn a semi-circle. Also draw a line passing through B perpendicular to OB. Suppose it cuts the semi-circle at D.
Consider triangle OBD, it is right angled at B.

$BD^2 = OD^2 - OB^2$

$\Rightarrow BD^2 = OC^2 - (OC - BC)^2$ $\quad [\because OC = OD = \text{radius}]$

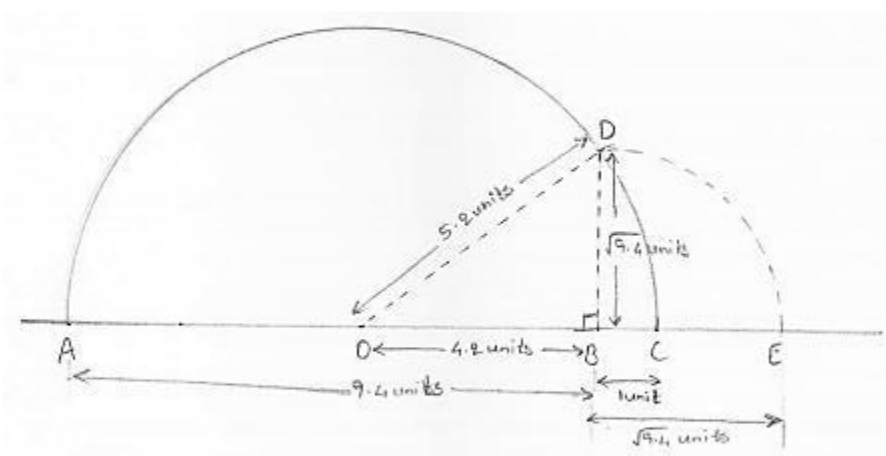
$\Rightarrow BD^2 = 2OC \cdot BC - (BC)^2$

$\Rightarrow BD = \sqrt{2 \times 2\cdot25 \times 1 - (1)^2} \Rightarrow BD = \sqrt{35}$

(vi) Taking B as the center and BD as radius draw an arc cutting OC produced at E. point E so obtained represents $\sqrt{3\cdot5}$ as $BD = BE = \sqrt{3\cdot5} = \text{radius}$
Thus, E represents the required point on the real number line.

Representation of $\sqrt{9\cdot4}$ on real number line steps involved:

(i) Draw and line and mark A on it



(ii) Mark a point B on the line drawn in step (i) such that $AB = 9\cdot 4$ units

(iii) Mark a point C on AB produced such that $BC = 1$ unit.

(iv) Find midpoint of AC. Let the midpoint be O.

$\Rightarrow AC = AB + BC = 9\cdot 4 + 1 = 10\cdot 4$ units

$\Rightarrow AD = OC = \dfrac{AC}{2} = \dfrac{10\cdot 4}{2} = 5\cdot 2$ units

(v) Taking O as the center and $OC = OA$ as radius draw a semi-circle. Also draw a line passing through B perpendicular to OB. Suppose it cuts the semi-circle at D. Consider triangle OBD, it is right angled at B.

$\Rightarrow BD^2 = OD^2 - OB^2$

$\Rightarrow BD^2 = OC^2 - (OC - BC)^2 \qquad [\because OC = OD = \text{radius}]$

$\Rightarrow BD^2 = OC^2 - \left(OC^2 - 2OC\cdot BC + (BC)^2\right)$

$\Rightarrow BD^2 = 2OC\cdot\left(BC - \left(BC^2\right)\right)$

$\Rightarrow BD^2 = \sqrt{2\times(5\cdot 2)\times 1 - 1^2} \Rightarrow BD = \sqrt{9\cdot 4}$ units

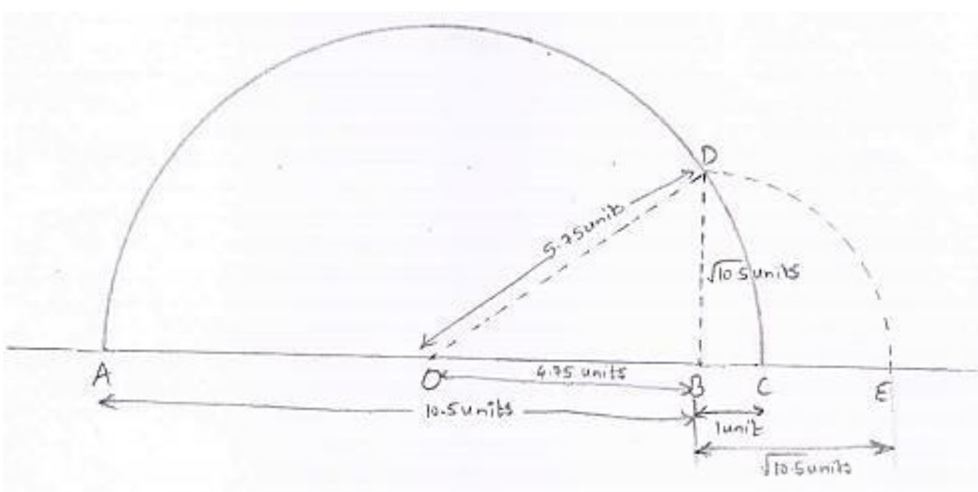
(vi) Taking B as center and BD as radius draw an arc cutting OC produced at E so obtained represents $\sqrt{9\cdot 4}$ as $BD = BE = \sqrt{9\cdot 4} = \text{radius}$

Thus, E represents the required point on the real number line.

Representation of $\sqrt{10\cdot 5}$ on the real number line:

Steps involved:

(i) Draw a line and mark A on it



(ii) Mark a point B on the line drawn in step (i) such that $AB = 10\cdot 5$ units

(iii) Mark a point C on AB produced such that $BC = 1$ unit

(iv) Find midpoint of AC. Let the midpoint be 0.

$\Rightarrow AC = AB + BC = 10\cdot 5 + 1 = 11\cdot 5$ units

$\Rightarrow AO = OC = \dfrac{AC}{2} = \dfrac{11\cdot 5}{2} = 5\cdot 75$ units

(v) Taking O as the center and $OC = OA$ as radius, draw a semi-circle. Also draw a line passing through B perpendicular to DB. Suppose it cuts the semi-circle at D. consider triangle OBD, it is right angled at B

$\Rightarrow BD^2 = OD^2 - OB^2$

$\Rightarrow BD^2 = OC^2 - (OC - BC)^2$ $\quad [\because OC = OD = \text{radius}]$

$\Rightarrow BD^2 = OC^2 - \left[OC^2 - 2OC\cdot BC + (BC)^2\right]$

$\Rightarrow BD^2 = 2OC\cdot BC - BC^2$

$\Rightarrow BC^2 = OD^2 - OB^2$

$\Rightarrow BD^2 = OC^2 - (OC - BC)^2$ $\quad [\because OC = OD = \text{radius}]$

$\Rightarrow BD^2 = OC^2 - \left[OC^2 - 2OC\cdot BC + (BC)^2\right]$

$\Rightarrow BD^2 = 2OC\cdot BC - BC^2$

$\Rightarrow BD = \sqrt{2\times 575\times 1 - (1)^2} \Rightarrow BD = \sqrt{10\cdot 5}$

(vi) Taking B as the center and BD as radius draw on arc cutting OC produced at E. point E so obtained represents $\sqrt{10\cdot 5}$ as $BD = BE = \sqrt{10\cdot 5} =$ radius arc

Thus, E represents the required point on the real number line

4. Find whether the following statements are true or false.

(i) Every real number is either rational or irrational.

(ii) it is an irrational number.

(iii) Irrational numbers cannot be represented by points on the number line.

Sol:

(i) True

As we know that rational and irrational numbers taken together from the set of real numbers.

(ii) True

As, π is ratio of the circumference of a circle to its diameter, it is an irrational number

$\Rightarrow \pi = \dfrac{2\pi r}{2r}$

(iii) False

Irrational numbers can be represented by points on the number line.

Exercise -1.6

Mark the correct alternative in each of the following:

1. Which one of the following is a correct statement?

(a) Decimal expansion of a rational number is terminating

(b) Decimal expansion of a rational number is non-terminating

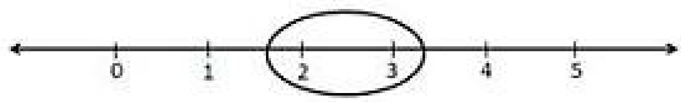
(c) Decimal expansion of an irrational number is terminating

(d) Decimal expansion of an irrational number is non-terminating and non-repeating

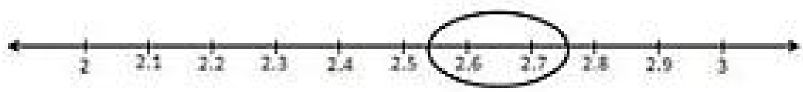
Sol:

The following steps for successive magnification to visualise 2.665 are:

(1) We observe that 2.665 is located somewhere between 2 and 3 on the number line. So, let us look at the portion of the number line between 2 and 3.

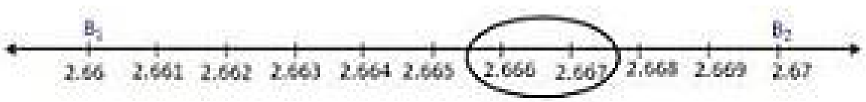


(2) We divide this portion into 10 equal parts and mark each point of division. The first mark to the right of 2 will represent 2.1, the next 2.2 and soon. Again we observe that 2.665 lies between 2.6 and 2.7.



(3) We mark these points A_1 and A_2 respectively. The first mark on the right side of A_1, will represent 2.61, the number 2.62, and soon. We observe 2.665 lies between 2.66 and 2.67.

(4) Let us mark 2.66 as B_1 and 2.67 as B_2. Again divide the B_1B_2 into ten equal parts. The first mark on the right side of B_1 will represent 2.661. Then next 2.662, and so on. Clearly, fifth point will represent 2.665.



2. Which one of the following statements is true?

(a) The sum of two irrational numbers is always an irrational number

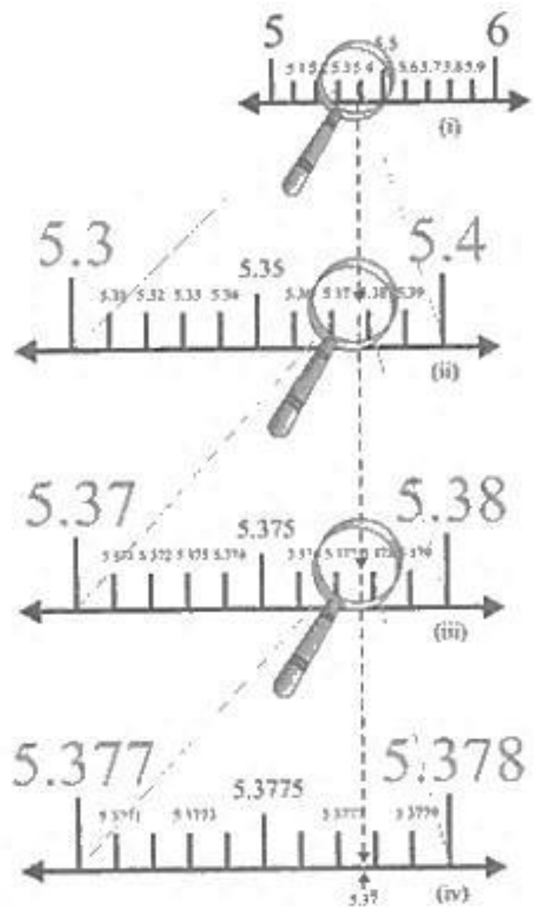
(b) The sum of two irrational numbers is always a rational number

(c) The sum of two irrational numbers may be a rational number or an irrational number

(d) The sum of two irrational numbers is always an integer

Sol:

Once again we proceed by successive magnification, and successively decrease the lengths of the portions of the number line in which $5.3\overline{7}$ is located. First, we see that $5.3\overline{7}$ is located between 5 and 6. In the next step, we locate $5.3\overline{7}$ between 5.3 and 5.4. To get a more accurate visualization of the representation, we divide this portion of the number line into lo equal parts and use a magnifying glass to visualize that $5\cdot 3\overline{7}$ lies between 5.37 and 5.38. To visualize $5.3\overline{7}$ more accurately, we again divide the portion between 5.37 and 5.38 into ten equal parts and use a magnifying glass to visualize that S.S lies between 5.377 and 5.378. Now to visualize $5.3\overline{7}$ still more accurately, we divide the portion between 5.377 and 5.378 into 10 equal parts, and visualize the representation of $5.3\overline{7}$ as in fig.,(iv) . Notice that $5.3\overline{7}$ is located closer to 5.3778 than to 5.3777(iv)



Exercise – 2.1

1. Assuming that x, y, z are positive real numbers, simplify each of the following:

(i) $\left(\sqrt{x^{-3}}\right)^5$

(ii) $\sqrt{x^3y^{-2}}$

(iii) $\left(x^{-\frac{2}{3}}y^{-\frac{1}{2}}\right)^2$

(iv) $\left(\sqrt{x}\right)^{-\frac{2}{3}}\sqrt{y^4}\div\sqrt{xy^{-\frac{1}{2}}}$

(v) $\sqrt[5]{243x^{10}y^5z^{10}}$

(vi) $\left(\frac{x^{-4}}{y^{-10}}\right)^{\frac{5}{4}}$

Sol:

We have,

$$\left(\sqrt{x^{-3}}\right)^5=\left(\sqrt{\frac{1}{x^3}}\right)^5$$

$$=\left(\frac{1}{x^{\frac{3}{2}}}\right)^5$$

$$=\frac{1}{x^{\frac{3}{2}\times5}}$$

$$=\frac{1}{x^{\frac{15}{2}}}$$

$$\Rightarrow\left(\sqrt{x^{-3}}\right)^5=\frac{1}{x^{\frac{15}{2}}}$$

We have,

$$\sqrt{x^3y^{-2}}=\sqrt{\frac{x^3}{y^2}}$$

$$=\left(\frac{x^3}{y^2}\right)^{\frac{1}{2}}$$

$$= \frac{x^{3 \times \frac{1}{2}}}{y^{2 \times \frac{1}{2}}}$$

$$= \frac{x^{\frac{3}{2}}}{y}$$

$$\Rightarrow \sqrt{x^3 y^{-2}} = \frac{x^{\frac{3}{2}}}{y}$$

We have,

$$\left(x^{-\frac{2}{3}} y^{-\frac{1}{2}}\right)^2 = \left(\frac{1}{x^{\frac{2}{3}} y^{\frac{1}{2}}}\right)^2$$

$$= \left(\frac{1}{x^{\frac{2}{3} \times 2} y^{2 \times \frac{1}{2}}}\right)^2$$

$$= \frac{1}{x^{\frac{4}{3}} y^1}$$

$$= \frac{1}{x^{\frac{4}{3}} y}$$

$$\Rightarrow \left(x^{-\frac{2}{3}} y^{-\frac{1}{2}}\right)^2 = \frac{1}{x^{\frac{4}{3}} y}$$

We have,

$$\left(\sqrt{x}\right)^{-\frac{2}{3}} \sqrt{y^4} \div \sqrt{xy^{-\frac{1}{2}}}$$

$$= \left(x^{\frac{1}{2}}\right)^{-\frac{2}{3}} \left(y^2\right) \div \sqrt{xy^{-\frac{1}{2}}}$$

$$= \frac{x^{\frac{1}{2} \times \frac{2}{3}} y^2}{\left(xy^{-\frac{1}{2}}\right)^{\frac{1}{2}}}$$

$$= \frac{x^{-\frac{1}{3}} y^2}{x^{\frac{1}{2}} y^{-\frac{1}{2} \times \frac{1}{2}}}$$

$$= \left(x^{-\frac{1}{3}} \times x^{-\frac{1}{2}} \right) \times \left(y^2 \times y^{\frac{1}{4}} \right)$$

$$= \left(x^{-\frac{1}{3}-\frac{1}{2}} \right) \left(y^{2+\frac{1}{4}} \right)$$

$$= \left(x^{\frac{-2-3}{6}} \right) \left(y^{\frac{8+1}{4}} \right)$$

$$= \left(x^{-\frac{5}{6}} \right) \left(y^{\frac{9}{4}} \right)$$

$$= \frac{y^{\frac{9}{4}}}{x^{\frac{5}{6}}}$$

$$\Rightarrow \left(\sqrt{x} \right)^{-\frac{2}{3}} \sqrt{y^4} \div \sqrt{xy^{-\frac{1}{2}}} = \frac{y^{\frac{9}{4}}}{x^{\frac{5}{6}}}$$

We have,

$$\sqrt[5]{243x^{10}y^5z^{10}} = \left(243x^{10}y^5z^{10} \right)^{\frac{1}{5}}$$

$$= (243)^{\frac{1}{5}} x^{\frac{10}{5}} y^{\frac{5}{5}} z^{\frac{10}{5}}$$

$$= \left(3^5 \right)^{\frac{1}{5}} x^2 y^1 z^2$$

$$= 3^{5 \times \frac{1}{5}} x^2 y z^2$$

$$= 3x^2yz^2$$

$$\Rightarrow \sqrt[5]{243x^{10}y^5z^{10}} = 3x^2yz^2$$

We have,

$$\left(\frac{x^{-4}}{y^{-10}} \right)^{\frac{5}{4}} = \left(\frac{y^{10}}{x^4} \right)^{\frac{5}{4}}$$

$$= \frac{y^{10 \times \frac{5}{4}}}{x^{4 \times \frac{5}{4}}}$$

$$= \frac{y^{\frac{25}{2}}}{x^5}$$

$$\Rightarrow \left(\frac{x^{-4}}{y^{-10}} \right)^{\frac{5}{4}} = \frac{y^{\frac{25}{2}}}{x^5}$$

2. Simplify:

(i) $\left(16^{-\frac{1}{5}}\right)^{\frac{5}{2}}$

(ii) $\sqrt[3]{(342)^{-2}}$

(iii) $(0.001)^{\frac{1}{3}}$

(iv) $\dfrac{(25)^{\frac{3}{2}}\times(243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}}\times(8)^{\frac{4}{3}}}$

(v) $\left(\dfrac{\sqrt{2}}{5}\right)^{8}\div\left(\dfrac{\sqrt{2}}{5}\right)^{13}$

(vi) $\left[\dfrac{5^{-1}\times7^{2}}{5^{2}\times7^{-4}}\right]^{\frac{7}{2}}\times\left[\dfrac{5^{-2}\times7^{3}}{5^{3}\times7^{-5}}\right]^{-\frac{5}{2}}$

Sol:

(i) We have

$$\left(16^{-\frac{1}{5}}\right)^{\frac{5}{2}}=(16)^{-\frac{1}{5}\times\frac{5}{2}}=(16)^{-\frac{1}{2}}=\left(4^{2}\right)^{-\frac{1}{2}}=4^{2x-\frac{1}{2}}=4^{-1}=\frac{1}{4}$$

Hence, $\boxed{\left(16^{-\frac{1}{5}}\right)^{\frac{5}{2}}=\frac{1}{4}}$

(ii) We have,

$$\sqrt[3]{(342)^{-2}}=\left[(343)^{-2}\right]^{\frac{1}{3}}=(343)^{-2\times\frac{1}{3}}$$

$$=\left(7^{3}\right)^{-\frac{2}{3}}$$

$$=7^{3\times\frac{-2}{3}}$$

$$=7^{-2}=\frac{1}{7^{2}}=\frac{1}{49}$$

Hence, $\boxed{\sqrt[3]{(343)^{-2}}=\frac{1}{49}}$

(iii) We have,

$$(0.001)^{\frac{1}{3}}=\left(\frac{1}{1000}\right)^{\frac{1}{3}}=\left(\frac{1}{10^{3}}\right)^{\frac{1}{3}}$$

$$= \frac{1^{\frac{1}{3}}}{\left(10^3\right)^{\frac{1}{3}}} = \frac{1}{10^{3\times\frac{1}{3}}} = \frac{1}{10} = 0.01$$

Hence, $\boxed{(0.001)^{\frac{1}{3}} = 0\cdot 1}$

(iv) We have,

$$\frac{(25)^{\frac{3}{2}} \times (243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}} \times (8)^{\frac{4}{3}}} = \frac{\left(5^2\right)^{\frac{3}{2}} \times \left(3^5\right)^{\frac{3}{5}}}{\left(2^4\right)^{\frac{5}{4}} \times \left(2^3\right)^{\frac{4}{3}}}$$

$$= \frac{5^{2\times\frac{3}{2}} \times 3^{5\times\frac{3}{5}}}{2^{4\times\frac{5}{4}} \times 2^{3\times\frac{4}{3}}}$$

$$= \frac{5^3 \times 3^3}{2^5 \times 2^4} = \frac{125 \times 27}{32 \times 16} = \frac{3375}{512}$$

Hence, $\boxed{\frac{(25)^{\frac{3}{2}} \times (243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}} \times (8)^{\frac{4}{3}}} = \frac{3375}{512}}$

(v) We have,

$$\left(\frac{\sqrt{2}}{5}\right)^8 \div \left(\frac{\sqrt{2}}{5}\right)^{13} = \frac{\left(\frac{\sqrt{2}}{5}\right)^8}{\left(\frac{\sqrt{2}}{5}\right)^{13}}$$

$$\Rightarrow \left(\frac{\sqrt{2}}{5}\right)^{8-13} = \left(\frac{\sqrt{2}}{5}\right)^{-5} = \frac{\left(2^{\frac{1}{2}}\right)^{-5}}{(5)^{-5}} = \frac{2^{\frac{1}{2}x-5}}{5^{-5}} = \frac{2^{-\frac{5}{2}}}{5^{-5}}$$

$$\Rightarrow \frac{1}{2^{\frac{5}{2}}} \times \frac{5^5}{1} = \frac{5^5}{2^{\frac{5}{2}}} = \frac{3125}{4\sqrt{2}}$$

Hence $\boxed{\left(\frac{\sqrt{2}}{5}\right)^8 \div \left(\frac{\sqrt{2}}{5}\right)^{13} = \frac{3125}{4\sqrt{2}}}$

(vi) We have,

$$\left[\frac{5^{-1} \times 7^2}{5^2 \times 7^{-4}}\right]^{\frac{7}{2}} \times \left[\frac{5^{-2} \times 7^3}{5^3 \times 7^{-5}}\right]^{-\frac{5}{2}}$$

$$\Rightarrow \frac{\left(5^1\times7^2\right)^{\frac{7}{2}}}{\left(5^2\times7^{-4}\right)^{\frac{7}{2}}}\times\frac{\left(5^{-2}\times7^3\right)^{-\frac{5}{2}}}{\left(5^3\times7^{-5}\right)^{-\frac{5}{2}}}$$

$$\Rightarrow \frac{\left(5^{-1}\right)^{\frac{7}{2}}\times\left(7^2\right)^{\frac{7}{2}}}{\left(5^2\right)^{\frac{7}{2}}\times\left(7^{-4}\right)^{\frac{7}{2}}}\times\frac{\left(5^{-2}\right)^{-\frac{5}{2}}\times\left(7^3\right)^{-\frac{5}{2}}}{\left(5^3\right)^{-\frac{5}{2}}\times\left(7^{-5}\right)^{-\frac{5}{2}}}$$

$$\Rightarrow \frac{5^{-1\times\frac{7}{2}}\times7^{2\times\frac{7}{2}}}{5^{2\times\frac{7}{2}}\times7^{-4\times\frac{7}{2}}}\times\frac{5^{-2\times\frac{-5}{2}}\times7^{3\times\frac{-5}{2}}}{5^{3\times\frac{-5}{2}}\times7^{-5x-\frac{5}{2}}}$$

$$\Rightarrow \frac{5^{\frac{-7}{2}}\times7^{7}}{5^{7}\times7^{-14}}\times\frac{5^{5}\times7^{\frac{-15}{2}}}{5^{\frac{-15}{2}}\times7^{\frac{25}{2}}}$$

$$\Rightarrow \frac{7^{7+14}}{5^{7+\frac{7}{2}}}\times\frac{5^{5+\frac{15}{2}}}{7^{\frac{25}{2}+\frac{15}{2}}}$$

$$\Rightarrow \frac{5^{\frac{-7}{2}}\times7^{7}}{5^{7}\times7^{-14}}\times\frac{5^{5}\times7^{\frac{-15}{2}}}{5^{\frac{-15}{2}}\times7^{\frac{25}{2}}}$$

$$\Rightarrow \frac{7^{7+14}}{5^{7+\frac{7}{2}}}\times\frac{5^{5+\frac{15}{2}}}{7^{\frac{25}{2}+\frac{15}{2}}}$$

$$\Rightarrow \frac{7^{21}}{5^{\frac{21}{2}}}\times\frac{5^{\frac{25}{2}}}{7^{\frac{40}{2}}}$$

$$\Rightarrow \frac{7^{21}}{7^{20}}\times\frac{5^{\frac{25}{2}}}{5^{\frac{21}{2}}}$$

$$\Rightarrow 7^{21-20}\times5^{\frac{25}{2}-\frac{21}{2}}$$

$$\Rightarrow 7^{1}\times5^{\frac{4}{2}}$$

$$\Rightarrow 7^{1}\times5^{2}\Rightarrow 7\times25\Rightarrow 175$$

Hence, $\left[\frac{5^{-1}\times7^{2}}{5^{2}\times7^{-4}}\right]^{\frac{7}{2}}\times\left[\frac{5^{-2}\times7^{3}}{5^{3}\times7^{-5}}\right]^{-\frac{5}{2}}=175$

3. Prove that:

(i) $9^{\frac{3}{2}} - 3\times5^0 - \left(\frac{1}{81}\right)^{-\frac{1}{2}} = 15$

(ii) $\left(\frac{1}{4}\right)^{-2} - 3\times8^{\frac{2}{3}}\times4^0 + \left(\frac{9}{16}\right)^{-\frac{1}{2}} = \frac{16}{3}$

(iii) $\dfrac{2^{\frac{1}{2}}\times3^{\frac{1}{3}}\times4^{\frac{1}{4}}}{10^{-\frac{1}{5}}\times5^{\frac{3}{5}}} \div \dfrac{3^{\frac{4}{3}}\times5^{\frac{-7}{5}}}{4^{\frac{-3}{5}}\times6} = 10$

(iv) $\dfrac{(0.6)^0 - (0.1)^{-1}}{\left(\frac{3}{8}\right)^{-1}\left(\frac{3}{2}\right)^3 + \left(-\frac{1}{3}\right)^{-1}} = -\frac{3}{2}$

(v) $\sqrt{\frac{1}{4}} + (0.01)^{-\frac{1}{2}} - (27)^{\frac{2}{3}} = \frac{3}{2}$

(vi) $\dfrac{2^n + 2^{n-1}}{2^{n+1} - 2^n} = \dfrac{2^n + 2^n\times2^{-1}}{2^n\times2^1 - 2^n}$

(vii) $\left(\frac{64}{125}\right)^{-\frac{2}{3}} + \dfrac{1}{\left(\frac{256}{625}\right)^{\frac{1}{4}}} + \left(\dfrac{\sqrt{25}}{\sqrt[3]{64}}\right)$

(viii) $\dfrac{3^{-3}\times6^2\times\sqrt{98}}{5^2\times\sqrt[3]{\frac{1}{25}}\times(15)^{-\frac{4}{3}}\times3^{\frac{1}{3}}}$

Sol:

(i) We have,

$$9^{\frac{3}{2}} - 3\times5^0 - \left(\frac{1}{81}\right)^{-\frac{1}{2}}$$

$$= \left(3^2\right)^{\frac{3}{2}} - 3\times1 - \left(\frac{1}{9^2}\right)^{-\frac{1}{2}}$$

$$= 3^{2\times\frac{3}{2}} - 3 - \left(9^{-2}\right)^{-\frac{1}{2}}$$

$$= 3^3 - 3 - 9^{-2x\left(-\frac{1}{2}\right)}$$

$$= 3^3 - 3 - 9$$

$$= 27 - 3 - 9$$

$$= 27 - 12$$

$$= 15$$

$$\Rightarrow 9^{\frac{3}{2}} - 3\times 5^0 - \left(\frac{1}{81}\right)^{-\frac{1}{2}} = 15$$

(ii) We have,

$$\left(\frac{1}{4}\right)^{-2} - 3\times 8^{\frac{2}{3}} \times 4^0 + \left(\frac{9}{16}\right)^{-\frac{1}{2}}$$

$$= \left(\frac{1}{2^2}\right)^{-2} - 3\times 8^{\frac{2}{3}} \times 1 + \left(\frac{3^2}{4^2}\right)^{-\frac{1}{2}}$$

$$= \left(2^{-2}\right)^{-2} - 3\times 8^{\frac{2}{3}} \times 1 + \left(\frac{3^{2\times\frac{-1}{2}}}{4^{2\times\frac{-1}{2}}}\right)$$

$$= 2^{(-2)\times(-2)} - 3\times 8^{\frac{2}{3}} + \left(\frac{3^{-1}}{4^{-1}}\right)$$

$$= 2^4 - 3\times 2^{3\times\frac{2}{3}} + \frac{4}{3}$$

$$= 2^4 - 3\times 2^2 + \frac{4}{3}$$

$$= 2^4 - 3\times 4 + \frac{4}{3}$$

$$= 16 - 12 + \frac{4}{3}$$

$$= 4 + \frac{4}{3} = \frac{12+4}{3}$$

$$= \frac{16}{3}$$

$$\Rightarrow \left(\frac{1}{4}\right)^{-2} - 3\times 8^{\frac{2}{3}} \times 4^0 + \left(\frac{9}{16}\right)^{-\frac{1}{2}} = \frac{16}{3}$$

(iii) We have,

$$\frac{2^{\frac{1}{2}} \times 3^{\frac{1}{3}} \times 4^{\frac{1}{4}}}{10^{-\frac{1}{5}} \times 5^{\frac{3}{5}}} \div \frac{3^{\frac{4}{3}} \times 5^{-\frac{7}{5}}}{4^{-\frac{3}{5}} \times 6}$$

$$= \frac{2^{\frac{1}{2}} \times 3^{\frac{1}{3}} \times 4^{\frac{1}{4}}}{10^{-\frac{1}{5}} \times 5^{\frac{3}{5}}} \times \frac{4^{-\frac{3}{5}} \times 6}{3^{\frac{4}{3}} \times 5^{-\frac{7}{5}}}$$

$$= \frac{2^{\frac{1}{2}} \times 3^{\frac{1}{3}} \times \left(2^2\right)^{\frac{1}{4}} \times \left(2^2\right)^{-\frac{3}{5}} \times (2\times 3)}{(2\times 5)^{-\frac{1}{5}} \times 5^{\frac{3}{5}} \times 3^{\frac{4}{3}} \times 5^{-\frac{7}{5}}}$$

$$= \frac{\left(2^{\frac{1}{2}} \times 2^{\frac{1}{2}} \times 2^{-\frac{6}{5}} \times 2^1\right) \times \left(3^{\frac{1}{3}} \times 3^1\right)}{2^{-\frac{1}{5}} \times 5^{-\frac{1}{5}} \times 5^{\frac{3}{5}} \times 3^{\frac{4}{3}} \times 5^{-\frac{7}{5}}}$$

$$= \frac{\left(2 \times 2^{\frac{-6}{5}} \times 2\right) \times \left(3^{\frac{1}{3}} \times 3^1 \times 3^{-\frac{4}{3}}\right)}{2^{-\frac{1}{5}} \times \left(5^{-\frac{1}{5}} \times 5^{\frac{3}{5}} \times 5^{-\frac{7}{5}}\right)}$$

$$= \frac{\left(2 \times 2^{-\frac{6}{5}} \times 2 \times 2^{\frac{1}{5}}\right) \times \left(3^{\frac{1}{3}} \times 3^1 \times 3^{-\frac{4}{3}}\right)}{\left(5^{-\frac{1}{5}} \times 5^{\frac{3}{5}} \times 5^{-\frac{7}{5}}\right)}$$

$$= \frac{(2)^{1-\frac{6}{5}+1+\frac{1}{5}} \times (3)^{\frac{1}{3}+1-\frac{4}{3}}}{(5)^{-\frac{1}{5}+\frac{3}{5}-\frac{7}{5}}}$$

$$= \frac{(2)^{2-\frac{6}{5}+\frac{1}{5}} \times (3)^{\frac{1+3-4}{3}}}{(5)^{\frac{-1+3-7}{5}}}$$

$$= \frac{(2)^{2-\frac{5}{5}} \times (3)^{\frac{0}{3}}}{(5)^{-\frac{5}{5}}}$$

$$= \frac{(2)^{2-1} \times (3)^0}{(5)^{-1}}$$

$$= 2^1 \times 1 \times 5^1$$

$$= 10$$

$$\Rightarrow \frac{2^{\frac{1}{2}} \times 3^{\frac{1}{3}} \times 4^{\frac{1}{4}}}{10^{-\frac{1}{5}} \times 5^{\frac{3}{5}}} \div \frac{3^{\frac{4}{3}} \times 5^{-\frac{7}{5}}}{4^{-\frac{3}{5}} \times 6} = 10$$

(iv) We have,

$$\frac{(0.6)^0 - (0.1)^{-1}}{\left(\frac{3}{8}\right)^{-1}\left(\frac{3}{2}\right)^3 + \left(-\frac{1}{3}\right)^{-1}}$$

$$= \frac{1 - \frac{1}{0.1}}{\left(\frac{8}{3}\right)\left(\frac{3}{2}\right)^3 + (-3)^1}$$

$$= \frac{1-10}{\frac{8}{3} \times \frac{3^3}{2^3} - 3}$$

$$= \frac{-9}{3^2 - 3}$$

$$= \frac{-9}{9-3} = -\frac{9}{6} = -\frac{3}{2}$$

(v) We have,

$$\sqrt{\frac{1}{4}} + (0.01)^{-\frac{1}{2}} - (27)^{\frac{2}{3}}$$

$$= \frac{1}{2} + \frac{1}{(0.01)^{\frac{1}{2}}} - \left(3^3\right)^{\frac{2}{3}}$$

$$= \frac{1}{2} + \frac{1}{(0.1)^{2\times\frac{1}{2}}} - 3^{3\times\frac{2}{3}}$$

$$= \frac{1}{2} + \frac{1}{0.1} - 3^2$$

$$= \frac{1}{2} + 10 - 9$$

$$= \frac{1}{2} + 1 = \frac{3}{2}$$

$$\Rightarrow \sqrt{\frac{1}{4}}(0.01)^{-\frac{1}{2}} - (27)^{\frac{2}{3}} = \frac{3}{2}$$

(vi) We have,

$$\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n} = \frac{2^n + 2^n \times 2^{-1}}{2^n \times 2^1 - 2^n}$$

$$= \frac{2^n\left[1 + 2^{-1}\right]}{2^n\left[2-1\right]}$$

$$= \frac{1 + \frac{1}{2}}{1}$$

$$= 1 + \frac{1}{2}$$

$$= \frac{3}{2}$$

$$\Rightarrow \frac{2^n + 2^{n-1}}{2^{n+1} - 2^n} = \frac{3}{2}$$

(vii) We have,

$$\left(\frac{64}{125}\right)^{-\frac{2}{3}} + \frac{1}{\left(\frac{256}{625}\right)^{\frac{1}{4}}} + \left(\frac{\sqrt{25}}{\sqrt[3]{64}}\right)$$

$$= \left(\frac{125}{64}\right)^{\frac{2}{3}} + \frac{1}{\left(\frac{4^4}{5^4}\right)^{\frac{1}{4}}} + \left(\frac{5}{(64)^{\frac{1}{3}}}\right)$$

$$= \left(\frac{5^3}{4^3}\right)^{\frac{2}{3}} + \frac{1}{\frac{4}{5}} + \left(\frac{5}{4^{3\times\frac{1}{3}}}\right)$$

$$= \frac{5^2}{4^2} + \frac{5}{4} + \frac{5}{4}$$

$$= \frac{25}{16} + \frac{10}{4}$$

$$= \frac{25+40}{16} = \frac{65}{16}$$

(viii) We have,

$$\frac{3^{-3} \times 6^2 \times \sqrt{98}}{5^2 \times \sqrt[3]{\frac{1}{25}} \times (15)^{-\frac{4}{3}} \times 3^{\frac{1}{3}}}$$

$$= \frac{3^{-3} \times 36 \times \sqrt{7 \times 7 \times 2}}{5^2 \times \left(\frac{1}{25}\right)^{\frac{1}{3}} \times (15)^{-\frac{4}{3}} \times 3^{\frac{1}{3}}}$$

$$= \frac{3^{-3} \times 36 \times 7\sqrt{2}}{5^2 \times \left(\frac{1}{5^{2\times\frac{1}{3}}}\right) \times \frac{1}{(15)^{\frac{4}{3}}} \times 3^{\frac{1}{3}}}$$

$$= \frac{3^{-3} \times 36 \times 7\sqrt{2}}{5^2 \times 5^{-\frac{2}{3}} \times \frac{1}{(5\times 3)^{\frac{4}{3}}} \times 3^{\frac{1}{3}}}$$

$$= \frac{3^{-3} \times 36 \times 7\sqrt{2}}{\left(5^2 \times 5^{-\frac{2}{3}} \times 5^{-\frac{4}{3}}\right) \times 3^{-\frac{4}{3}} \times 3^{\frac{1}{3}}}$$

$$= \frac{3^{-3} \times 36 \times 7\sqrt{2} \times 3^{\frac{4}{3}} \times 3^{-\frac{1}{3}}}{(5)^{2-\frac{2}{3}-\frac{4}{3}}}$$

$$= \frac{3^{-3} \times 36 \times 7\sqrt{2} \times 3^{\frac{4}{3}} \times 3^{-\frac{1}{3}}}{(5)^{\frac{6-2-4}{3}}}$$

$$= \frac{3^{-3+\frac{4}{3}-\frac{1}{3}} \times 36 \times 7\sqrt{2}}{5^0}$$

$$= 3^{-3+\left(\frac{4-1}{3}\right)} \times 36 \times 7\sqrt{2}$$

$$= 3^{-3+\frac{3}{3}} \times 36 \times 7\sqrt{2}$$

$$= 3^{-3+1} \times 36 \times 7\sqrt{2}$$

$$= 3^{-2} \times 36 \times 7\sqrt{2}$$

$$= \frac{1}{9} \times 36 \times 7\sqrt{2}$$

$$= 4 \times 7\sqrt{2}$$

$$= 28\sqrt{2}$$

$$\Rightarrow \frac{3^{-3} \times 6^2 \times \sqrt{98}}{5^2 \times \sqrt[3]{\frac{1}{25}} \times (15)^{-\frac{4}{3}} \times 3^{\frac{1}{3}}} = 28\sqrt{2}$$

4. If $27^x = \frac{9}{3^x}$, find x

Sol:

We have,

$$\left(27^x\right) = \frac{9}{3^x}$$

$$\Rightarrow \quad \left(3^3\right)^x = \frac{9}{3^x}$$

$$\Rightarrow \quad 3^{3\times x} = \frac{3^2}{3^x}$$

$$\Rightarrow \quad 3^{3x} = 3^{2-x}$$

$$\Rightarrow \quad 3x = 2 - x \qquad \text{[On equating exponents]}$$

$\Rightarrow 3x + x = 2$

$\Rightarrow 4x = 2 \Rightarrow x = \frac{2}{4} \Rightarrow \boxed{x = \frac{1}{2}}$

Hence, value of x is $\frac{1}{2}$

5. Find the values of x in each of the following:

(i) $2^{5x} \div 2^{x} = \sqrt[5]{2^{20}}$

(ii) $\left(2^3\right)^4 = \left(2^2\right)^x$

(iii) $\left(\frac{3}{5}\right)^x \left(\frac{5}{x}\right)^{2x} = \frac{125}{27}$

(iv) $5^{x-2} \times 3^{2x-3} = 135$

(v) $2^{x-5} \times 5^{x-4} = 5$

(vi) $2^{x-7} \times 5^{x-4} = 1250$

Sol:

(i) We have

$2^{5x} \div 2^{x} = \sqrt[5]{2^{20}}$

$\frac{2^{5x}}{2^x} = \left(2^{20}\right)^{\frac{1}{5}}$

$2^{5x-x} = 2^{20\times\frac{1}{5}}$

$2^{4x} = 2^4$

$\Rightarrow 4x = 4$ [On equating exponents]

$\Rightarrow \boxed{x = 1}$

Hence value of x is 1

(ii) We have,

$\left(2^3\right)^4 = \left(2^2\right)^x$

$\Rightarrow 2^{3\times4} = 2^{2\times x}$

$\Rightarrow 12 = 2x$ [On equating exponents]

$\Rightarrow 2x = 12$

$\Rightarrow \boxed{x = 6}$

Hence, value of x is 6.

(iii) We have,

$$\left(\frac{3}{5}\right)^x\left(\frac{5}{x}\right)^{2x}=\frac{125}{27}$$

$$\Rightarrow \frac{(3)^x}{(5)^x}\frac{(5)^{2x}}{(3)^{2x}}=\frac{5^3}{3^3}$$

$$\Rightarrow \frac{5^{2x-x}}{3^{2x-x}}=\left(\frac{5}{3}\right)^3$$

$$\Rightarrow \frac{5^x}{3^x}=\left(\frac{5}{3}\right)^3$$

$$\Rightarrow \left(\frac{5}{3}\right)^x=\left(\frac{5}{3}\right)^3$$ [On equating exponents]

$$\Rightarrow \boxed{x=3}$$

Hence, value of x is 3

(iv) We have

$$5^{x-2}\times 3^{2x-3}=135$$

$$\Rightarrow 5^{x-2}\times 3^{2x-3}=5\times 27$$

$$\Rightarrow 5^{x-2}\times 3^{2x-3}=5^1\times 3^3$$

$$\Rightarrow x-2=1,\quad 2x-3=3$$ [On equating exponents]

$$\Rightarrow x=2+1,\quad 2x=3+3$$

$$\Rightarrow x=3,\qquad 2x=6\Rightarrow x=3$$

Hence, the value of x is 3

(v) We have,

$$2^{x-5}\times 5^{x-4}=5$$

$$\Rightarrow 2^{x-5}\times 5^{x-4}=5^1\times 2^0$$

$$\Rightarrow x-5=0,\quad x-4=1$$

$$\Rightarrow x=5,\qquad x=4+1$$

$$\Rightarrow \boxed{x=5}$$

Hence, the value of x is 5

(vi) We have,

$$2^{x-7}\times 5^{x-4}=1250$$

$$\Rightarrow 2^{x-7}\times 5^{x-4}=2\times 625$$

$$\Rightarrow 2^{x-7}\times 5^{x-4}=2^1\times 5^4$$

$$\Rightarrow x-7=1$$

$$\Rightarrow x=8,\quad x-4=4$$

$$\Rightarrow \boxed{x=8}$$

Hence, the value of x is 8

Exercise – 3.1

1. Simplify each of the following:

(i) $\sqrt[3]{4}\times\sqrt[3]{16}$

(ii) $\dfrac{\sqrt[4]{1250}}{\sqrt[4]{2}}$

Sol:

(i) $\sqrt[3]{4}\times\sqrt[3]{16}$

$\Rightarrow \sqrt[3]{4\times16}$ $\qquad$ $\boxed{\because \sqrt[n]{a}\times\sqrt[n]{b}=\sqrt[n]{ab}}$

$\Rightarrow \sqrt[3]{64}$

$\Rightarrow \sqrt[3]{4^3} \Rightarrow \left(4^3\right)^{\frac{1}{3}} \Rightarrow 4^{3\times\frac{1}{3}} \Rightarrow 4^1 \Rightarrow 4$

$\therefore \sqrt[3]{4}\times\sqrt[3]{16}=4$

(ii) $\dfrac{\sqrt[4]{1250}}{\sqrt[4]{2}}$

$\Rightarrow \sqrt[4]{\dfrac{1250}{2}}$ $\qquad$ $\boxed{\because \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}=\sqrt[n]{\dfrac{a}{b}}}$

$\Rightarrow \sqrt[4]{\dfrac{625\times2}{2}} \Rightarrow \sqrt[4]{625} \Rightarrow \sqrt[4]{5^4} \Rightarrow \left(5^4\right)^{\frac{1}{4}}$

$\Rightarrow 5^{4\times\frac{1}{4}} \Rightarrow 5^1=5$

$\therefore \sqrt[4]{\dfrac{1250}{2}}=5$

2. Simplify the following expressions:

(i) $\left(4+\sqrt{7}\right)\left(3+\sqrt{2}\right)$

(ii) $\left(3+\sqrt{3}\right)\left(5-\sqrt{2}\right)$

(iii) $\left(\sqrt{5}-2\right)\left(\sqrt{3}-\sqrt{5}\right)$

Sol:

(i) We have

$$\left(4+\sqrt{7}\right)\left(3+\sqrt{2}\right)=4\times3+4\times\sqrt{2}+\sqrt{7}\times3+\sqrt{7}\times\sqrt{2}$$

$$=12+4\sqrt{2}+3\sqrt{7}+\sqrt{7\times2}$$

$$= 12 + 4\sqrt{2} + 3\sqrt{7} + \sqrt{14} \qquad \boxed{\because \sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}}$$

$$\therefore \left(4+\sqrt{7}\right)\left(3+\sqrt{2}\right) = 12 + 4\sqrt{2} + 3\sqrt{7} + \sqrt{14}$$

(ii) We have,

$$\left(3+\sqrt{3}\right)\left(5-\sqrt{2}\right) = 3\times5 + 3\times\left(-\sqrt{2}\right) + \sqrt{3}\times5 + \sqrt{3}\times\left(-\sqrt{2}\right)$$

$$= 15 - 3\sqrt{2} + 5\sqrt{3} - \sqrt{3\times2}$$

$$= 15 - 3\sqrt{2} + 5\sqrt{3} - \sqrt{6} \qquad \boxed{\because \sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}}$$

$$\therefore \left(3+\sqrt{3}\right)\left(5-\sqrt{2}\right) = 15 - 3\sqrt{2} + 5\sqrt{3} - \sqrt{6}$$

(iii) We have

$$\left(\sqrt{5}-2\right)\left(\sqrt{3}-\sqrt{5}\right) = \sqrt{5}\times\sqrt{3} + \sqrt{5}\times\left(-\sqrt{5}\right) + (-2)\times\sqrt{3}$$

$$= \sqrt{5\times3} - \sqrt{5\times5} - 2\sqrt{3} + 2\sqrt{5}$$

$$= \sqrt{15} - \sqrt{5^2} - 2\sqrt{3} + 2\sqrt{5} \qquad \boxed{\because \sqrt[n]{a} + \sqrt[n]{b} = \sqrt[n]{ab}}$$

$$= \sqrt{15} - \sqrt{5^2} - 2\sqrt{3} + 2\sqrt{5}$$

$$= \sqrt{15} - 5 - 2\sqrt{3} + 2\sqrt{5}$$

$$\therefore \left(\sqrt{5}-2\right)\left(\sqrt{3}-\sqrt{5}\right) = \sqrt{15} - 2\sqrt{3} + 2\sqrt{5} - 5$$

3. Simplify the following expressions:

(i) $\left(11+\sqrt{11}\right)\left(11-\sqrt{11}\right)$

(ii) $\left(5+\sqrt{7}\right)\left(5-\sqrt{7}\right)$

(iii) $\left(\sqrt{8}-\sqrt{2}\right)\left(\sqrt{8}+\sqrt{2}\right)$

(iv) $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$

(v) $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)$

Sol:

(i) We have,

$$\left(11+\sqrt{11}\right)\left(11-\sqrt{11}\right) = (11)^2 - \left(\sqrt{11}\right)^2 \qquad \boxed{\because (a+b)(a-b) = a^2 - b^2}$$

$$= 121 - 11$$

$$= 110$$

$$\therefore \left(11+\sqrt{11}\right)\left(11-\sqrt{11}\right) = 110$$

(ii) We have,

$$\left(5+\sqrt{7}\right)\left(5-\sqrt{7}\right) = 5^2 - \left(\sqrt{7}\right)^2 \qquad \boxed{\because (a+b)(a-b) = a^2 - b^2}$$

$= 25 - 7 = 18$

$\therefore \left(5+\sqrt{7}\right)\left(5-\sqrt{7}\right) = 18$

(iii) We have,

$\left(\sqrt{8}-\sqrt{2}\right)\left(\sqrt{8}+\sqrt{2}\right) = \left(\sqrt{8}\right)^2 - \left(\sqrt{2}\right)^2$ $\boxed{\because (a+b)(a-b) = a^2 - b^2}$

$= 8 - 2 = 6$

$\therefore \left(\sqrt{8}-\sqrt{2}\right)\left(\sqrt{8}+\sqrt{2}\right) = 6$

(iv) We have,

$\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right) = (3)^2 - \left(\sqrt{3}\right)^2$ $\boxed{\because (a+b)(a-b) = a^2 - b^2}$

$= 9 - 3 = 6$

$\therefore \left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right) = 6$

(v) We have,

$\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right) = \left(\sqrt{5}\right)^2 - \left(\sqrt{2}\right)^2$ $\boxed{\because (a-b)(a+b) = a^2 - b^2}$

$= 5 - 2 = 3$

$\therefore \left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right) = 3$

4. Simplify the following expressions:

(i) $\left(\sqrt{3}+\sqrt{7}\right)^2$

(ii) $\left(\sqrt{5}-\sqrt{3}\right)^2$

(iii) $\left(2\sqrt{5}+3\sqrt{2}\right)^2$

Sol:

(i) $\left(\sqrt{3}+\sqrt{7}\right)^2 = \left(\sqrt{3}\right)^2 + 2\times\sqrt{3}\times\sqrt{7} + \left(\sqrt{7}\right)^2$ $\boxed{\because (a+b)^2 = a^2 + 2ab + b^2}$

$= 3 + 2\sqrt{3\times 7} + 7$ $\boxed{\because \sqrt[n]{a}\times\sqrt[n]{b} = \sqrt[n]{ab}}$

$= 10 + 2\sqrt{21}$

$\therefore \left(\sqrt{3}+\sqrt{7}\right)^2 = 10 + 2\sqrt{21}$

(ii) We have

$\left(\sqrt{5}-\sqrt{3}\right)^2 = \left(\sqrt{5}\right)^2 - 2\times\sqrt{5}\times\sqrt{3} + \left(\sqrt{3}\right)^2$

$= 5 - 2\sqrt{5\times 3} + 3$ $\boxed{\because (a-b)^2 = a^2 - 2ab + b^2}$

$= 8 - 2\sqrt{15}$ $\boxed{\because \sqrt[n]{a}\times\sqrt[n]{b} = \sqrt[n]{ab}}$

$$\therefore\left(\sqrt{5}-\sqrt{3}\right)^2=8-2\sqrt{15}$$

(iii) We have

$$\left(2\sqrt{5}+3\sqrt{2}\right)^2=\left(2\sqrt{5}\right)^2+2\times\left(2\sqrt{5}\right)\times\left(3\sqrt{2}\right)+\left(3\sqrt{2}\right)^2 \quad \left[\because\left(a+b\right)^2=a^2+2ab+b^2\right]$$

$$=2^2\times\left(\sqrt{5}\right)^2+\left(2\times2\times3\right)\times\sqrt{5\times2}+3^2\left(\sqrt{2}\right)^2$$

$$=4\times5+12\times\sqrt{10}+9\times2 \quad \boxed{\because\left(ab\right)^n=a^n\times b^n \text{ and}}$$

$$=20+12\sqrt{10}+18 \quad \boxed{\sqrt[n]{a}\times\sqrt[n]{b}=\sqrt[n]{ab}}$$

$$=38+12\sqrt{10}$$

$$\therefore\left(2\sqrt{5}+3\sqrt{2}\right)^2=38+12\sqrt{10}$$

Exercise – 3.2

1. Rationalise the denominator of each of the following (i – vii) :

(i) $\frac{3}{\sqrt{5}}$

(ii) $\frac{3}{2\sqrt{5}}$

(iii) $\frac{1}{\sqrt{12}}$

(iv) $\frac{\sqrt{2}}{\sqrt{5}}$

(v) $\frac{\sqrt{3}+1}{\sqrt{2}}$

(vi) $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}$

(vii) $\frac{3\sqrt{2}}{\sqrt{5}}$

Sol:

(i) $\frac{3}{5}\sqrt{5}$

(ii) $\frac{3}{10}\sqrt{5}$

(iii) $\frac{\sqrt{3}}{6}$

(iv) $\frac{1}{5}\sqrt{10}$

(v) $\frac{\sqrt{6}+\sqrt{2}}{2}$

(vi) $\frac{\sqrt{6}+\sqrt{5}}{3}$

(vii) $\frac{3\sqrt{10}}{5}$

2. Find the value to three places of decimals of each of the following. It is given that $\sqrt{2}=1.414, \sqrt{3}=1.732, \sqrt{5}=2.236$ *and* $\sqrt{10}=3.162.$

(i) $\frac{2}{\sqrt{3}}$

(ii) $\frac{3}{\sqrt{10}}$

(iii) $\frac{\sqrt{5}+1}{\sqrt{2}}$

(iv) $\frac{\sqrt{10}+\sqrt{15}}{\sqrt{2}}$

(v) $\frac{2+\sqrt{3}}{3}$

(vi) $\frac{\sqrt{2}-1}{\sqrt{5}}$

Sol:

Given that $\sqrt{2}=1.414, \sqrt{3}=1.732, \sqrt{5}=2.236$ and $\sqrt{10}=3.162.$

(i) We have $\frac{2}{\sqrt{3}}$

Rationalising factor of denominator is $\sqrt{3}$

$$\therefore \frac{2}{\sqrt{3}}=\frac{2}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{2\sqrt{3}}{\left(\sqrt{3}\right)^2}=\frac{2\sqrt{3}}{3}=\frac{2\times1.732}{3}=\frac{3.464}{3}$$

$=1.15466667$

$=1.154$

(ii) We have $\frac{3}{\sqrt{10}}$

Rationalising factor of denominator is $\sqrt{10}$

$$\therefore \frac{3}{\sqrt{10}}=\frac{3}{\sqrt{10}}\times\frac{\sqrt{10}}{\sqrt{10}}=\frac{3\sqrt{10}}{\left(\sqrt{10}\right)^2}=\frac{3\sqrt{10}}{10}=\frac{3\times3.162}{10}=\frac{9.486}{10}$$

$=0.9486$

$=0.948$

$$\therefore \frac{3}{\sqrt{10}}=0.948$$

(iii) We have $\frac{\sqrt{5}+1}{\sqrt{2}}$

Rationalising factor of denominator is $\sqrt{2}$.

$$\therefore \frac{\sqrt{5}+1}{\sqrt{2}}=\frac{\sqrt{5}+1}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$$

$$=\frac{\left(\sqrt{5}+1\right)\sqrt{2}}{\left(\sqrt{2}\right)^2}$$

$$=\frac{\sqrt{5}\times\sqrt{2}+1\times\sqrt{2}}{2}$$

$$=\frac{\sqrt{5\times2}+\sqrt{2}}{2}=\frac{\sqrt{10}+\sqrt{2}}{2}$$

$$=\frac{3.162+1.414}{2}=\frac{4.576}{2}=2.288$$

$$\therefore \frac{\sqrt{5}+1}{\sqrt{2}}=2.288$$

(iv) We have $\dfrac{\sqrt{10}+\sqrt{15}}{\sqrt{2}}$

Rationalising factor of denominator is $\sqrt{2}$

$$\therefore \frac{\sqrt{10}+\sqrt{15}}{\sqrt{2}} = \frac{\sqrt{10}+\sqrt{15}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\left(\sqrt{10}+\sqrt{15}\right)\sqrt{2}}{\left(\sqrt{2}\right)^2}$$

$$= \frac{\sqrt{10}\times\sqrt{2}+\sqrt{15}\times\sqrt{2}}{2}$$

$$= \frac{\sqrt{10\times 2}+\sqrt{15\times 2}}{2}$$

$$= \frac{\sqrt{20}+\sqrt{30}}{2} = \frac{\sqrt{2\times 10}+\sqrt{3\times 10}}{2}$$

$$= \frac{\sqrt{2}\times\sqrt{10}+\sqrt{3}\times\sqrt{10}}{2}$$

$$= \frac{(1\cdot 414\times 3\cdot 162)+(1\cdot 732\times 3\cdot 162)}{2}$$

$$= \frac{4\cdot 471068+5\cdot 476584}{2}$$

$$\Rightarrow \frac{4\cdot 471068+5\cdot 476584}{2}$$

$$\Rightarrow \frac{9\cdot 947652}{2} = 4.973826 \simeq 4.973$$

(v) We have $\dfrac{2+\sqrt{3}}{3}$

$$\Rightarrow \frac{2+1\cdot 732}{3} = \frac{3\cdot 732}{3} = 1\cdot 244$$

(vi) We have $\dfrac{\sqrt{2}-1}{\sqrt{5}}$

Rationalising factor for $\dfrac{1}{\sqrt{5}}$ is $\sqrt{5}$

$$\Rightarrow \frac{\sqrt{2}-1}{\sqrt{5}} = \frac{\sqrt{2}-1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\left(\sqrt{2}\times\sqrt{5}\right)-\left(1\times\sqrt{5}\right)}{\left(\sqrt{5}\right)^2}$$

$$= \frac{\sqrt{2\times5}-\sqrt{5}}{5}$$

$$= \frac{\sqrt{10}-\sqrt{5}}{5}$$

$$= \frac{3\cdot162-2\cdot236}{5} = \frac{0\cdot926}{5} = 0\cdot1852$$

$$\simeq 0.185$$

$$\therefore \frac{\sqrt{2}-1}{\sqrt{5}} = 0\cdot185$$

3. Express each one of the following with rational denominator:

(i) $\frac{1}{3+\sqrt{2}}$

(ii) $\frac{1}{\sqrt{6}-\sqrt{5}}$

(iii) $\frac{16}{\sqrt{41}-5}$

(iv) $\frac{30}{5\sqrt{3}-3\sqrt{5}}$

(v) $\frac{1}{2\sqrt{5}-\sqrt{3}}$

(vi) $\frac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}}$

(vii) $\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$

(viii) $\frac{3\sqrt{2}+1}{2\sqrt{5}-3}$

(ix) $\frac{b^2}{\sqrt{a^2+b^2}+a}$

Sol:

(i) We have $\frac{1}{3+\sqrt{2}}$

Rationalising factor for $\frac{1}{3+\sqrt{2}}$ is $3-\sqrt{2}$

$$\Rightarrow \frac{1}{3+\sqrt{2}} = \frac{1}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}}$$

$$= \frac{3-\sqrt{2}}{(3)^2 - \left(\sqrt{2}\right)^2} \qquad \boxed{\because (a+b)(a-b) = a^2 - b^2}$$

$$= \frac{3-\sqrt{2}}{9-2} = \frac{3-\sqrt{2}}{7} \qquad \therefore \frac{1}{3+\sqrt{2}} = \frac{3-\sqrt{2}}{7}$$

(ii) We have $\frac{1}{\sqrt{6}-\sqrt{5}}$

Rationalising factor for $\frac{1}{\sqrt{6}-\sqrt{5}}$ is $\sqrt{6}+\sqrt{5}$

$$\Rightarrow \frac{1}{\sqrt{6}-\sqrt{5}} = \frac{1}{\sqrt{6}-\sqrt{5}} \times \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}$$

$$= \frac{\sqrt{6}+\sqrt{5}}{\left(\sqrt{6}-\sqrt{5}\right)\left(\sqrt{6}+\sqrt{5}\right)}$$

$$= \frac{\sqrt{6}+\sqrt{5}}{\left(\sqrt{6}\right)^2 - \left(\sqrt{5}\right)^2} \qquad \boxed{\because (a+b)(a-b) = a^2 - b^2}$$

$$= \frac{\sqrt{6}+\sqrt{5}}{6-5}$$

$$= \frac{\sqrt{6}+\sqrt{5}}{1} = \sqrt{6}+\sqrt{5}$$

$$\therefore \frac{1}{\sqrt{6}-\sqrt{5}} = \sqrt{6}+\sqrt{5}$$

(iii) We have $\frac{16}{\sqrt{41}-5}$

Rationalisation factor for $\frac{1}{\sqrt{41}-5}$ is $\left(\sqrt{41}-5\right)$

$$\Rightarrow \frac{16}{\sqrt{41}-5} = \frac{16}{\sqrt{41}-5} \times \frac{\sqrt{41}+5}{\sqrt{41}+5}$$

$$= \frac{16\left(\sqrt{41}+5\right)}{\left(\sqrt{41}-5\right)\left(\sqrt{41}+5\right)}$$

$$= \frac{16\left(\sqrt{41}+5\right)}{\left(\sqrt{41}\right)^2-(5)^2} \qquad \boxed{\because (a-b)(a+b)=a^2-b^2}$$

$$= \frac{16\left(\sqrt{41}+5\right)}{41-25} = \frac{16\left(\sqrt{41}+5\right)}{16} = \sqrt{41}+5$$

$$\therefore \frac{16}{41-5} = \sqrt{41}+5$$

(iv) We have $\frac{30}{5\sqrt{3}-3\sqrt{5}}$

Rationalisation factor for $\frac{1}{5\sqrt{3}-3\sqrt{5}}$ is $5\sqrt{3}+3\sqrt{5}$

$$\Rightarrow \frac{30}{5\sqrt{3}-3\sqrt{5}} = \frac{30}{5\sqrt{3}-3\sqrt{5}} \times \frac{5\sqrt{3}+3\sqrt{5}}{5\sqrt{3}+3\sqrt{5}}$$

$$= \frac{30\left(5\sqrt{3}+3\sqrt{5}\right)}{\left(5\sqrt{3}-3\sqrt{5}\right)\left(5\sqrt{3}+3\sqrt{5}\right)}$$

$$= \frac{30\left(5\sqrt{3}+3\sqrt{5}\right)}{\left(5\sqrt{3}\right)^2-\left(3\sqrt{5}\right)^2} \qquad \boxed{\because (a-b)(a-b)=a^2-b^2}$$

$$= \frac{30\left(5\sqrt{3}+3\sqrt{5}\right)}{5^2\left(\sqrt{3}\right)^2-3^2\left(\sqrt{5}\right)^2}$$

$$= \frac{30\left(5\sqrt{3}+3\sqrt{5}\right)}{25\times3-9\times5} = \frac{30\left(5\sqrt{3}+3\sqrt{5}\right)}{75-45} = \frac{30\left(5\sqrt{3}+3\sqrt{5}\right)}{30}$$

$$= 5\sqrt{3}+3\sqrt{5}$$

$$\therefore \frac{30}{5\sqrt{3}-3\sqrt{5}} = 5\sqrt{3}+3\sqrt{5}$$

(v) We have $\frac{1}{2\sqrt{5}-\sqrt{3}}$

Rationalisation factor for $\frac{1}{2\sqrt{5}-\sqrt{3}}$ is $2\sqrt{5}+\sqrt{3}$

$$\Rightarrow \frac{1}{2\sqrt{5}-\sqrt{3}} = \frac{1}{2\sqrt{5}-\sqrt{3}} \times \frac{2\sqrt{5}+\sqrt{3}}{2\sqrt{5}+\sqrt{3}}$$

$$= \frac{2\sqrt{5}+\sqrt{3}}{\left(2\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2} \qquad \boxed{\because (a-b)(a+b)=a^2-b^2}$$

$$= \frac{2\sqrt{5}+\sqrt{3}}{2^2\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$$

$$= \frac{2\sqrt{5}+\sqrt{3}}{4\times5-3} = \frac{2\sqrt{5}+\sqrt{3}}{20-3} = \frac{2\sqrt{5}+\sqrt{3}}{17}$$

$$\therefore \frac{1}{2\sqrt{5}-\sqrt{3}} = \frac{2\sqrt{5}+\sqrt{3}}{17}$$

(vi) We have $\frac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}}$

Rationalisation factor for $\frac{1}{2\sqrt{2}-\sqrt{3}}$ is $2\sqrt{2}+\sqrt{3}$

$$\Rightarrow \frac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}} = \frac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}} \times \frac{2\sqrt{2}+\sqrt{3}}{2\sqrt{2}+\sqrt{3}}$$

$$= \frac{\left(\sqrt{3}+1\right)\left(2\sqrt{2}+\sqrt{3}\right)}{\left(2\sqrt{2}-\sqrt{3}\right)\left(2\sqrt{2}+\sqrt{3}\right)}$$

$$= \frac{\sqrt{3}\times2\sqrt{2}+\sqrt{3}\times\sqrt{3}+1\times2\sqrt{2}+1\times\sqrt{3}}{\left(2\sqrt{2}\right)^2-\left(\sqrt{3}\right)^2}$$

$\because (a-b)(a+b)=a^2-b^2$

$$= \frac{2\sqrt{2\times3}+\sqrt{3\times3}+2\sqrt{2}+\sqrt{3}}{2^2\left(\sqrt{2}\right)^2-\left(\sqrt{3}\right)^2}$$

$$= \frac{2\sqrt{6}+3+2\sqrt{2}+\sqrt{3}}{4\times2-3} = \frac{2\sqrt{6}+3+2\sqrt{2}+\sqrt{3}}{8-3}$$

$$= \frac{2\sqrt{6}+3+2\sqrt{2}+\sqrt{3}}{5}$$

$$\therefore \frac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}} = \frac{2\sqrt{6}+3+2\sqrt{2}+\sqrt{3}}{5}$$

(vii) We have $\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$

Rationalisation factor for $\frac{1}{6+4\sqrt{2}}$ is $6-4\sqrt{2}$

$$\Rightarrow \frac{6-4\sqrt{2}}{6+4\sqrt{2}} = \frac{6-4\sqrt{2}}{6+4\sqrt{2}} \times \frac{6-4\sqrt{2}}{6+4\sqrt{2}}$$

$$= \frac{\left(6-4\sqrt{2}\right)^2}{(6)^2-\left(4\sqrt{2}\right)^2}$$

$\because (a+b)(a-b)=a^2-b^2$

$(a-b)(a-b)=(a-b)^2$

$$= \frac{6^2 - 2\times 6\times 4\sqrt{2} + \left(4\sqrt{2}\right)^2}{36 - 4^2\left(\sqrt{2}\right)^2} \qquad \boxed{(a-b)^2 = a^2 - 2ab + b^2}$$

$$= \frac{36 - 48\sqrt{2} + 32}{36 - 32}$$

$$= \frac{68 - 48\sqrt{2}}{4} = \frac{4\left(17 - 12\sqrt{2}\right)}{4} = 17 - 12\sqrt{2}$$

$$\therefore \frac{6 - 4\sqrt{2}}{6 + 4\sqrt{2}} = 17 - 12\sqrt{2}$$

(viii) We have $\frac{3\sqrt{2}+1}{2\sqrt{5}-3}$.

Rationalising factor for $\frac{1}{2\sqrt{5}-3}$ is $2\sqrt{5}+3$

$$\Rightarrow \frac{3\sqrt{2}+1}{2\sqrt{5}-3} = \frac{3\sqrt{2}+1}{2\sqrt{5}-3} \times \frac{2\sqrt{5}+3}{2\sqrt{5}+3}$$

$$= \frac{\left(3\sqrt{2}+1\right)\left(2\sqrt{5}+3\right)}{\left(2\sqrt{5}-3\right)\left(2\sqrt{5}+3\right)}$$

$$= \frac{3\sqrt{2}\times 2\sqrt{5} + 3\sqrt{2}\times 3 + 1\times 2\sqrt{5} + 1\times 3}{\left(2\sqrt{5}\right)^2 - (3)^2} \qquad \boxed{(a-b)(a+b) = a^2 - b^2}$$

$$= \frac{6\sqrt{10} + 9\sqrt{2} + 2\sqrt{5} + 3}{20 - 9} = \frac{6\sqrt{10} + 9\sqrt{2} + 2\sqrt{5} + 3}{11}$$

$$\therefore \frac{3\sqrt{2}+1}{2\sqrt{5}-3} = \frac{6\sqrt{10} + 9\sqrt{2} + 2\sqrt{5} + 3}{11}$$

(ix) We have $\frac{b^2}{\sqrt{a^2+b^2}+a}$

Rationalisation factor for $\frac{1}{\sqrt{a^2+b^2}+a}$ is $\sqrt{a^2+b^2}-a$

$$\Rightarrow \frac{b^2}{\sqrt{a^2+b^2}+a} = \frac{b^2}{\sqrt{a^2+b^2}+a} \times \frac{\sqrt{a^2+b^2}-a}{\sqrt{a^2+b^2}-a}$$

$$= \frac{b^2\left(\sqrt{a^2+b^2}-a\right)}{\left(\sqrt{a^2+b^2}\right)^2 - (a)^2} \qquad \boxed{(x+y)(x-y) = x^2 - y^2}$$

$$= \frac{b^2\left(\sqrt{a^2+b^2}-a\right)}{a^2+b^2-a^2}$$

$$= \frac{b^2\left(\sqrt{a^2+b^2}-a\right)}{b^2}$$

$$= \left(\sqrt{a^2+b^2}-a\right)$$

$$\therefore \frac{b^2}{\sqrt{a^2+b^2}+a} = \sqrt{a^2+b^2}-a$$

4. Rationalize the denominator and simplify:

(i) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

(ii) $\frac{5+2\sqrt{3}}{7+4\sqrt{3}}$

(iii) $\frac{1+\sqrt{2}}{3-2\sqrt{2}}$

(iv) $\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}$

(v) $\frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$

(vi) $\frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}$

Sol:

(i) We have $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

Rationalisation factor for $\frac{1}{\sqrt{a}+\sqrt{b}}$ is $\sqrt{a}-\sqrt{b}$

$\Rightarrow$ for $\frac{1}{\sqrt{3}+\sqrt{2}}$ it is $\sqrt{3}-\sqrt{2}$

$$\therefore \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{\left(\sqrt{3}-\sqrt{2}\right)^2}{\left(\sqrt{3}\right)^2+\left(\sqrt{2}\right)^2}$$

$\because (a-b)(a-b) = (a-b)^2$
and $(a+b)(a-b) = a^2-b^2$

$$= \frac{\left(\sqrt{3}\right)^2 - 2\times\sqrt{3}\times\sqrt{2} + \left(\sqrt{2}\right)^2}{3-2} \qquad \boxed{\because (a-b)^2 = a^2 - 2ab + b^2}$$

$$= \frac{3 - 2\sqrt{6} + 2}{1} = 5 - 2\sqrt{6}$$

$$\therefore \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} = 5 - 2\sqrt{6}$$

(ii) We have $\frac{5+2\sqrt{3}}{7+4\sqrt{3}}$

Rationalising factor for $\frac{1}{a+b\sqrt{c}}$ is $a-b\sqrt{c}$

$\Rightarrow$ for $\frac{1}{7+4\sqrt{3}}$ is $7-4\sqrt{3}$

$$\therefore \frac{5+2\sqrt{3}}{7+4\sqrt{3}} = \frac{5+2\sqrt{3}}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}}$$

$$= \frac{5\times7 + 5\times\left(-4\sqrt{3}\right) + 2\sqrt{3}\times7 + 2\sqrt{3}\times\left(-4\sqrt{3}\right)}{7^2 - \left(4\sqrt{3}\right)^2} \qquad \boxed{\because (a+b)(a-b) = a^2 - b^2}$$

$$= \frac{35 - 20\sqrt{3} + 14\sqrt{3} - 8\times3}{49-48}$$

$$= \frac{35 - 24 - 6\sqrt{3}}{1} = 11 - 6\sqrt{3}$$

$$\therefore \frac{5+2\sqrt{3}}{7+4\sqrt{3}} = 11 - 6\sqrt{3}$$

(iii) We have $\frac{1+\sqrt{2}}{3-2\sqrt{2}}$

Rationalisation factor for $\frac{1}{a-b\sqrt{c}}$ is $a+b\sqrt{c}$

$\Rightarrow$ for $\frac{1}{3-2\sqrt{2}}$ is $3+2\sqrt{2}$

$$\therefore \frac{1+\sqrt{2}}{3-2\sqrt{2}} = \frac{1+\sqrt{2}}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}}$$

$$= \frac{1\times3 + 1\times2\sqrt{2} + \sqrt{2}\times3 + \sqrt{2}\times2\sqrt{2}}{3^2 - \left(2\sqrt{2}\right)^2} \qquad \boxed{\because (a-b)(a+b) = a^2 - b^2}$$

$$= \frac{3+2\sqrt{2}+3\sqrt{2}+2\times 2}{9-8}$$

$$= \frac{3+4+5\sqrt{2}}{1} = \frac{7+5\sqrt{2}}{1} = 7+5\sqrt{2}$$

$$\therefore \frac{1+\sqrt{2}}{3-2\sqrt{2}} = 7+5\sqrt{2}$$

(iv) We have $\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}$

Rationalisation factor for $\frac{1}{a\sqrt{b}-c\sqrt{d}}$ is $a\sqrt{b}+c\sqrt{d}$

$\Rightarrow$ for $\frac{1}{3\sqrt{5}-2\sqrt{6}}$ is $3\sqrt{5}+2\sqrt{6}$

$$\therefore \frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}} = \frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}} \times \frac{3\sqrt{5}+2\sqrt{6}}{3\sqrt{5}+2\sqrt{6}}$$

$$= \frac{2\sqrt{6}\times 3\sqrt{5}+2\sqrt{6}\times 2\sqrt{6}+\left(-\sqrt{5}\right)\left(3\sqrt{5}\right)+\left(-\sqrt{5}\right)\left(2\sqrt{6}\right)}{\left(3\sqrt{5}\right)^2-\left(2\sqrt{6}\right)^2}$$

$\boxed{\because (a-b)(a+b) = a^2-b^2}$

$$= \frac{6\sqrt{30}+4\times 6-3\times 5-2\sqrt{5\times 6}}{45-24}$$

$$= \frac{6\sqrt{30}+24-15-2\sqrt{30}}{21}$$

$$= \frac{9+4\sqrt{30}}{21}$$

$$\therefore \frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}} = \frac{9+4\sqrt{30}}{21}$$

(v) We have $\frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$

Rationalisation factor for $\frac{1}{\sqrt{a}+\sqrt{b}}$ is $\sqrt{a}-\sqrt{b}$

$\Rightarrow$ for $\frac{1}{\sqrt{48}+\sqrt{18}}$ is $\sqrt{48}-\sqrt{18}$

$$\therefore \frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}} = \frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}} \times \frac{\sqrt{48}-\sqrt{18}}{\sqrt{48}-\sqrt{18}}$$

$$= \frac{4\sqrt{3}\times\sqrt{48}+4\sqrt{3}\times\left(-\sqrt{18}\right)+5\sqrt{2}\times\sqrt{48}+5\sqrt{2}\times\left(-\sqrt{18}\right)}{\left(\sqrt{48}\right)^2-\left(\sqrt{18}\right)^2} \quad \boxed{\because (a+b)(a-b)=a^2-b^2}$$

$$= \frac{4\sqrt{3}\times\sqrt{3\times16}-4\sqrt{3}\times\sqrt{2\times9}+5\sqrt{2}\times\sqrt{3\times16}-5\sqrt{2}\times\sqrt{2\times9}}{48-18}$$

$$= \frac{4\sqrt{3}\times4\sqrt{3}-4\sqrt{3}\times3\sqrt{2}+5\sqrt{2}\times4\sqrt{3}-5\sqrt{2}\times3\sqrt{2}}{30}$$

$$= \frac{48-12\sqrt{6}+20\sqrt{6}-30}{30}$$

$$= \frac{18+8\sqrt{6}}{30} = \frac{2\left(9+4\sqrt{6}\right)}{30} = \frac{9+4\sqrt{6}}{15}$$

$$\therefore \frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}} = \frac{9+4\sqrt{6}}{\sqrt{15}}$$

(vi) We have $\frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}$

Rationalisation factor for $\frac{1}{a\sqrt{b}+c\sqrt{d}}$ is $a\sqrt{b}-c\sqrt{d}$

$\Rightarrow$ for $\frac{1}{2\sqrt{2}+3\sqrt{3}}$ is $2\sqrt{2}-3\sqrt{3}$

$$\therefore \frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}} = \frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}\times\frac{2\sqrt{2}-3\sqrt{3}}{2\sqrt{2}+3\sqrt{3}}$$

$$= \frac{2\sqrt{3}\times2\sqrt{2}+2\sqrt{3}\times\left(-3\sqrt{3}\right)+\left(-\sqrt{5}\right)\left(2\sqrt{2}\right)+\left(-\sqrt{5}\right)\left(-3\sqrt{3}\right)}{\left(2\sqrt{2}\right)^2-\left(3\sqrt{3}\right)^2} \quad \boxed{\because (a+b)(a-b)=a^2-b^2}$$

$$= \frac{4\sqrt{6}-6\sqrt{3^2}-2\sqrt{10}+3\sqrt{15}}{8-27}$$

$$= \frac{4\sqrt{6}-18-2\sqrt{10}+3\sqrt{15}}{-19}$$

$$= \frac{-\left(18-3\sqrt{15}+2\sqrt{10}-4\sqrt{6}\right)}{-19} = \frac{18-3\sqrt{15}+2\sqrt{10}-4\sqrt{6}}{19}$$

$$\therefore \frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}} = \frac{18-3\sqrt{15}+2\sqrt{10}-4\sqrt{6}}{19}$$

5. Simplify:

(i) $\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}+\frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}$

(ii) $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

(iii) $\frac{7+3\sqrt{5}}{3+\sqrt{5}}-\frac{7-3\sqrt{5}}{3-\sqrt{5}}$

(iv) $\frac{1}{2+\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}+\frac{1}{2-\sqrt{5}}$

(v) $\frac{2}{\sqrt{5}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{2}}-\frac{3}{\sqrt{5}+\sqrt{2}}$

Sol:

(i) We have $\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}+\frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}$

Rationalisation factor for $\frac{1}{3\sqrt{2}+2\sqrt{3}}$ is $3\sqrt{2}-2\sqrt{3}$ and for $\frac{1}{\sqrt{3}-\sqrt{2}}$ is $\sqrt{3}+\sqrt{2}$

$$\Rightarrow \frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}\times\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}+\frac{\sqrt{4\times3}}{\sqrt{3}-\sqrt{2}}\times\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$\Rightarrow \frac{\left(3\sqrt{2}-2\sqrt{3}\right)^2}{\left(3\sqrt{2}\right)^2-\left(2\sqrt{3}\right)^2}+\frac{\left(\sqrt{4}\times\sqrt{3}\right)\left(\sqrt{3}+\sqrt{2}\right)}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}$$

$$\because (a-b)(a-b)=(a-b)^2$$
$$(a+b)(a-b)=a^2-b^2$$

$$\Rightarrow \frac{\left(3\sqrt{2}\right)^2-2\times3\sqrt{2}\times2\sqrt{3}+\left(2\sqrt{3}\right)^2}{3^2\left(\sqrt{2}\right)^2-(2)^2\left(\sqrt{3}\right)^2}+\frac{2\sqrt{3}\times\sqrt{3}+2\sqrt{3}\times\sqrt{2}}{3-2}$$

$$\Rightarrow \frac{18-12\sqrt{6}+12}{18-12}+\frac{6+2\sqrt{6}}{1}$$

$$\Rightarrow \frac{30-12\sqrt{6}}{6}+6+2\sqrt{6}$$

$$\Rightarrow \frac{6\left(5-2\sqrt{6}\right)}{6}+6+2\sqrt{6}$$

$$\Rightarrow 5-2\sqrt{6}+6+2\sqrt{6}\Rightarrow 5+6\Rightarrow 11$$

$$\therefore \frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}+\frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}=11$$

(ii) we have $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

Rationalisation factor for $\frac{1}{\sqrt{a}\pm\sqrt{b}}$ is $\sqrt{a}\mp\sqrt{b}$

$\Rightarrow$ for $\frac{1}{\sqrt{5}-\sqrt{3}}$ it is $\sqrt{5}+\sqrt{3}$ and for $\frac{1}{\sqrt{5}+\sqrt{3}}$ it is $\sqrt{5}-\sqrt{3}$

$$\Rightarrow \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}+\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}\times\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$$

$$\Rightarrow \frac{\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)}+\frac{\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)}$$

$$\Rightarrow \frac{\left(\sqrt{5}+\sqrt{3}\right)^2}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}+\frac{\left(\sqrt{5}+\sqrt{3}\right)^2}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$$

$\because (a+b)(a+b)=(a+b)^2$
$(a-b)(a-b)=(a-b)^2$
$(a+b)(a-b)=a^2-b^2$

$$\Rightarrow \frac{\left(\sqrt{5}\right)^2+2\sqrt{5}\sqrt{3}+\left(\sqrt{3}\right)^2}{5-3}+\frac{\left(\sqrt{5}\right)^2-2\sqrt{5}\sqrt{3}+\left(\sqrt{3}\right)^2}{5-3}$$

$$\Rightarrow \frac{5+2\sqrt{15}+3}{2}+\frac{5-2\sqrt{15}+3}{2}$$

$$\Rightarrow \frac{8+2\sqrt{15}}{2}+\frac{8-2\sqrt{15}}{2}$$

$$\Rightarrow \frac{2\left(4+\sqrt{15}\right)}{2}+\frac{2\left(4-\sqrt{15}\right)}{2}$$

$$\Rightarrow 4+\sqrt{15}+4-\sqrt{15}$$

$$\Rightarrow 4+4\Rightarrow 8$$

$$\therefore \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}=8$$

(iii) We have $\frac{7+3\sqrt{5}}{3+\sqrt{5}}-\frac{7-3\sqrt{5}}{3-\sqrt{5}}$

Rationalisation factor for $\frac{1}{a\pm\sqrt{b}}$ is $a\mp\sqrt{b}$

$\Rightarrow$ for $\frac{1}{3+\sqrt{5}}$ it is $3-\sqrt{5}$ and for $\frac{1}{3-\sqrt{5}}$ it is $3+\sqrt{5}$

$$\Rightarrow \frac{7+3\sqrt{5}}{3+\sqrt{5}}-\frac{7-3\sqrt{5}}{3-\sqrt{5}}$$

$$\Rightarrow \frac{7+3\sqrt{5}}{3+\sqrt{5}}\times\frac{3-\sqrt{5}}{3-\sqrt{5}}-\frac{7-3\sqrt{5}}{3-\sqrt{5}}\times\frac{3+\sqrt{5}}{3+\sqrt{5}}$$

$$\Rightarrow \frac{\left(7+3\sqrt{5}\right)\left(3-\sqrt{5}\right)}{\left(3+\sqrt{5}\right)\left(3-\sqrt{5}\right)}-\frac{\left(7-3\sqrt{5}\right)\left(3+\sqrt{5}\right)}{\left(3-\sqrt{5}\right)\left(3+\sqrt{5}\right)}$$

$$\Rightarrow \frac{7\times3+7\times\left(-\sqrt{5}\right)+3\sqrt{5}\times3+3\sqrt{5}\times\left(-\sqrt{5}\right)}{3^2-\left(\sqrt{5}\right)^2}$$

$$-\frac{7\times3+7\times\sqrt{5}+\left(-3\sqrt{5}\right)\times3+\left(-3\sqrt{5}\right)\times\sqrt{5}}{3^2-\left(\sqrt{5}\right)^2} \qquad \boxed{\because\left(a-b\right)\left(a+b\right)=a^2-b^2}$$

$$\Rightarrow \frac{21-7\sqrt{5}+9\sqrt{5}-3\times5}{9-5}-\frac{21+7\sqrt{5}-9\sqrt{5}-3\times5}{9-5}$$

$$\Rightarrow \frac{21-15+2\sqrt{5}}{4}-\frac{21-15+2\sqrt{5}}{4}$$

$$\Rightarrow \frac{6+2\sqrt{5}}{4}-\frac{6-2\sqrt{5}}{4}$$

$$\Rightarrow \frac{6+2\sqrt{5}-\left(6-2\sqrt{5}\right)}{4}\Rightarrow\frac{6+2\sqrt{5}-6+2\sqrt{5}}{4}\Rightarrow\frac{4\sqrt{5}}{4}\Rightarrow\sqrt{5}$$

$$\therefore \frac{7+3\sqrt{5}}{3+\sqrt{5}}-\frac{7-3\sqrt{5}}{3-\sqrt{5}}=\sqrt{5}$$

(iv) We have $\frac{1}{2+\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}+\frac{1}{2-\sqrt{5}}$

Rationalisation factor for $\frac{1}{a\pm\sqrt{b}}$ is $a\mp\sqrt{b}$

$\Rightarrow$ for $\frac{1}{2+\sqrt{3}}$ it is $2-\sqrt{3}$ and for $\frac{1}{2-\sqrt{5}}$ it is $2+\sqrt{5}$

And also, rationalisation factor for $\frac{1}{\sqrt{a}\pm\sqrt{b}}$ is $\sqrt{a}\mp\sqrt{b}\Rightarrow$ for $\frac{1}{\sqrt{5}-\sqrt{3}}$ it is $\sqrt{5}+\sqrt{3}$

$$\Rightarrow \frac{1}{2+\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}+\frac{1}{2-\sqrt{5}}$$

$$\Rightarrow \frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}+\frac{1}{2-\sqrt{5}}\times\frac{2+\sqrt{5}}{2+\sqrt{5}}$$

$$\Rightarrow \frac{2-\sqrt{3}}{2^2-\left(\sqrt{3}\right)^2}+\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}+\frac{2+\sqrt{5}}{2^2-\left(\sqrt{5}\right)^2} \qquad \boxed{\because\left(a+b\right)\left(a-b\right)=a^2-b^2}$$

$$\Rightarrow \frac{2-\sqrt{3}}{4-3}+\frac{2\sqrt{5}+2\sqrt{3}}{5-3}+\frac{2+\sqrt{5}}{4-5}$$

$$\Rightarrow \frac{2-\sqrt{3}}{1}+\frac{2\sqrt{5}+2\sqrt{3}}{2}+\frac{2+\sqrt{5}}{-1}$$

$$\Rightarrow 2-\sqrt{3}+\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{2}-\left(2+\sqrt{3}\right)$$

$$\Rightarrow 2-\sqrt{3}+\sqrt{5}+\sqrt{3}-2-\sqrt{3}$$

$$\Rightarrow 0$$

$$\therefore \frac{1}{2+\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}+\frac{1}{2-\sqrt{5}}=0$$

(v) We have,

$$\frac{2}{\sqrt{5}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{2}}-\frac{3}{\sqrt{5}+\sqrt{2}}$$

Rationalisation factor for $\frac{1}{\sqrt{a}\pm\sqrt{b}}$ is $\sqrt{a}\mp\sqrt{b} \Rightarrow$ for $\frac{1}{\sqrt{5}+\sqrt{3}}$ it is $\sqrt{5}-\sqrt{3}$

$\Rightarrow$ for $\frac{1}{\sqrt{3}+\sqrt{2}}$ it is $\sqrt{3}-\sqrt{2} \Rightarrow$ for $\frac{1}{\sqrt{5}+\sqrt{2}}$ it is $\sqrt{5}-\sqrt{2}$

$$\Rightarrow \frac{2}{\sqrt{5}+\sqrt{3}}\times\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}-\frac{3}{\sqrt{5}+\sqrt{2}}\times\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

$$\Rightarrow \frac{2\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}+\frac{\sqrt{3}-\sqrt{2}}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}-\frac{3\left(\sqrt{5}-\sqrt{2}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}$$

$$\Rightarrow \frac{2\left(\sqrt{5}-\sqrt{3}\right)}{5-3}+\frac{\sqrt{3}-\sqrt{2}}{3-2}-\frac{3\left(\sqrt{5}-\sqrt{2}\right)}{5-2}$$

$$\Rightarrow \frac{2\left(\sqrt{5}-\sqrt{3}\right)}{2}+\frac{\sqrt{3}-\sqrt{2}}{1}-\frac{3\left(\sqrt{5}-\sqrt{2}\right)}{3}$$

$$\Rightarrow \sqrt{5}-\sqrt{3}+\sqrt{3}-\sqrt{2}-\sqrt{5}+\sqrt{2}$$

$$\Rightarrow 0$$

$$\therefore \frac{2}{5+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{2}}-\frac{3}{\sqrt{5}+\sqrt{2}}=0$$

6. In each of the following determine rational numbers a and b:

(i) $\frac{\sqrt{3}-1}{\sqrt{3}+1}=a-b\sqrt{3}$

(ii) $\frac{4+\sqrt{2}}{2+\sqrt{2}}=a-\sqrt{b}$

(iii) $\frac{3+\sqrt{2}}{3-\sqrt{2}}=a+b\sqrt{2}$

(iv) $\frac{5+3\sqrt{3}}{7+4\sqrt{3}}=a+b\sqrt{3}$

(v) $\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}}=a-b\sqrt{77}$

(vi) $\frac{4+3\sqrt{5}}{4-3\sqrt{5}}=a+b\sqrt{5}$

Sol:

(i) Given $\frac{\sqrt{3}-1}{\sqrt{3}+1}=a-b\sqrt{3}$

Rationalisation factor for $\frac{1}{\sqrt{x}+y}$ is $\sqrt{x}-y \Rightarrow$ for $\frac{1}{\sqrt{3}+1}$ is $\sqrt{3}-1$

$\Rightarrow \frac{\sqrt{3}-1}{\sqrt{3}+1}=\frac{\sqrt{3}-1}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}$

$=\frac{\left(\sqrt{3}-1\right)^2}{\left(\sqrt{3}\right)^2-(1)^2}$ $\quad$ $\because (a-b)(a-b)=(a-b)^2$, $(a+b)(a-b)=a^2-b^2$

$=\frac{\left(\sqrt{3}\right)^2-2\sqrt{3}\times 1+(1)^2}{3-1}$ $\quad$ $\because (a-b)^2=a^2-2ab+b^2$

$=\frac{3-2\sqrt{3}+1}{2}$

$=\frac{4-2\sqrt{3}}{2}=\frac{2\left(2-\sqrt{3}\right)}{2}=2-\sqrt{3}$

We have

$\frac{\sqrt{3}-1}{\sqrt{3}+1}=a-b\sqrt{3}$

$\Rightarrow 2-\sqrt{3}=a-b\sqrt{3}\Rightarrow 2-(1)\sqrt{3}=a-b\sqrt{3}$

On equating rational and irrational parts, we get $a=2$ and $b=1$

(ii) Given that

$$\frac{4+\sqrt{2}}{2+\sqrt{2}}=a-\sqrt{b}$$

Rationalisation factor for $\frac{1}{a\pm b}$ is $a\mp\sqrt{b}\Rightarrow$ for $\frac{1}{2+\sqrt{2}}$ it is $2-\sqrt{2}$

$$\Rightarrow\frac{4+\sqrt{2}}{2+\sqrt{2}}=\frac{4+\sqrt{2}}{2+\sqrt{2}}\times\frac{2-\sqrt{2}}{2-\sqrt{2}}$$

$$=\frac{4\times2+\sqrt{2}\times2+4\times\left(-\sqrt{2}\right)+\sqrt{2}\times\left(-\sqrt{2}\right)}{2^2-\left(\sqrt{2}\right)^2}$$

$$\boxed{\because\left(a+b\right)\left(a-b\right)=a^2-b^2}$$

$$=\frac{8+2\sqrt{2}-4\sqrt{2}-2}{4-2}$$

$$=\frac{6-2\sqrt{2}}{2}=\frac{2\left(3-\sqrt{2}\right)}{2}=3-\sqrt{2}$$

We have,

$$\frac{4+\sqrt{2}}{2+\sqrt{2}}=a-\sqrt{b}$$

$$\Rightarrow3-\sqrt{2}=a-\sqrt{b}$$

On equating rational and irrational parts

We get

$a=3$ *and* $b=2$

(iii) Given that

$$\frac{3+\sqrt{2}}{3-\sqrt{2}}=a+b\sqrt{2}$$

The rationalisation factor for $\frac{1}{a-\sqrt{b}}$ is $a+\sqrt{b}\Rightarrow$ for $\frac{1}{3-\sqrt{2}}$ it is $3+\sqrt{2}$

$$\Rightarrow\frac{3+\sqrt{2}}{3-\sqrt{2}}=\frac{3+\sqrt{2}}{3-\sqrt{2}}\times\frac{3+\sqrt{2}}{3+\sqrt{2}}$$

$$=\frac{\left(3+\sqrt{2}\right)^2}{3^2-\left(\sqrt{2}\right)^2}$$

$$\boxed{\begin{aligned}\because\left(a+b\right)\left(a+b\right)&=\left(a+b\right)^2\\ \left(a+b\right)\left(a-b\right)&=a^2-b^2\end{aligned}}$$

$$=\frac{3^2+2\times3\sqrt{2}+\left(\sqrt{2}\right)^2}{9-2}$$

$$\boxed{\left(a+b\right)^2=a^2+2ab+b^2}$$

$$=\frac{9+6\sqrt{2}+2}{7}=\frac{11+6\sqrt{2}}{7}=\frac{11}{7}+\frac{6}{7}\sqrt{2}$$

We have

$$\frac{3+\sqrt{2}}{3-\sqrt{2}}=a+b\sqrt{2}$$

$$\Rightarrow \frac{11}{7}+\frac{6}{7}\sqrt{2}=a+b\sqrt{2}$$

On equating rational and irrational parts

We get

$a=\frac{11}{7}$ and $b=\frac{6}{7}$

(iv) Given that

$$\frac{5+3\sqrt{3}}{7+4\sqrt{3}}=a+b\sqrt{3}$$

Rationalisation factor for $\frac{1}{a+b\sqrt{c}}$ is $a-b\sqrt{c}\Rightarrow$ for $\frac{1}{7+4\sqrt{3}}$ it is $7-4\sqrt{3}$

$$\Rightarrow \frac{5+3\sqrt{3}}{7+4\sqrt{3}}=\frac{5+3\sqrt{3}}{7+4\sqrt{3}}\times\frac{7-4\sqrt{3}}{7-4\sqrt{3}}$$

$$=\frac{5\times7+5\times\left(-4\sqrt{3}\right)+3\sqrt{3}\times7+3\sqrt{3}\left(-4\sqrt{3}\right)}{\left(7\right)^2-\left(4\sqrt{3}\right)^2}$$

$\boxed{\because (a+b)(a-b)=a^2-b^2}$

$$=\frac{35-20\sqrt{3}+21\sqrt{3}-12\times3}{49-48}$$

$$=\frac{35-36+\sqrt{3}}{1}=\frac{\sqrt{3}-1}{1}=\sqrt{3}-1$$

We have

$$\frac{5+3\sqrt{3}}{7+4\sqrt{3}}=a+b\sqrt{3}$$

$$\Rightarrow \sqrt{3}-1=a+b\sqrt{3}$$

$$\Rightarrow -1+(1)\sqrt{3}=a+b\sqrt{3}$$

On equating the rational and irrational parts

We get

$a=-1$ and $b=1$

(v) Given that,

$$\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}}=a-b\sqrt{77}$$

We know that rationalisation factor for $\frac{1}{\sqrt{a}+\sqrt{b}}$ is $\sqrt{a}-\sqrt{b}\Rightarrow$ for $\frac{1}{\sqrt{11}+\sqrt{7}}$ it is

$\sqrt{11}-\sqrt{7}$

$$\Rightarrow \frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}} = \frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}} \times \frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}-\sqrt{7}}$$

$$= \frac{\left(\sqrt{11}-\sqrt{7}\right)^2}{\left(\sqrt{11}\right)^2 + \left(\sqrt{7}\right)^2}$$

$$\boxed{\begin{array}{l} \because (a-b)(a-b) = (a-b)^2 \\ (a-b)(a+b) = a^2 - b^2 \end{array}}$$

$$= \frac{\left(\sqrt{11}\right)^2 - 2\sqrt{11}\times\sqrt{7} + \left(\sqrt{7}\right)^2}{11-2}$$

$$\boxed{\because (a-b)^2 = a^2 - 2ab + b^2}$$

$$= \frac{11 - 2\sqrt{11\times 7} + 7}{4}$$

$$= \frac{18 - 2\sqrt{77}}{4} = \frac{2\left(9-\sqrt{77}\right)}{4} = \frac{9-\sqrt{27}}{2} = \frac{9}{2} - \frac{\sqrt{77}}{2}$$

We have,

$$\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}} = a - b\sqrt{77}$$

$$\Rightarrow \frac{9}{2} - \frac{\sqrt{77}}{2} = a - b\sqrt{77}$$

$$\Rightarrow \frac{9}{2} - \frac{1}{2}\sqrt{77} = a - b\sqrt{77}$$

On equating the rational and irrational posts

We have

$a = \frac{9}{2}$ and $b = \frac{1}{2}$

(vi) Given that

$$\frac{4+3\sqrt{5}}{4-3\sqrt{5}} = a + b\sqrt{5}$$

Rationalisation factor for $\frac{1}{a-b\sqrt{c}}$ is $a+b\sqrt{c} \Rightarrow$ for $\frac{1}{4-3\sqrt{5}}$ it is $4+3\sqrt{5}$

$$\Rightarrow \frac{4+3\sqrt{5}}{4-3\sqrt{5}} = \frac{4+3\sqrt{5}}{4-3\sqrt{5}} \times \frac{4+3\sqrt{5}}{4+3\sqrt{5}}$$

$$= \frac{\left(4+3\sqrt{5}\right)^2}{4^2 - \left(3\sqrt{5}\right)^2}$$

$$\boxed{\begin{array}{l} \because (a+b)(a+b) = (a+b)^2 \\ (a-b)(a+b) = a^2 - b^2 \end{array}}$$

$$= \frac{4^2 \times 2 \times 4 \times 3\sqrt{5} + \left(3\sqrt{5}\right)^2}{16 - 3^2\left(\sqrt{5}\right)^2}$$

$$\boxed{\because (a+b)^2 = a^2 + 2ab + b^2}$$

$$= \frac{16+24\sqrt{5}+45}{16-45} = \frac{61+24\sqrt{5}}{-29} = \frac{-\left(61+24\sqrt{5}\right)}{29}$$

$$= \frac{-61}{29} - \frac{24}{29}\sqrt{5}$$

7. If x = 2 + $\sqrt{3}$, find the value of $x^3 + \frac{1}{x^3}$

Sol:

Given $x = 2+\sqrt{3}$ and given to find the value of $x^3 + \frac{1}{x^3}$

We have $x = 2+\sqrt{3}$

$$\Rightarrow \frac{1}{x} = \frac{1}{2+\sqrt{3}}$$

Rationalization factor for $\frac{1}{a+\sqrt{b}}$ is $a-\sqrt{b}$

$$\Rightarrow \frac{1}{x} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$= \frac{2-\sqrt{3}}{2^2-\left(\sqrt{3}\right)^2} \qquad \boxed{\because (a-b)(a+b) = a^2-b^2}$$

$$= \frac{2-\sqrt{3}}{4-3} = \frac{2-\sqrt{3}}{1} = 2-\sqrt{3}$$

$$\therefore \frac{1}{x} = 2-\sqrt{3}$$

And also, $\left(x+\frac{1}{x}\right) = 2+\sqrt{3}+2-\sqrt{3} = 2+2 = 4$

$$\therefore \boxed{\left(x+\frac{1}{x}\right) = 4} \qquad \ldots\ldots\ldots\ldots\ldots(1)$$

We know that

$$x^3 + \frac{1}{x^3} = \left(x+\frac{1}{x}\right)\left(x^2 - x\times\frac{1}{x} + \frac{1}{x^2}\right)$$

$$= \left(x+\frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} - 1\right)$$

$$= \left(x+\frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} + 2 - 2 - 1\right)$$

$$= \left(x+\frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} + 2.x.\frac{1}{x} - 3\right)$$

$$\Rightarrow x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}+2.x.\frac{1}{x}-3\right)$$

$$=\left(x+\frac{1}{x}\right)\left(\left(x+\frac{1}{x}\right)^2-3\right) \qquad \left[\because (a+b)^2=a^2+2ab+b^2\right]$$

By putting $\left(x+\frac{1}{x}\right)=4,$ we get

$$x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)\left[\left(x+\frac{1}{x}\right)^2-3\right]$$

$$=(4)\left(4^2-3\right)$$

$$=4(16-3)$$

$$=4(13)$$

$$=52$$

$\therefore$ The value off $x^3+\frac{1}{x^3}$ is 52

8. If x = 3 + $\sqrt{8}$, *find the value of* $x^2+\frac{1}{x^2}$

Sol:

$$\Rightarrow \frac{1}{x}=\frac{1}{3+\sqrt{8}}\times\frac{3-\sqrt{8}}{3-\sqrt{8}}$$

$$=\frac{3-\sqrt{8}}{3^2-\left(\sqrt{8}\right)^2}=\frac{3-\sqrt{8}}{9-8}=\frac{3-\sqrt{8}}{1}=3-\sqrt{8}$$

$$\therefore \boxed{\frac{1}{x}=3-\sqrt{8}}$$

And also,

$$\left(x+\frac{1}{x}\right)=3+\sqrt{8}+3-\sqrt{8}=3+3=6$$

$$\therefore \boxed{\left(x+\frac{1}{x}\right)=6}$$

We know that

$$x^2+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)^2-2$$

By putting $x+\frac{1}{x}=6$ in the above

We get,

$x^2+\frac{1}{x^2}=(6)^2-2$

$=36-2$

$=34$

$\therefore$ The value of $x^2+\frac{1}{x^2}$ is 34.

Given that $x=3+\sqrt{8}$ and given to find the value of $x^2+\frac{1}{x^2}$

We have $x=3+\sqrt{8}$

$\Rightarrow \frac{1}{x}=\frac{1}{3+\sqrt{8}}$

The rationalization factor for $\frac{1}{a+\sqrt{b}}$ is $a-\sqrt{b}$

$\Rightarrow$ For $\frac{1}{3+\sqrt{8}}$ is $3-\sqrt{8}$

9. Find the value of $\frac{6}{\sqrt{5}-\sqrt{3}}$, it being given that $\sqrt{3}=1.732$ and $\sqrt{5}=2.236$.

Sol:

Given to find the value of $\frac{6}{\sqrt{5}-\sqrt{3}}$

Rationalisation factor for $\frac{1}{\sqrt{a}-\sqrt{b}}$ is $\sqrt{a}+\sqrt{b}\Rightarrow$ for $\frac{1}{\sqrt{5}-\sqrt{3}}$ is $\sqrt{5}+\sqrt{3}$

$\Rightarrow \frac{6}{\sqrt{5}-\sqrt{3}}=\frac{6}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$=\frac{6\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$ $\quad\left[\because (a-b)(a+b)=a^2-b^2\right]$

$=\frac{6\left(\sqrt{5}+\sqrt{3}\right)}{5-3}$

$=\frac{6\left(\sqrt{5}+\sqrt{3}\right)}{2}$

$=3\left(\sqrt{5}+\sqrt{3}\right)$

We have $\sqrt{3}=1\cdot 732, \sqrt{5}=2\cdot 236$

$\Rightarrow \frac{6}{5-\sqrt{3}} = 3(2\cdot 236 + 1\cdot 732)$

$= 3(3\cdot 968)$

$= 11.904$

$\therefore$ Value of $\frac{6}{5-\sqrt{3}}$ is $11\cdot 904$

10. Find the values of each of the following correct to three places of decimals, it being given that $\sqrt{2} = 1.4142$, $\sqrt{3} = 1.732$, $\sqrt{5} = 2.2360$, $\sqrt{6} = 2.4495$ and $\sqrt{10} = 3.162$

(i) $\frac{3-\sqrt{5}}{3+2\sqrt{5}}$

Sol:

(i) We have $\frac{3-\sqrt{5}}{3+2\sqrt{5}}$

Rationalization factor for $\frac{1}{a+b\sqrt{c}}$ is $a-b\sqrt{c} \Rightarrow$ for $\frac{1}{3+2\sqrt{5}}$ it is $3-2\sqrt{5}$

$\Rightarrow \frac{3-\sqrt{5}}{3+2\sqrt{5}} = \frac{3-\sqrt{5}}{3+2\sqrt{5}} \times \frac{3-2\sqrt{5}}{3-2\sqrt{5}}$

$= \frac{3\times 3 + 3\times(-2\sqrt{5}) + (-\sqrt{5})(3) + (-\sqrt{5})(-2\sqrt{5})}{(3)^2 - (2\sqrt{5})^2}$ $\boxed{\because (a+b)(a-b) = a^2 - b^2}$

$= \frac{9 - 6\sqrt{5} - 3\sqrt{5} + 2\times 5}{9-20}$

$= \frac{9+10-9\sqrt{5}}{-11} = \frac{19-9\sqrt{5}}{-11} = \frac{9\sqrt{5}-19}{11}$

We have $\sqrt{5} = 2.2360$

$\Rightarrow \frac{3-\sqrt{5}}{3+2\sqrt{5}} = \frac{9(2.2360)-19}{11}$

$= \frac{20\cdot 124 - 19}{11}$

$= \frac{1.124}{11}$

$= 0.102181818$

$\simeq 0.102$ (upto 3 decimals)

$\therefore$ The value of $\frac{3-\sqrt{5}}{3+2\sqrt{5}} = 0.102$

11. If $x = \frac{\sqrt{3}+1}{2}$, find the value of $4x^3 + 2x^2 - 8x + 7$.

Sol:

Given $x = \frac{\sqrt{3}+1}{2}$ and given to find the value of $4x^3 + 2x^2 - 8x + 7$

Now, $x = \frac{\sqrt{3}+1}{2}$

$\Rightarrow 2x = \sqrt{3}+1 \Rightarrow (2x-1) = \sqrt{3}$

Squaring on both sides we get

$(2x-1)^2 = \left(\sqrt{3}\right)^2$

$\Rightarrow (2x)^2 - 2.2x.1 + (1)^2 = 3$ $\quad \boxed{\because (a-b)^2 = a^2 - 2ab + b^2}$

$\Rightarrow 4x^2 - 4x + 1 = 3$

$\Rightarrow 4x^2 - 4x + 1 - 3 = 0$

$\Rightarrow 4x^2 - 4x - 2 = 0$

$\Rightarrow 2\left(2x^2 - 2x - 1\right) = 0$

$\Rightarrow \boxed{2x^2 - 2x - 1 = 0}$

Now take $4x^3 + 2x^2 - 8x + 7$

$\Rightarrow 2x\left(2x^2 - 2x - 1\right) + 4x^2 + 2x + 2x^2 - 8x + 7$

$\Rightarrow 2x\left(2x^2 - 2x - 1\right) + 6x^2 - 6x + 7$

$\Rightarrow 2x(0) + 3\left(2x^2 - 2x - 1\right) + 7 + 3$

$\Rightarrow 0 + 3(0) + 10$

$\Rightarrow 10$

$\therefore$ The value of $4x^3 + 2x^2 - 8x + 7$ is 10.

Exercise – 4.1

1. Evaluate each of the following using identities:

(i) $\left(2x-\frac{1}{x}\right)^2$

(ii) $(2x+y)(2x-y)$

(iii) $\left(a^2b-ab^2\right)^2$

(iv) $(a-0\cdot1)(a+0\cdot1)$

(v) $\left[1\cdot5x^2-0\cdot3y^2\right]\left[1\cdot5x^2+0\cdot3y^2\right]$

Sol:

(i) We have,

$\left(2x-\frac{1}{x}\right)^2=(2x)^2+\left(\frac{1}{x}\right)^2-2\cdot2x\cdot\frac{1}{x}$

$\left(2x-\frac{1}{x}\right)^2=4x^2+\frac{1}{x^2}-4$ $\left[\because(a-b)^2=a^2+b^2-2ab \text{ where } a=2x \text{ and } b=\frac{1}{x}\right]$

$\therefore\left(2x-\frac{1}{x}\right)^2=4x^2+\frac{1}{x^2}-4.$

(ii) We have

$(2x+y)(2x-y)$

$=(2x)^2-(y)^2$ $\left[\because(a+b)(a-b)=a^2-b^2\right]$ where $a=2x$ and $b=y$.

$=4x^2-y^2$

$(2x+y)(2x-y)=4x^2-y^2$

(iii) We have

$\left(a^2b-ab^2\right)^2$

$=\left(a^2b\right)^2+\left(ab^2\right)^2-2\times a^2b\times ab^2$ $\left[\because(a-b)^2=a^2+b^2-2ab\right]$

$=a^4b^2+b^4a^2-2a^3b^3$ where $=a^2b$ and $b=ab^2$

$\therefore\left(a^2b-b^2a\right)^2=a^4b^2+b^4a^2-2a^3b^3$

(iv) We have

$(a-0\cdot1)(a+0\cdot1)=a^2-(0\cdot1)^2$ $\left[\because(a-b)(a+b)=a^2-b^2\right]$

$=a^2-0\cdot01$ $[a=a;b=0\cdot1]$

$(a-0\cdot1)(a+0\cdot1)=a^2-0\cdot01$

(v) We have

$\left[1\cdot 5x^2 - 0\cdot 3y^2\right]\left[1\cdot 5x^2 + 0\cdot 3y^2\right]$

$= \left[1\cdot 5x^2\right]^2 - \left[0\cdot 3y^2\right]^2$ $\quad\left[\because (a+b)(a-b) = a^2 - b^2\right]$

$= 2\cdot 25x^4 - 0\cdot 09y^4$ $\quad\left[\because a = 1\cdot 5x^2 \text{ and } b = 0\cdot 3y^2\right]$

$\left[1\cdot 5x^2 - 0\cdot 3y^2\right]\left[1\cdot 5x^2 + 0\cdot 3y^2\right] = 2\cdot 25x^4 - 0\cdot 09y^4.$

2. Evaluate each of the following using identities:

(i) $(399)^2$

(ii) $(0\cdot 98)^2$

(iii) 991×1009

(iv) 117×83

Sol:

(i) We have

$(399)^2 = (400-1)^2$

$= (400)^2 + (1)^2 - 2(400)(1)$ $\quad\left[\because (a-b)^2 = a^2 + b^2 - 2ab\right]$

$= 1,60,000 + 1 - 8,000$ $\quad\left[\because a = 400 \;\&\; b = 1\right]$

$= 159201$

$(399)^2 = 159201$

(ii) We have

$(0\cdot 98)^2 = [1 - 0\cdot 02]^2$

$= (1)^2 + (0\cdot 02)^2 - 2\times 1\times 0\cdot 02$

$= 1 + 0\cdot 0004 - 0\cdot 04$ $\quad\left[\because a = 1; b = 0\cdot 02\right]$

$= 1\cdot 0004 - 0\cdot 04$

$= 0\cdot 9604$

$\therefore (0\cdot 98)^2 = 0\cdot 9604.$

(iii) We have

991×1009

$= (1000-9)(1000+9)$

$= (1000)^2 - (9)^2$ $\quad\left[\because (a-b)(a+b) = a^2 - b^2\right]$

$= 1000000 - 81$ $\quad\left[\because a = 1000; b = 9\right]$

$= 999919$

$991 \times 1009 = 999919$

(iv) We have

117×83

$=(100+17)(100-17)$

$=(100)^2-(17)^2$ $\left[\because (a+b)(a-b)=a^2-b^2\right]$

$=10000-289$ $[\because a=100; b=17]$

$=9711$

$117\times 83=9711$

3. Simplify each of the following:

(i) $175\times 175+2\times 175\times 25+25\times 25$

(ii) $322\times 322-2\times 322\times 22+22\times 22$

(iii) $0\cdot 76\times 0\cdot 76+2\times 0\cdot 76\times 0\cdot 24+0\cdot 24\times 0\cdot 24$

(iv) $\dfrac{7\cdot 83+7\cdot 83-1\cdot 17\times 1\cdot 17}{6\cdot 66}$

Sol:

(i) We have

$175\times 175+2\times 175\times 25+25\times 25=(175)^2+2(175)(25)+(25)^2$

$=(175+25)^2$ $\left[\because a^2+b^2+2ab=(a+b)^2\right]$

$=(200)^2=40000$ [here $a=175$ and $b=25$]

$\therefore 175\times 175+2\times 175\times 25+25\times 25=40000$

(ii) We have

$322\times 322-2\times 322\times 22+22\times 22$

$=(322-22)^2$ $\left[\because (a-b)^2=a^2-2ab+b^2\right]$

$=(300)^2$

$=90000$

$\therefore 322\times 322-2\times 322\times 22+22\times 22=90000$

(iii) We have

$0\cdot 76\times 0\cdot 76+2\times 0\cdot 76\times 0\cdot 24+0\cdot 24\times 0\cdot 24$

$=[0\cdot 76+0\cdot 24]^2$ $\left[\because a^2+b^2+2ab=(a+b)^2\right]$

$=[1\cdot 00]^2$

$=1$

$\therefore 0\cdot 76\times 0\cdot 76+2\times 0\cdot 76\times 0\cdot 24+0\cdot 24\times 0\cdot 24=1$

(iv) We have

$$\frac{7\cdot83+7\cdot83-1\cdot17\times1\cdot17}{6\cdot66}$$

$$=\frac{(7\cdot83+1\cdot17)(7\cdot83-1\cdot17)}{6\cdot66} \qquad \left[\because (a^2-b^2)=(a+b)(a-b)\right]$$

$$=\frac{(9\cdot00)(6\cdot66)}{(6\cdot66)}$$

$$=9$$

$$\therefore \frac{7\cdot83\times7\cdot83-1\cdot17\times1\cdot17}{6\cdot66}=9$$

4. If $x+\frac{1}{x}=11$, find the value of $x^2+\frac{1}{x^2}$

Sol:

We have $x+\frac{1}{x}=11$

Now, $\left(x+\frac{1}{x}\right)^2=x^2+\left(\frac{1}{x}\right)^2+2\times x\times\frac{1}{x}$

$$\Rightarrow \left(x+\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}+2$$

$$\Rightarrow (11)^2=x^2+\frac{1}{x^2}+2 \qquad \left[\because x+\frac{1}{x}=11\right]$$

$$\Rightarrow 121=x^2+\frac{1}{x^2}+2$$

$$\Rightarrow x^2+\frac{1}{x^2}=119.$$

5. If $x-\frac{1}{x}=-1$, find the value of $x^2+\frac{1}{x^2}$

Sol:

We have

$$\left(x-\frac{1}{x}\right)^2=x^2+\left(\frac{1}{x}\right)^2-2\times x\times\frac{1}{x}$$

$$\Rightarrow \left(x-\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}-2$$

$$\Rightarrow (-1)^2=x^2+\frac{1}{x^2}-2 \qquad \left[\because x-\frac{1}{x}=-1\right]$$

$$\Rightarrow 2+1=x^2+\frac{1}{x^2}$$

$$\therefore x^2+\frac{1}{x^2}=3.$$

6. If $x+\frac{1}{x}=\sqrt{5}$, find the values of $x^2+\frac{1}{x^2}$ and $x^4+\frac{1}{x^4}$

Sol:

We have

$$\left(x+\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}+2\times x\times\frac{1}{x}$$

$$\Rightarrow\left(x+\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}+2$$

$$\Rightarrow\left(\sqrt{5}\right)^2=x^2+\frac{1}{x^2}+2 \qquad \left[\because x+\frac{1}{x}=\sqrt{5}\right]$$

$$\Rightarrow 5=x^2+\frac{1}{x^2}+2$$

$$\Rightarrow x^2+\frac{1}{x^2}=3 \qquad \text{........}(1)$$

Now, $\left(x^2+\frac{1}{x^2}\right)^2=x^4+\frac{1}{x^4}+2\times x^2\times\frac{1}{x^2}$

$$\Rightarrow\left(x^2+\frac{1}{x^2}\right)^2=x^4+\frac{1}{x^4}+2$$

$$\Rightarrow 9=x^4+\frac{1}{x^4}+2 \qquad \left[\because x^2+\frac{1}{x^2}=3\right]$$

$$\Rightarrow x^4+\frac{1}{x^4}=7$$

Hence, $x^2+\frac{1}{x^2}=3; x^4+\frac{1}{x^4}=7.$

7. If $x^2+\frac{1}{x^2}=66$, find the value of $x-\frac{1}{x}$

Sol:

We have

$$\left(x-\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}-2\times x\times\frac{1}{x}$$

$$\left(x-\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}-2$$

$$\Rightarrow\left(x-\frac{1}{x}\right)^2=66-2 \qquad \left[\because x^2+\frac{1}{x^2}=66\right]$$

$$\Rightarrow\left(x-\frac{1}{x}\right)^2=64$$

$$\Rightarrow\left(x-\frac{1}{x}\right)^2=(\pm 8)^2$$

$$\Rightarrow x-\frac{1}{x}=\pm 8$$

8. If $x^2+\frac{1}{x^2}=79$, find the value of $x+\frac{1}{x}$

Sol:

We have

$$\left(x+\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}+2\times x\times\frac{1}{x}$$

$$\Rightarrow\left(x+\frac{1}{x}\right)^2=\left(x^2+\frac{1}{x^2}\right)+2$$

$$\Rightarrow\left(x+\frac{1}{x}\right)^2=79+2$$

$$\Rightarrow\left(x+\frac{1}{x}\right)^2=81$$

$$\Rightarrow\left(x+\frac{1}{x}\right)^2=(\pm 9)^2$$

$$\Rightarrow x+\frac{1}{x}=\pm 9.$$

9. If $9x^2+25y^2=181$ and xy = -6, find the value of 3x + 5y

Sol:

We have,

$$(3x+5y)^2=(3x)^2+(5y)^2+2\times 3x\times 5y$$

$$\Rightarrow(3x+5y)^2=9x^2+25y^2+30xy$$

$$=181+30(-6) \qquad \left[\because 9x^2+25y^2=181 \text{ and } xy=-6\right]$$

$$=181-180$$

$$\Rightarrow(3x+5y)^2=1$$

$$\Rightarrow(3x+5y)^2=(\mp 1)^2$$

$$\Rightarrow 3x+5y=\pm 1$$

10. If 2x + 3y = 8 and xy = 2, find the value of $4x^2 + 9y^2$

Sol:

We have

$$(2x+3y)^2 = (2x)^2 + (3y)^2 + 2(2x)(3y)$$

$$\Rightarrow (2x+3y)^2 = 4x^2 - 9y^2 + 12xy$$

$$\Rightarrow (8)^2 = 4x^2 + 9y^2 + 24 \qquad [\because 2x+3y=8, xy=24]$$

$$\Rightarrow 64 - 24 = 4x^2 + 9y^2$$

$$\Rightarrow 4x^2 + 9y^2 = 40.$$

11. If 3x $-7y$ = 10 and xy = -1, find the value of $9x^2 + 49y^2$

Sol:

We have,

$$(3x-7y)^2 = (3x)^2 + (-7y)^2 - 2(3x)(7y)$$

$$= 9x^2 + 49y^2 - 42xy$$

$$\Rightarrow [10]^2 = 9x^2 + 49y^2 - 42xy \qquad [\because 3x - 7y = 10]$$

$$\Rightarrow 100 = 9x^2 + 49y^2 - 42[-1] \qquad [\because xy = -1]$$

$$\Rightarrow 100 = 9x^2 + 49y^2 + 42$$

$$\Rightarrow 100 - 42 = 9x^2 + 49y^2$$

$$\Rightarrow 9x^2 + 49y^2 = 58.$$

12. Simplify each of the following products:

(i) $\left(\frac{1}{2}a - 3b\right)\left(\frac{1}{2}a + 3b\right)\left(\frac{1}{4}a^2 + 9b^2\right)$

(ii) $\left(m + \frac{n}{7}\right)^3 \left(m - \frac{n}{7}\right)$

(iii) $\left(\frac{x}{2} - \frac{2}{5}\right)\left(\frac{2}{5} - \frac{x}{2}\right) - x^2 + 2x$

(iv) $(x^2 + x - 2)(x^2 - x + 2)$

(v) $(x^3 - 3x^2 - x)(x^2 - 3x + 1)$

(vi) $[2x^4 - 4x^2 + 1][2x^4 - 4x^2 - 1]$

Sol:

(i) $\left(\frac{1}{2}a - 3b\right)\left(\frac{1}{2}a + 3b\right)\left(\frac{1}{4}a^2 + 9b^2\right)$

$$\Rightarrow\left[\left(\frac{1}{2}a\right)^2-(3b)^2\right]\left[\frac{1}{4}a^2+9b^2\right] \qquad \left[\because (a+b)(a-b)=a^2-b^2\right]$$

$$=\left[\left(\frac{1}{4}a^2\right)-9b^2\right]\left[\frac{1}{4}a^2+9b^2\right] \qquad \left[\because (ab)^2=a^2b^2\right]$$

$$=\left[\frac{1}{4}a^2\right]^2-\left[9b^2\right]^2 \qquad \left[\because (a-b)(a+b)=a^2-b^2\right]$$

$$=\frac{1}{16}a^4-81b^4$$

$$\therefore\left(\frac{1}{2}a-3b\right)\left(\frac{1}{2}a+3b\right)\left(\frac{1}{4}a^2+9b^2\right)=\frac{1}{16}a^4-81b^4$$

(ii) We have

$$\left(m+\frac{n}{7}\right)^3\left(m-\frac{n}{7}\right)$$

$$=\left(m+\frac{n}{7}\right)\left(m+\frac{n}{7}\right)\left(m+\frac{n}{7}\right)\left(m-\frac{n}{7}\right)$$

$$=\left(m+\frac{n}{7}\right)^2\left((m)^2-\left(\frac{n}{7}\right)^2\right) \qquad \left[\because (a+b)(a+b)=(a+b)^2 \ \& \ (a+b)(a-b)=a^2-b^2\right]$$

$$=\left(m+\frac{n}{7}\right)^2\left[m^2-\frac{n^2}{49}\right]$$

$$\therefore\left(m+\frac{n}{7}\right)^3\left(m-\frac{n}{7}\right)=\left(m+\frac{n}{7}\right)^2\left[m^2-\frac{n^2}{49}\right]$$

(iii) We have

$$\left(\frac{x}{2}-\frac{2}{5}\right)\left(\frac{2}{5}-\frac{x}{2}\right)-x^2+2x$$

$$\Rightarrow-\left(\frac{2}{5}-\frac{x}{2}\right)\left(\frac{2}{5}-\frac{x}{2}\right)-x^2+2x$$

$$\Rightarrow-\left(\frac{2}{5}-\frac{x}{2}\right)^2-x^2+2x \qquad \left[\because (a-b)(a-b)=(a-b)^2\right]$$

$$\Rightarrow-\left[\left(\frac{2}{5}\right)^2+\left(\frac{x}{2}\right)^2-2\left(\frac{2}{5}\right)\left(\frac{x}{2}\right)\right]-x^2+2x$$

$$\Rightarrow-\left[\frac{4}{25}+\frac{x^2}{4}-\frac{2x}{5}\right]-x^2+2x$$

$$\Rightarrow-\frac{x^2}{4}+\frac{2x}{5}-x^2+2x-\frac{4}{25}\Rightarrow-\frac{x^2}{4}-x^2+\frac{2x}{5}+2x-\frac{4}{25}$$

$$\Rightarrow -\frac{5x^2}{4}+\frac{2x}{5}+2x-\frac{4}{25}$$

$$\Rightarrow -\frac{5x^2}{4}+\frac{2x+10x}{5}-\frac{4}{25}$$

$$\Rightarrow \frac{-5x^2}{4}+\frac{12x}{5}-\frac{4}{25}$$

$$\therefore \left(\frac{x}{2}-\frac{2}{5}\right)\left(\frac{2}{5}-\frac{x}{2}\right)-x^2+2x$$

$$=\frac{-5x^2}{4}+\frac{12x}{5}-\frac{4}{25}$$

(iv) We have,

$$\left(x^2+x-2\right)\left(x^2-x+2\right)$$

$$\left[(x)^2+(x-2)\right]\left[x^2-(x-2)\right]$$

$$\Rightarrow \left[x^2\right]^2-(x-2)^2 \qquad \left[(a-b)(a+b)=a^2-b^2\right]$$

$$\Rightarrow x^4-\left(x^2+4-4x\right) \qquad \left[\because (a-b)^2=a^2+b^2-2ab\right]$$

$$\Rightarrow x^4-x^2-4+4x$$

$$\Rightarrow x^4-x^2+4x-4$$

$$\therefore \left(x^2+x-2\right)\left(x^2-x+2\right)=x^4-x^2+4x-4$$

(v) We have,

$$\left(x^3-3x^2-x\right)\left(x^2-3x+1\right)$$

$$\Rightarrow x\left(x^2-3x-1\right)\left(x^2-3x+1\right)$$

$$\Rightarrow x\left[\left[x^2-3x\right]^2-[1]^2\right] \qquad \left[\because (a-b)(a+b)=a^2-b^2\right]$$

$$\Rightarrow x\left[\left(x^2\right)^2+(-3x)^2-2(+3x)x^2\right]-1\Big]$$

$$\Rightarrow x\left[x^4+9x^2-6x^3-1\right]$$

$$\Rightarrow x^5-6x^4+9x^3-x$$

$$\therefore \left(x^3-3x^2-2\right)\left(x^2-3x+1\right)=x^5-6x^4+9x^3-x.$$

(vi) We have

$$\left[2x^4-4x^2+1\right]\left[2x^4-4x^2-1\right]$$

$$\Rightarrow \left[\left(2x^4-4x^2\right)^2-(1)^2\right] \qquad \left[\because (a+b)(a-b)=a^2-b^2\right]$$

$$\Rightarrow \left[\left(2x^4\right)^2+\left(4x^2\right)^2-2\left(2x^4\right)\left(4x^2\right)-1\right]$$

$$\Rightarrow 4x^8 + 16^4 - 16x^6 - 1 \qquad \left[\because (a-b)^2 = a^2 + b^2 - 2ab\right]$$

$$\Rightarrow 4x^8 - 16x^6 + 16x^4 - 1$$

$$\therefore \left[2x^4 - 4x^2 + 1\right]\left[2x^4 - 4x^2 - 1\right] = 4x^8 - 16x^6 + 16x^4 - 1.$$

13. Prove that $a^2 + b^2 + c^2 - ab - bc - ca$ is always non-negative for all values of a, b and c

Sol:

We have

$$a^2 + b^2 + c^2 - ab - bc - ca$$

$$= \frac{2}{2}\left[a^2 + b^2 + c^2 - ab - bc - ca\right] \qquad \text{[Multiply and divide by ‘2’]}$$

$$= \frac{1}{2}\left[2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca\right]$$

$$= \frac{1}{2}\left[a^2 + a^2 + b^2 + b^2 + c^2 + c^2 - 2ab - 2bc - 2ca\right]$$

$$= \frac{1}{2}\left[\left(a^2 + b^2 - 2ab\right) + \left(a^2 + c^2 - 2ac\right) + \left(b^2 + c^2 - 2bc\right)\right]$$

$$= \frac{1}{2}\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right] \qquad \left[\because (a-b)^2 = a^2 + b^2 - 2ab\right]$$

$$= \frac{(a-b)^2 + (b-c)^2 + (c-a)^2}{2} \geq 0$$

$$\therefore a^2 + b^2 + c^2 - ab - bc - ca \geq 0$$

Hence, $a^2 + b^2 + c^2 - ab - bc - ca$ is always non negative for all values of a, b and c.

Exercise – 4.2

1. Write the following in the expanded form:

(i) $(a+2b+c)^2$

(ii) $(2a-3b-c)^2$

(iii) $(-3x+y+z)^2$

(iv) $(m+2n-5p)^2$

(v) $(2+x-2y)^2$

(vi) $(a^2+b^2+c^2)^2$

(vii) $(ab+bc+ca)^2$

(viii) $\left(\frac{x}{y}+\frac{y}{z}+\frac{z}{x}\right)^2$

(ix) $\left(\frac{a}{bc}+\frac{b}{ca}+\frac{c}{ab}\right)^2$

(x) $(x+2y+4z)^2$

(xi) $(2x-y+z)^2$

(xii) $(-2x+3y+2z)^2$

Sol:

(i) We have, $(a+2b+c)^2$

$=a^2+(2b)^2+(c)^2+2(a)(2b)+2ac+(2b)2c$

$\left[\because (a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca\right]$

$\therefore (a+2b+c)^2=a^2+4b^2+c^2+4ab+2ac+4bc.$

(ii) We have

$(2a-3b-c)^2=\left[(2a)+(-3b)+(-c)\right]^2$

$=(2a)^2+(-3b)^2+(-c)^2+2(2a)(-3b)+2(-3b)(-c)+2(2a)(-c)$

$\left[\because (a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca\right]$

$=4a^2+9b^2+c^2-12ab+6bc-4ac$

$\therefore (2a-3b-c)^2=4a^2+9b^2+c^2-12ab+6bc-4ca.$

(iii) $(-3x+y+z)^2 = [(-3x)+y+z]^2$

$= (-3x)^2 + y^2 + z^2 + 2(-3x)y + 2yz + 2(-3x)z$

$[\because (a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca]$

$= 9x^2 + y^2 + z^2 - 6xy + 2yz - 6xz$

$\therefore (-3x+y+z)^2 = 9x^2 + y^2 + z^2 - 6xy + 2xy - 6xz$

(iv) We have,

$(m+2n-5p)^2 = m^2 + (2n)^2 + (-5p)^2 + 2(m)(2n) + 2(2n)(-5p) + 2(m)(-5p)$

$[\because (a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca]$

$= m^2 + 4n^2 + 25p^2 + 4mn - 20np - 10pm$

$\therefore (m+n-5p)^2 = m^2 + 4n^2 + 25p^2 + 4mn - 20np - 10pm.$

(v) We have,

$(2+x-2y)^2 = [2+x+(-2y)]^2$

$= (2)^2 + x^2 + (-2y)^2 + 2(2)(x) + 2(x)(-2y) + 2(2)(-2y)$

$[\because (a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca]$

$= 4 + x^2 + 4y^2 + 4x - 4xy - 8y$

$\therefore (2+x-2y)^2 = 4 + x^2 + 4y^2 + 4x - 4xy - 8y$

(vi) We have

$(a^2+b^2+c^2)^2 = (a^2)^2 + (b^2)^2 + (c^2)^2 + 2a^2b^2 + 2b^2c^2 + 2a^2c^2$

$[\because (a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca]$

$= a^4 + b^4 + c^4 + 2a^2b^2 + 2b^2c^2 + 2a^2c^2$

$\therefore (a^2+b^2+c^2)^2 = a^4 + b^4 + c^4 + 2a^2b^2 + 2b^2c^2 + 2a^2c^2$

(vii) We have

$(ab+bc+ca)^2 = (ab)^2 + (bc)^2 + (ca)^2 + 2(ab)(bc) + 2(bc)ca + 2(ab)(ca)$

$[\because (a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca]$

$= a^2b^2 + b^2c^2 + c^2a^2 + 2ab^2c + 2bc^2a + 2a^2bc$

$\therefore (ab+bc+ca)^2 = a^2b^2 + b^2c^2 + c^2a^2 + 2ab^2c + 2bc^2a + 2a^2bc$

(viii) We have

$$\left(\frac{x}{y}+\frac{y}{z}+\frac{z}{x}\right)^2=\left(\frac{x}{y}\right)^2+\left(\frac{y}{z}\right)^2+\left(\frac{z}{x}\right)^2+2\cdot\frac{x}{y}\cdot\frac{y}{z}+2\frac{y}{z}\cdot\frac{z}{x}+2\cdot\frac{z}{x}\cdot\frac{x}{y}$$

$$\left[\because (a+b+c)^2=a^2+b^2+2ab+c^2+2bc+2ca\right]$$

$$\therefore\left(\frac{x}{y}+\frac{y}{z}+\frac{z}{x}\right)^2=\frac{x^2}{y^2}+\frac{y^2}{z^2}+\frac{z^2}{x^2}+2\frac{x}{z}+2\frac{y}{x}+2\frac{z}{y}$$

(ix) We have

$$\left(\frac{a}{bc}+\frac{b}{ca}+\frac{c}{ab}\right)^2=\left(\frac{a}{bc}\right)^2+\left(\frac{b}{ca}\right)^2+\left(\frac{c}{ab}\right)^2+2\left(\frac{a}{bc}\right)\left(\frac{b}{ca}\right)+2\left(\frac{b}{ca}\right)\left(\frac{c}{ab}\right)+2\left(\frac{a}{bc}\right)\left(\frac{c}{ab}\right)$$

$$\left[\because (a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca\right]$$

$$\therefore\left(\frac{a}{bc}+\frac{b}{ca}+\frac{c}{ab}\right)^2=\frac{a^2}{b^2c^2}+\frac{b^2}{c^2a^2}+\frac{c^2}{a^2b^2}+\frac{2}{a^2}+\frac{2}{b^2}+\frac{2}{c^2}$$

(x) $(x+2y+4z)^2=x^2+(2y)^2+(4z)^2+2x(2y)+2(2y)(4z)+2x(4z)$

$=x^2+4y^2+16z^2+4xy+16yz+4xz.$

(xi) $(2x-y+z)^2=\left[(2x)+(-y)+z\right]^2$

$=(2x)^2+(-y)^2+(z)^2+2(2x)(-y)+2(-y)(z)+2(2x)(z)$

$=4x^2+y^2+z^2+4x(-y)-2yz+4xz$

$\therefore (2x-y+z)^2=4x^2+y^2+z^2-4xy-2yz+4xz$

(xii) $(-2x+3y+2z)^2=((-2x)+3y+2z)^2$

$=(-2x)^2+(3y)^2+(2z)^2+2(-2x)(3y)+2(3y)(2z)+2(2z)(-2x)$

$=4x^2+9y^2+4z^2-12xy+12yz-8xz$

2. Simplify:

(i) $(a+b+c)^2+(a-b+c)^2$

(ii) $(a+b+c)^2-(a-b+c)^2$

(iii) $(a+b+c)^2+(a-b+c)^2+(a+b-c)^2$

(iv) $(2x+p-c)^2-(2x-p+c)^2$

(v) $(x^2+y^2-z)^2-(x^2-y^2+z^2)^2$

Sol:

(i) We have

$$(a+b+c)^2+(a-b+c)^2$$
$$=(a^2+b^2+c^2+2ab+2bc+2ca)+(a^2+b^2+c^2-2ab-2bc+2ac)$$
$$\left[\because (x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx\right]$$
$$=2a^2+2b^2+2c^2+4ca$$
$$\therefore (a+b+c)^2+(a-b+c)^2=2a^2+2b^2+2c^2+4ca.$$

(ii) We have

$$(a+b+c)^2-(a-b+c)^2$$
$$=\left[(a+b+c)^2\right]-[a-b+c]^2$$
$$=a^2+b^2+c^2+2ab+2bc+2ca-\left[a^2+b^2+c^2-2ab-2bc+2ca\right]$$
$$=a^2+b^2+c^2+2ab+2bc+2ca-a^2-b^2-c^2+2ab+2bc-2ca$$
$$=4ab+4bc$$
$$\therefore (a+b+c)^2-(a-b+c)^2=4ab+4bc$$

(iii) We have

$$(a+b+c)^2+(a-b+c)^2+(a+b-c)^2$$
$$=\left[a^2+b^2+c^2+2ab+2bc+2ca\right]+\left[a^2+b^2+c^2-2bc-2ab+2ca\right]$$
$$+\left[a^2+b^2+c^2-2ca-2bc+2ab\right]$$
$$\left[\because (x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx\right]$$
$$=3a^2+3b^2+3c^2+2ab+2bc+2ca-2bc-2ab+2ca-2ca-2bc+2ab$$
$$=3a^2+3b^2+3c^2+2ab-2bc+2ca$$
$$=3(a^2+b^2+c^2)+2(ab-bc+ca)$$
$$\therefore (a+b+c)^2+(a-b+c)^2+(a+b-c)^2=3(a^2+b^2+c^2)+2[ab-bc+ca]$$

(iv) We have

$$(2x+p-c)^2-(2x-p+c)^2$$
$$=\left[(2x)^2+(p)^2+(-c)^2+2(2x)(p)+2(p)(-c)+2(2x)(-c)\right]$$
$$-\left[(2x)^2+(-p)^2+c^2+2(2x)(-p)+2(2x)(c)+2(-p)c\right]$$
$$=\left[4x^2+p^2+c^2+4xp-2pc-4cx\right]-\left[4x^2+p^2+c^2-4xp-2pc+4cx\right]$$
$$=4x^2+p^2+c^2+4xp-2pc-4cx-4x^2-p^2-c^2+4xp+2pc-4cx$$

$=8xp-8xc$

$=8x(p-c)$

$\therefore (2x+p-c)^2-(2x-p+c)^2=8x(p-c)$

(v) We have

$(x^2+y^2-z)^2-(x^2-y^2+z^2)^2$

$=\left[x^2+y^2+(-z)^2\right]^2-\left[x^2+(-y^2)+(z^2)\right]^2$

$=\left[(x^2)^2+(y^2)^2+(-z^2)^2+2(x^2)(y^2)+2(y^2)(-z^2)+2(x^2)(-z^2)\right]$

$-\left[(x^2)^2+(-y^2)^2+(z^2)^2+2(x^2)(-y^2)+2(-y^2)z^2+2x^2z^2\right]$

$\left[\because (a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca\right]$

$=x^4+y^4+z^4+2x^2y^2-2y^2z^2-2z^2x^2-x^4-y^4-z^4+2x^2y^2+2y^2z^2-2z^2x^2$

$=4x^2y^2-4z^2x^2$

$\therefore (x^2+y^2-z^2)^2-(x^2-y^2+z^2)^2=4x^2y^2-4z^2x^2$

3. If a + b + c = 0 and $a^2+b^2+c^2=16$, find the value of ab + bc + ca.

Sol:

We know that,

$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

$\Rightarrow (0)^2=16+2(ab+bc+ca)$ $\left[\because a+b+c= \text{ and } a^2+b^2+c^2=16\right]$

$\Rightarrow 2(ab+bc+ca)=-16$

$\Rightarrow ab+bc+ca=-8$

4. If $a^2+b^2+c^2=16$ and ab + bc + ca = 10, find the value of a + b + c.

Sol:

We know that,

$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

$\Rightarrow (a+b+c)^2=16+2(10)$ $\left[\because a^2+b^2+c^2=16 \text{ and } ab+bc+ca=10\right]$

$\Rightarrow (a+b+c)^2=16+20$

$\Rightarrow (a+b+c)=\sqrt{36}$

$\Rightarrow a+b+c=\pm 6$

5. If a + b + c = 9 and ab + bc + ca = 23, find the value of $a^2 + b^2 + c^2$.

Sol:

We know that,

$$(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$$

$$\Rightarrow (9)^2 = a^2+b^2+c^2+2(23)$$

$$\Rightarrow 81 = a^2+b^2+c^2+46 \qquad [\because a+b+c=9 \text{ and } (ab+bc+ca=23)]$$

$$\Rightarrow a^2+b^2+c^2 = 81-46$$

$$\Rightarrow a^2+b^2+c^2 = 35.$$

6. Find the value of $4x^2 + y^2 + 25z^2 + 4xy - 10yz - 20zx$ when x = 4, y = 3 and z = 2.

Sol:

We have,

$$4x^2+y^2+25z^2+4xy-10yz-20zx$$

$$\Rightarrow (2x)^2+(y)^2+(-5z)^2+2(2x)(y)+2(y)(-5z)+2(-5z)(2x)$$

$$\Rightarrow (2x+y-5z)^2$$

$$\Rightarrow [2[4]+3-5(2)]^2 \qquad [\because x=4, y=3 \text{ and } z=2]$$

$$=[8+3-10]^2$$

$$=[1]^2$$

$$=1$$

$$\therefore 4x^2+y^2+25z^2+4xy-10yz-20zx=1.$$

7. Simplify each of the following expressions:

(i) $(x+y+z)^2+\left(x+\frac{y}{2}+\frac{2}{3}\right)^2-\left(\frac{x}{2}+\frac{y}{3}+\frac{z}{4}\right)^2$

(ii) $(x+y-2z)^2-x^2-y^2-3z^2+4xy$

(iii) $(x^2-x+1)^2-(x^2+x+1)^2$

Sol:

(i) We have,

$$(x+y+z)^2+\left(x+\frac{y}{2}+\frac{2}{3}\right)^2-\left(\frac{x}{2}+\frac{y}{3}+\frac{z}{4}\right)^2$$

$$=\left[x^2+y^2+z^2+2xy+2yz+2zx\right]+\left[x^2+\frac{y^2}{4}+\frac{z^2}{9}+2x\cdot\frac{y}{2}+2\frac{zx}{3}+\frac{yz}{3}\right]$$

$$-\left[\frac{x^2}{4}+\frac{y^2}{9}+\frac{z^2}{10}+\frac{xy}{3}+\frac{xz}{4}+\frac{yz}{6}\right]$$

$$=x^2+y^2+z^2+x^2+\frac{y^2}{4}+\frac{z^2}{9}-\frac{x^2}{4}-\frac{y^2}{9}-\frac{z^2}{16}+2xy+2x\cdot\frac{y}{2}-\frac{xy}{3}+2yz+\frac{yz}{3}-\frac{yz}{6}+2zx+\frac{2zx}{3}-\frac{xz}{4}$$

$$=\frac{8x^2-x^2}{4}+\frac{36y^2+9y^2-4y^2}{36}+\frac{144z^2+16z^2-9z^2}{144}+\frac{6xy+3xy-xy}{3}+\frac{13yz}{6}+\frac{29xz}{12}$$

$$=\frac{7x^2}{4}+\frac{41y^2}{36}+\frac{151z^2}{144}+\frac{8xy}{3}+\frac{13yz}{6}+\frac{29zx}{12}$$

$$\therefore (x+y+z)^2+\left(x+\frac{y}{2}+\frac{z}{3}\right)^2-\left(\frac{x}{2}+\frac{y}{3}+\frac{z}{4}\right)^2$$

$$=\frac{7x^2}{4}+\frac{41y^2}{36}+\frac{151Z^2}{144}+\frac{8xy}{3}+\frac{13yz}{6}+\frac{29zx}{12}$$

(ii) We have,

$$(x+y-2z)^2-x^2-y^2-3z^2+4xy$$

$$=\left[x^2+y^2+(-2z)^2+2xy+2(y)(-2z)+2x(-2z)\right]-x^2-y^2-3z^2+4xy$$

$$=x^2+y^2+4z^2+2xy-4yz-4xz-x^2-y^2-3z^2+4xy$$

$$=z^2+6xy-4yz-4zx$$

$$\therefore (x+y-2z)^2-x^2-y^2-3z^2+4xy=z^2+6xy-4yz-4zx$$

(iii) We have,

$$\left[x^2-x+1\right]^2-\left[x^2+x+1\right]^2$$

$$=\left[(x^2)^2+(-x)^2+1^2+2(x^2)(-x)+2(-x)(1)+2x^2(1)\right]$$

$$-\left[(x^2)^2+(x)^2+(1)^2+2x^2(x)+2(x)(1)+2(x^2)(1)\right]$$

$$=x^4+x^2+1-2x^3-2x+2x^2-x^2-x^4-1-2x^3-2x-2x^2$$

$$\left[\because (a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca\right]$$

$$=-4x^3-4x$$

$$=-4x\left[x^2+1\right]$$

$$\therefore \left[x^2-x+1\right]^2-\left[x^2+x+1\right]^2=-4x\left[x^2+1\right]$$

Exercise – 5.1

Factorize:

1. $x^3 + x - 3x^2 - 3$

Sol:

$x^3 + x - 3x^2 - 3$

Taking x common in $\left(x^3 + x\right)$

$= x\left(x^2 + 1\right) - 3x^2 - 3$

Taking –3 common in $\left(-3x^2 - 3\right)$

$= x\left(x^2 + 1\right) - 3\left(x^2 + 1\right)$

Now, we take $\left(x^2 + 1\right)$ common

$= \left(x^2 + 1\right)\left(x - 3\right)$

$\therefore x^3 + x - 3x^2 - 3 = \left(x^2 + 1\right)\left(x - 3\right)$

2. $a\left(a+b\right)^3 - 3a^2b\left(a+b\right)$

Sol:

Taking $\left(a+b\right)$ common in two terms

$= \left(a+b\right)\left\{a\left(a+b\right)^2 - 3a^2b\right\}$

Now, using $\left(a+b\right)^2 = a^2 + b^2 + 2ab$

$= \left(a+b\right)\left\{a\left(a^2 + b^2 + 2ab\right) - 3a^2b\right\}$

$= \left(a+b\right)\left\{a^3 + ab^2 + 2a^2b - 3a^2b\right\}$

$= \left(a+b\right)\left\{a^3 + ab^2 - a^2b\right\}$

$= \left(a+b\right)a\left\{a^2 + b^2 - ab\right\}$

$= a\left(a+b\right)\left(a^2 + b^2 - ab\right)$

$\therefore a\left(a+b\right)^3 - 3a^2b\left(a+b\right) = a\left(a+b\right)\left(a^2 + b^2 - ab\right)$

3. $x\left(x^3-y^3\right)+3xy\left(x-y\right)$

Sol:

Elaborating x^3-y^3 using identity $a^3-b^3=\left(a-b\right)\left(a^2+ab+b^2\right)$

$=x\left(x-y\right)\left(x^2+xy+y^2\right)+3xy\left(x-y\right)$

Taking common $x\left(x-y\right)$ in both the terms

$=x\left(x-y\right)\left\{x^2+xy+y^2+3y\right\}$

$\therefore x\left(x^3-y^3\right)+3xy\left(x-y\right)=x\left(x-y\right)\left(x^2+xy+y^2+3y\right)$

4. $a^2x^2+\left(ax^2+1\right)x+a$

Sol:

We multiply $x\left(ax^2+1\right)=ax^3+x$

$=a^2x^2+ax^3+x+a$

Taking common ax^2 in $\left(a^2x^2+ax^3\right)$ and 1 in $\left(x+a\right)$

$=ax^2\left(a+x\right)+1\left(x+a\right)$

$=ax^2\left(a+x\right)+1\left(a+x\right)$

Taking $\left(a+x\right)$ common in both the terms

$=\left(a+x\right)\left(ax^2+1\right)$

$\therefore a^2x^2+\left(ax^2+1\right)x+a=\left(a+x\right)\left(ax^2+1\right)$

5. $x^2+y-xy-x$

Sol:

On rearranging

$x^2-xy-x+y$

Taking *x* common in the $\left(x^2+y\right)$ and -1 in $\left(-x+y\right)$

$=x\left(x-y\right)-1\left(x-y\right)$

Taking $\left(x-y\right)$ common in both the terms

$=\left(x-y\right)\left(x-1\right)$

$\therefore x^2+y-xy-x=\left(x-y\right)\left(x-1\right)$

6. $x^3-2x^2y+3xy^2-6y^3$

Sol:

Taking x^2 common in $\left(x^3-2x^2y\right)$ and $+3y^2$ common in $\left(3xy^2-6y^3\right)$

$=x^2\left(x-2y\right)+3y^2\left(x-2y\right)$

$=\left(x-2y\right)\left(x^2+3y^2\right)$

$\therefore x^3-2x^2y+3xy^2-6y^3=\left(x-2y\right)\left(x^2+3y^2\right)$

7. $6ab-b^2+12ac-2bc$

Sol:

Taking common b in $\left(6ab-b^2\right)$ and 2c in $\left(12ac-2bc\right)$

$=b\left(6a-b\right)+2c\left(6a-b\right)$

Taking $\left(6a-b\right)$ common in both terms

$=\left(6a-b\right)\left(b+2c\right)$

$\therefore 6ab-b^2+12ac-2bc=\left(6a-b\right)\left(b+2c\right)$

8. $\left[x^2+\frac{1}{x^2}\right]-4\left[x+\frac{1}{x}\right]+6$

Sol:

$=x^2+\frac{1}{x^2}-4x-\frac{4}{x}+4+2$

$=x^2+\frac{1}{x^2}+4+2-\frac{4}{x}-4x$

$=\left(x^2\right)+\left(\frac{1}{x}\right)^2+\left(-2\right)^2+2\times x\times x\frac{1}{x}+2x\frac{1}{x}\times\left(-2\right)+2\left(-2\right)x$

Using identity

$a^2+b^2+c^2+2ab+2bc+2ca=\left(a+b+c\right)^2$

We get,

$=\left[x+\frac{1}{x}+\left(-2\right)\right]^2$

$=\left[x+\frac{1}{x}-2\right]^2$

$=\left[x+\frac{1}{x}-2\right]\left[x+\frac{1}{x}-2\right]$

$\therefore\left[x^2+\frac{1}{x^2}\right]-4\left[x+\frac{1}{x}\right]+6=\left[x+\frac{1}{x}-2\right]\left[x+\frac{1}{x}-2\right]$

9. $x(x-2)(x-4)+4x-8$

Sol:

$=x(x-2)(x-4)+4(x-2)$

Taking $(x-2)$ common in both terms

$=(x-2)\{x(x-4)+4\}$

$=(x-2)\{x^2-4x+4\}$

Now splitting middle term of x^2-4x+4

$=(x-2)\{x^2-2x-2x+4\}$

$=(x-2)\{x(x-2)-2(x-2)\}$

$=(x-2)\{(x-2)(x-2)\}$

$=(x-2)(x-2)(x-2)$

$=(x-2)^3$

$\therefore x(x-2)(x-4)+4x-8=(x-2)^3$

10. $(x+2)(x^2+25)-10x^2-20x$

Sol:

$=(x+2)(x^2+25)-10x(x+2)$

Taking $(x+2)$ common in both terms

$=(x+2)(x^2+25-10x)$

$=(x+2)(x^2-10x+25)$

Splitting middle term of $x^2-10x+25$

$=(x+2)\{x^2-5x-5x+25\}$

$=(x+2)\{x(x-5)-5(x-5)\}$

$=(x+2)(x-5)(x-5)$

$\therefore (x+2)(x^2+25)-10x^2-20x=(x+2)(x-5)(x-5)$

11. $2a^2+2\sqrt{6}ab+3b^2$

Sol:

$=\left(2\sqrt{a}\right)^2+2\times\sqrt{2}a\times\sqrt{3}b+\left(\sqrt{3}b\right)^2$

Using identity $a^2+2ab+b^2=(a+b)^2$

$=\left(\sqrt{2}a+\sqrt{3}b\right)^2$

$=\left(\sqrt{2}a+\sqrt{3}b\right)\left(\sqrt{2}a+\sqrt{3}b\right)$

$\therefore 2a^2+2\sqrt{6}ab+3b^2=\left(\sqrt{2}a+\sqrt{3}b\right)\left(\sqrt{2}a+\sqrt{3}b\right)$

12. $(a-b+c)^2+(b-c+a)^2+2(a-b+c)(b-c+a)$

Sol:

Let $(a-b+c)=x$ and $(b-c+a)=y$

$=x^2+y^2+2xy$

Using identity $a^2+b^2+2ab=(a+b)^2$

$=(x+y)^2$

Now, substituting x and y

$=(a-b+c+b-c+a)^2$

Cancelling $-b,+b$ and $+c,-c$

$=(2a)^2$

$=4a^2$

$\therefore (a-b+c)^2+(b-c+a)^2+2(a-b+c)(b-c+a)=4a^2$

13. $a^2+b^2+2(ab+bc+ca)$

Sol:

$=a^2+b^2+2ab+2bc+2ca$

Using identity $a^2+b^2+2ab=(a+b)^2$

We get,

$=(a+b)^2+2bc+2ca$

$=(a+b)^2+2c(b+a)$

or $(a+b)^2+2c(a+b)$

Taking $(a+b)$ common

$=(a+b)(a+b+2c)$

$\therefore a^2+b^2+2(ab+bc+ca)=(a+b)(a+b+2c)$

14. $4(x-y)^2-12(x-y)(x+y)+9(x+y)^2$

Sol:

Let $(x-y)=a,(x+y)=b$

$=4a^2-12ab+9b^2$

Splitting middle term $-12=-6-6$ also $4\times 9=-6\times -6$

$=4a^2-6ab-6ab+9b^2$

$=2a(2a-3b)-3b(2a-3b)$

$=(2a-3b)(2a-3b)$

$=(2a-3b)^2$

Substituting $a=x-y$ and $b=x+y$

$=[2(x-y)-3(x+y)]^2$

$=[2x-2y-3x-3y]^2$

$=[2x-3x-2y-3y]^2$

$=[-x-5y]^2$

$=[(-1)(x+5y)]^2$

$=(x+5y)^2$ $\quad\left[\because (-1)^2=1\right]$

$\therefore 4(x-y)^2-12(x-y)(x+y)+9(x+y)^2=(x+5y)^2$

15. $a^2-b^2+2bc-c^2$

Sol:

$=a^2-(b^2-2bc+c^2)$

Using identity $a^2-2ab+b^2=(a-b)^2$

$=a^2-(b-c)^2$

Using identity $a^2-b^2=(a+b)(a-b)$

$=(a+b-c)(a-(b-c))$

$=(a+b-c)(a-b+c)$

$\therefore a^2-b^2+2bc-c^2=(a+b-c)(a-b+c)$

16. $a^2+2ab+b^2-c^2$

Sol:

Using identity $a^2+2ab+b^2=(a+b)^2$

$=(a+b)^2-c^2$

Using identity $a^2-b^2=(a+b)(a-b)$

$=(a+b+c)(a+b-c)$

$\therefore a^2+2ab+b^2-c^2=(a+b+c)(a+b-c)$

17. $a^2+4b^2-4ab-4c^2$

Sol:

On rearranging

$=a^2-4ab+4b^2-4c^2$

$=(a)^2-2\times a\times 2b+(2b)^2-4c^2$

Using identity $a^2-2ab+b^2=(a-b)^2$

$=(a-2b)^2-4c^2$

$=(a-2b)^2-(2c)^2$

Using identity $a^2-b^2=(a+b)(a-b)$

$=(a-2b+2c)(a-2b-2c)$

$\therefore a^2+4b^2-4ab-4c^2=(a-2b+2c)(a-2b-2c)$

18. xy^9-yx^9

Sol:

xy^9-yx^9

$=xy\left(y^8-x^8\right)$

$=xy\left(\left(y^4\right)^2-\left(x^4\right)^2\right)$

Using identity $a^2-b^2=(a+b)(a-b)$

$=xy\left(y^4+x^4\right)\left(y^4-x^4\right)$

$=xy\left(y^4+x^4\right)\left(\left(y^2\right)^2-\left(x^2\right)^2\right)$

Using identity $a^2-b^2=(a+b)(a-b)$

$= xy\left(y^4 + x^4\right)\left(y^2 + x^2\right)\left(y^2 - x^2\right)$

$= xy\left(y^4 + x^4\right)\left(y^2 + x^2\right)\left(y + x\right)\left(y - x\right)$

$= xy\left(x^4 + y^4\right)\left(x^2 + y^2\right)\left(x + y\right)\left(-1\right)\left(x - y\right)$

$\because \left(b - a\right) = -1\left(a - b\right)$

$= -xy\left(x^4 + y^4\right)\left(x^2 + y^2\right)\left(x + y\right)\left(x - y\right)$

$\therefore xy^9 - yx^9 = -xy\left(x^4 + y^4\right)\left(x^2 + y^2\right)\left(x + y\right)\left(x - y\right)$

19. $x^4 + x^2y^2 + y^4$

Sol:

Adding x^2y^2 and subtracting x^2y^2 to the given equation

$= x^4 + x^2y^2 + y^4 + x^2y^2 - x^2y^2$

$= x^4 + 2x^2y^2 + y^4 - x^2y^2$

$= \left(x^2\right)^2 + 2 \times x^2 \times y^2 + \left(y^2\right)^2 - \left(xy\right)^2$

Using identity $a^2 + 2ab + b^2 = \left(a + b\right)^2$

$= \left(x^2 + y^2\right)^2 - \left(xy\right)^2$

Using identity $a^2 - b^2 = \left(a + b\right)\left(a - b\right)$

$= \left(x^2 + y^2 + xy\right)\left(x^2 + y^2 - xy\right)$

$\therefore x^4 + x^2y^2 + y^4 = \left(x^2 + y^2 + xy\right)\left(x^2 + y^2 - xy\right)$

20. $x^2 - y^2 - 4xz + 4z^2$

Sol:

On rearranging the terms

$= x^2 - 4xz + 4z^2 - y^2$

$= \left(x\right)^2 - 2 \times x \times 2z + \left(2z\right)^2 - y^2$

Using identity $a^2 - 2ab + b^2 = \left(a - b\right)^2$

$= \left(x - 2z\right)^2 - y^2$

Using identity $a^2 - b^2 = \left(a + b\right)\left(a - b\right)$

$= \left(x - 2z + y\right)\left(x - 2z - y\right)$

$\therefore x^2 - y^2 - 4xz + 4z^2 = \left(x - 2z + y\right)\left(x - 2z - y\right)$

21. $x^2+6\sqrt{2}x+10$

Sol:

Splitting middle term,

$=x^2+5\sqrt{2}x+\sqrt{2}x+10 \qquad \left[\because 6\sqrt{2}=5\sqrt{2}+\sqrt{2} \text{ and } 5\sqrt{2}\times\sqrt{2}=10\right]$

$=x\left(x+5\sqrt{2}\right)+\sqrt{2}\left(x+5\sqrt{2}\right)$

$=\left(x+5\sqrt{2}\right)\left(x+\sqrt{2}\right)$

$\therefore x^2+6\sqrt{2}x+10=\left(x+5\sqrt{2}\right)\left(x+\sqrt{2}\right)$

22. $x^2-2\sqrt{2}x-30$

Sol:

Splitting the middle term,

$=x^2-5\sqrt{2}x+3\sqrt{2}x-30 \qquad \left[\because -2\sqrt{2}=-5\sqrt{2}+3\sqrt{2} \text{ also } -5\sqrt{2}\times3\sqrt{2}=-30\right]$

$x\left(x-5\sqrt{2}\right)+3\sqrt{2}\left(x-5\sqrt{2}\right)$

$=\left(x-5\sqrt{2}\right)\left(x+3\sqrt{2}\right)$

$\therefore x^2-2\sqrt{2}x-30=\left(x-5\sqrt{2}\right)\left(x+3\sqrt{2}\right)$

23. $x^2-\sqrt{3}x-6$

Sol:

Splitting the middle term,

$=x^2-2\sqrt{3}x+\sqrt{3}x-6 \qquad \left[\because -\sqrt{3}=-2\sqrt{3}+\sqrt{3} \text{ also } -2\sqrt{3}\times\sqrt{3}=-6\right]$

$=x\left(x-2\sqrt{3}\right)+\sqrt{3}\left(x-2\sqrt{3}\right)$

$=\left(x-2\sqrt{3}\right)\left(x+\sqrt{3}\right)$

$\therefore x^2-\sqrt{3}x-6=\left(x-2\sqrt{3}\right)\left(x+\sqrt{3}\right)$

24. $x^2+5\sqrt{5}x+30$

Sol:

Splitting the middle term,

$=x^2+2\sqrt{5}x+3\sqrt{5}x+30 \qquad \left[\because 5\sqrt{5}=2\sqrt{5}+3\sqrt{5} \text{ also } 2\sqrt{5}\times\sqrt{3}=30\right]$

$=x\left(x+2\sqrt{5}\right)+3\sqrt{5}\left(x+2\sqrt{5}\right)$

$=\left(x+2\sqrt{5}\right)\left(x+3\sqrt{5}\right)$

$\therefore x^2+5\sqrt{5}x+30=\left(x+2\sqrt{5}\right)\left(x+3\sqrt{5}\right)$

25. $x^2+2\sqrt{3}x-24$

Sol:

Splitting the middle term,

$=x^2+4\sqrt{3}x-2\sqrt{3}x-24 \qquad \left[\because 2\sqrt{3}=4\sqrt{3}-2\sqrt{3} \text{ also } 4\sqrt{3}\left(-2\sqrt{3}\right)=-24\right]$

$=x\left(x+4\sqrt{3}\right)-2\sqrt{3}\left(x+4\sqrt{3}\right)$

$=\left(x+4\sqrt{3}\right)\left(x-2\sqrt{3}\right)$

$\therefore x^2+2\sqrt{3}x-24=\left(x+4\sqrt{3}\right)\left(x-2\sqrt{3}\right)$

26. $2x^2-\frac{5}{6}x+\frac{1}{12}$

Sol:

Splitting the middle term,

$=2x^2-\frac{x}{2}-\frac{x}{3}+\frac{1}{12} \qquad \left[\because -\frac{5}{6}=-\frac{1}{2}-\frac{1}{3} \text{ also } -\frac{1}{2}\times-\frac{1}{3}=2\times\frac{1}{12}\right]$

$=x\left(2x-\frac{1}{2}\right)-\frac{1}{6}\left(2x-\frac{1}{2}\right)$

$=\left(2x-\frac{1}{2}\right)\left(x-\frac{1}{6}\right)$

$\therefore 2x^2-\frac{5}{6}x+\frac{1}{12}=\left(2x-\frac{1}{2}\right)\left(x-\frac{1}{6}\right)$

27. $x^2+\frac{12}{35}x+\frac{1}{35}$

Sol:

Splitting the middle term,

$=x^2+\frac{5}{35}x+\frac{7}{35}x+\frac{1}{35} \qquad \left[\because \frac{12}{35}=\frac{5}{35}+\frac{7}{35} \text{ and } \frac{5}{35}\times\frac{7}{35}=\frac{1}{35}\right]$

$=x^2+\frac{x}{7}+\frac{x}{5}+\frac{1}{35}$

$=x\left(x+\frac{1}{7}\right)+\frac{1}{5}\left(x+\frac{1}{7}\right)$

$=\left(x+\frac{1}{7}\right)\left(x+\frac{1}{5}\right)$

$\therefore x^2+\frac{12}{35}x+\frac{1}{35}=\left(x+\frac{1}{7}\right)\left(x+\frac{1}{5}\right)$

28. $21x^2 - 2x + \frac{1}{21}$

Sol:

$= \left(\sqrt{21}x\right)^2 - 2 \times \sqrt{21}x \times \frac{1}{\sqrt{21}} + \left(\frac{1}{\sqrt{21}}\right)^2$

Using identity $a^2 - 2ab + b^2 = (a-b)^2$

$= \left(\sqrt{21}x - \frac{1}{\sqrt{21}}\right)^2$

$\therefore 21x^2 - 2x + \frac{1}{21} = \left(\sqrt{21}x - \frac{1}{\sqrt{21}}\right)^2$

29. $5\sqrt{5}x^2 + 20x + 3\sqrt{5}$

Sol:

Splitting the middle term,

$= 5\sqrt{5}x^2 + 15x + 5x + 3\sqrt{5}$ $\left[\because 20 = 15 + 5 \text{ and } 15 \times 5 = 5 = 5\sqrt{5} \times 3\sqrt{5}\right]$

$= 5x\left(\sqrt{5}x + 3\right) + \sqrt{5}\left(\sqrt{5}x + 3\right)$

$= \left(\sqrt{5}x + 3\right)\left(5x + \sqrt{5}\right)$

$\therefore 5\sqrt{5}x^2 + 20x + 3\sqrt{5} = \left(\sqrt{5}x + 3\right)\left(5x + \sqrt{5}\right)$

30. $2x^2 + 3\sqrt{5}x + 5$

Sol:

Splitting the middle term,

$= 2x^2 + 2\sqrt{5}x + \sqrt{5}x + 5$ $\left[\because 3\sqrt{5} = 2\sqrt{5} + \sqrt{5} \text{ also } 2\sqrt{5} \times \sqrt{5} = 2 \times 5\right]$

$= 2x\left(x + \sqrt{5}\right) + \sqrt{5}\left(x + \sqrt{5}\right)$

$= \left(x + \sqrt{5}\right)\left(2x + \sqrt{5}\right)$

$\therefore 2x^2 + 3\sqrt{5}x + 5 = \left(x + \sqrt{5}\right)\left(2x + \sqrt{5}\right)$

31. $9(2a-b)^2 - 4(2a-b) - 13$

Sol:

Let $2a - b = x$

$= 9x^2 - 4x - 13$

Splitting the middle term,

$= 9x^2 - 13x + 9x - 13$

$= x(9x-13) + 1(9x-13)$

$= (9x-13)(x+1)$

Substituting $x = 2a - b$

$= [9(2a-b)-13](2a-b+1)$

$= (18a-9b-13)(2a-b+1)$

$\therefore 9(2a-b)^2 - 4(2a-b) - 13 = (18a-9b-13)(2a-b+1)$

32. $7(x-2y)^2 - 25(x-2y) + 12$

Sol:

Let $x - 2y = P$

$= 7P^2 - 25P + 12$

Splitting the middle term,

$= 7P^2 - 21P - 4P + 12$

$= 7P(P-3) - 4(P-3)$

$= (P-3)(7P-4)$

Substituting $P = x - 2y$

$= (x-2y-3)(7(x-2y)-4)$

$= (x-2y-3)(7x-14y-4)$

$\therefore 7(x-2y)^2 - 25(x-2y) + 12 = (x-2y-3)(7x-14y-4)$

33. $2(x+y)^2 - 9(x+y) - 5$

Sol:

Let $x + y = z$

$= 2z^2 - 9z - 5$

Splitting the middle term,

$= 2z^2 - 10z + z - 5$

$= 2z(z-5) + 1(z-5)$

$= (z-5)(2z+1)$

Substituting $z = x + y$

$= (x+y-5)(2(x+y)+1)$

$= (x+y-5)(2x+2y+1)$

$\therefore 2(x+y)^2 - 9(x+y) - 5 = (x+y-5)(2x+2y+1)$

34. Given possible expressions for the length and breadth of the rectangle having $35y^2 + 13y - 12$ as its area.

Sol:

Area $= 35y^2 + 13y - 12$

Splitting the middle term,

Area $= 35y^2 + 28y - 15y - 12$

$= 7y(5y+4) - 3(5y+4)$

Area $= (5y+4)(7y-3)$

Also area of rectangle = Length $\times$ Breadth

$\therefore$ Possible length $= (5y+4)$ and breadth $= (7y-3)$

Or Possible length $= (7y-3)$ and breadth $= (5y+4)$

35. What are the possible expressions for the dimensions of the cuboid whose volume is $3x^2 - 12x$.

Sol:

Here volume $= 3x^2 - 12x$

$= 3x(x-4)$

$= 3 \times x(x-4)$

Also volume = Length $\times$ Breadth $\times$ Height

$\therefore$ Possible expressions for dimensions of the cuboid are $= 3, x, (x-4)$

Exercise – 5.2

Factorize each of the following expressions :

1. p^3+27

Sol:

p^3+27

$=p^3+3^3$

$=(p+3)(p^2-3p+3^2)$ $\left[\because a^3+b^3=(a+b)(a^2-ab+b^2)\right]$

$=(p+3)(p^2-3p+9)$

$\therefore p^3+27=(p+3)(p^2-3p+9)$

2. y^3+125

Sol:

$=y^3+5^3$

$=(y+5)(y^2-5y+5^2)$ $\left[\because a^3+b^3=(a+b)(a^2-ab+b^2)\right]$

$=(y+5)(y^2-5y+25)$

$\therefore y^3+125=(y+5)(y^2-5y+25)$

3. $1-27a^3$

Sol:

$=(1)^3-(3a)^3$

$=(1-3a)\left(1^2+1\times 3a+(3a)^2\right)$ $\left[\because a^3-b^3=(a-b)(a^2+ab+b^2)\right]$

$=(1-3a)(1+3a+9a^2)$

$\therefore 1-27a^3=(1-3a)(1+3a+9a^2)$

4. $8x^3y^3+27a^3$

Sol:

$=(2xy)^3+(3a)^3$

$=(2xy+3a)\left((2xy)^2-2xy\times 3a+(3a)^2\right)$ $\left[\because a^3+b^3=(a+b)(a^2-ab+b^2)\right]$

$=(2xy+3a)(4x^2y^2-6xya+9a^2)$

$\therefore 8x^3y^3+27a^3=(2xy+3a)(4x^2y^2-6xya+9a^2)$

5. $64a^3 - b^3$

Sol:

$= (4a)^3 - b^3$

$= (4a - b)\left((4a)^2 + 4a \times b + b^2\right)$ $\left[\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)\right]$

$= (4a - b)(16a^2 + 4ab + b^2)$

$\therefore 64a^3 - b^3 = (4a - b)(16a^2 + 4ab + b^2)$

6. $\frac{x^3}{216} - 8y^3$

Sol:

$= \left(\frac{x}{6}\right)^3 - (2y)^3$

$= \left(\frac{x}{6} - 2y\right)\left(\left(\frac{x}{6}\right)^2 + \frac{x}{6} \times 2y + (2y)^2\right)$ $\left[\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)\right]$

$= \left(\frac{x}{6} - 2y\right)\left(\frac{x^2}{36} + \frac{xy}{3} + 4y^2\right)$

$\therefore \frac{x^3}{216} - 8y^3 = \left(\frac{x}{6} - 2y\right)\left(\frac{x^2}{36} + \frac{xy}{3} + 4y^2\right)$

7. $10x^4y - 10xy^4$

Sol:

$10x^4y - 10xy^4$

$= 10xy(x^3 - y^3)$

$= 10xy(x - y)(x^2 + xy + y^2)$ $\left[\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)\right]$

$\therefore 10x^4y - 10xy^4 = 10xy(x - y)(x^2 + xy + y^2)$

8. $54x^6y + 2x^3y^4$

Sol:

$54x^6y+2x^3y^4$

$=2x^3y\left(27x^3+y^3\right)$

$=2x^3y\left(\left(3x\right)^3+y^3\right)$

$=2x^3y\left(3x+y\right)\left(\left(3x\right)^2-3\times xy+y^2\right)$ $\left[\because a^3+b^3=\left(a+b\right)\left(a^2-ab+b^2\right)\right]$

$=2x^3y\left(3x+y\right)\left(9x^2-3xy+y^2\right)$

$\therefore 54x^6y+2x^3y^4=2x^3y\left(3x+y\right)\left(9x^2-3xy+y^2\right)$

9. $32a^3 + 108b^3$

Sol:

$32a^3+108b^3$

$=4\left(8a^3+27b^3\right)$

$=4\left(\left(2a\right)^3+\left(3b\right)^3\right)$ [Using $a^3+b^3=\left(a+b\right)\left(a^2-ab+b^2\right)$]

$=4\left[\left(2a+3b\right)\left(\left(2a\right)^2-2a\times 3b+\left(3b\right)^2\right)\right]$

$=4\left(2a+3b\right)\left(4a^2-6ab+9b^2\right)$

$\therefore 32a^3+108b^3=4\left(2a+3b\right)\left(4a^2-6ab+9b^2\right)$

10. $\left(a-2b\right)^3-512b^3$

Sol:

$\left(a-2b\right)^3-512b^3$

$=\left(a-2b\right)^3-\left(8b\right)^3$

$=\left(a-2b-8b\right)\left(\left(a-2b\right)^2+\left(a-2b\right)8b+\left(8b\right)^2\right)$ $\left[\because a^3-b^3=\left(a-b\right)\left(a^2+ab+b^2\right)\right]$

$=\left(a-10b\right)\left(a^2+4b^2-4ab+8b\left(a-2b\right)+\left(8b\right)^2\right)$ $\left[\because \left(a-b\right)^2=a^2+b^2-2ab\right]$

$=\left(a-10b\right)\left(a^2+4b^2-4ab+8ab-16b^2+64b^2\right)$

$=\left(a=10b\right)\left(a^2+68b^2-16b^2-4ab+8ab\right)$

$=\left(a-10b\right)\left(a^2+52b^2+4ab\right)$

$\therefore \left(a-2b\right)^3-512b^3=\left(a-10b\right)\left(a^2+4ab+52b^2\right)$

11. $(a + b)^3 – 8(a – b)^3$

Sol:

$(a+b)^3-8(a-b)^3$

$=(a+b)^3-\left[2(a-b)\right]^3$

$=(a+b)^3-(2a-2b)^3$ [Using $a^3-b^3=(a-b)\left(a^2+ab+b^2\right)$]

$=(a+b-(2a-2b))\left((a+b)^2+(a+b)(2a-2b)+(2a-2b)^2\right)$

$=(a+b-2a+2b)\left(a^2+b^2+2ab+(a+b)(2a-2b)+(2a-2b)^2\right)$ $\left[\because (a+b)^2=a^2+b^2+2ab\right]$

$=(3b-a)\left(a^2+b^2+2ab+2a^2-2ab+2ab-2b^2+(2a-2b)^2\right)$

$=(3b-a)\left(3a^2+2ab-b^2+(2a-2b)^2\right)$

$=(3b-a)\left(3a^2+2ab-b^2+4a^2+4b^2-8ab\right)$ $\left[\because (a-b)^2=a^2+b^2-2ab\right]$

$=(3b-a)\left(3a^2+4a^2-b^2+4b^2+2ab-8ab\right)$

$=(3b-a)\left(7a^2+3b^2-6ab\right)$

$\therefore (a+b)^3-8(a-b)^3=(-a+3b)\left(7a^2-6ab+3b^2\right)$

12. $(x+2)^3+(x-2)^3$

Sol:

$=(x+2+x-2)\left((x+2)^2-(x+2)(x-2)+(x-2)^2\right)$ $\left[\because a^3+b^3-(a+b)\left(a^2-ab+b^2\right)\right]$

$=2x\left(x^2+4x+4-(x+2)(x-2)+x^2-4x+4\right)$ $\left[\because (a+b)^2=a^2+2ab+b^2,(a-b)^2=a^2-2ab+b^2\right]$

$=2x\left(2x^2+8-\left(x^2-2^2\right)\right)$ $\left[\because (a+b)(a-b)=a^2-b^2\right]$

$=2x\left(2x^2+8-x^2+4\right)$

$=2x\left(x^2+12\right)$

$\therefore (x+2)^3+(x-2)^3=2x\left(x^2+12\right)$

13. $8x^2y^3-x^5$

Sol:

$=x^2\left(8y^3-x^3\right)$

$=x^2\left((2y)^3-x^3\right)$

$= x^2(2y - x)((2y)^2 + 2y(x) + x^2)$ $\quad [\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)]$

$= x^2(2y - x)(4y^2 + 2xy + x^2)$

$\therefore 8x^2y^3 - x^5 = x^2(2y - x)(4y^2 + 2xy + x^2)$

14. $1029 - 3x^3$

Sol:

$= 3(343 - x^3)$

$= 3(7^3 - x^3)$

$= 3(7 - x)(7^2 + 7 \times x + x^2)$ $\quad [\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)]$

$= 3(7 - x)(49 + 7x + x^2)$

$\therefore 1029 - 3x^3 = 3(7 - x)(49 + 7x + x^2)$

15. $x^6 + y^6$

Sol:

$x^6 + y^6$

$= (x^2)^3 + (y^2)^3$

$= (x^2 + y^2)((x^2)^2 - x^2y^2 + (y^2)^2)$ $\quad [\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)]$

$= (x^2 + y^2)(x^4 - x^2y^2 + y^4)$

$\therefore x^6 + y^6 = (x^2 + y^2)(x^4 - x^2y^2 + y^4)$

16. $x^3y^3 + 1$

Sol:

$= (xy)^3 + 1^3$

$= (xy + 1)((xy)^2 - xy \times 1 + 1^2)$ $\quad [\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)]$

$= (xy + 1)(x^2y^2 - xy + 1)$

$\therefore x^3y^3 + 1 = (xy + 1)(x^2y^2 - xy + 1)$

17. $x^4y^4 - xy$

Sol:

$= xy\left(x^3y^3 - 1\right)$

$= xy\left(\left(xy\right)^3 - 1^3\right)$

$= xy\left(xy-1\right)\left(\left(xy\right)^2 + \left(xy\right)1 + 1^2\right)$ $\left[\because a^3 - b^3 = \left(a-b\right)\left(a^2+ab+b^2\right)\right]$

$= xy\left(xy-1\right)\left(x^2y^2 + xy + 1\right)$

$\therefore x^4y^4 - xy = xy\left(xy-1\right)\left(x^2y^2+xy+1\right)$

18. $a^{12} + b^{12}$

Sol:

$= \left(a^4\right)^3 + \left(b^4\right)^3$

$= \left(a^4\right)^3 + \left(b^4\right)^3$

$= \left(a^4+b^4\right)\left(\left(a^4\right)^2 - a^4 \times b^4 + \left(b^4\right)^2\right)$ $\left[\because a^3 + b^3 = \left(a+b\right)\left(a^2-ab+b^2\right)\right]$

$= \left(a^4+b^4\right)\left(a^8 - a^4b^4 + b^8\right)$

$\therefore a^{12} + b^{12} = \left(a^4+b^4\right)\left(a^8 - a^4b^4 + b^8\right)$

19. $x^3 + 6x^2 + 12x + 16$

Sol:

$= x^3 + 6x^2 + 12x + 8 + 8$

$= x^3 + 3\times x^2 \times 2 + 3 \times x \times 2^2 + 2^3 + 8$

$= \left(x = 2\right)^3 + 8$ $\left[\because a^3 + 3a^2b + 3ab^2 + b^3 = \left(a+b\right)^3\right]$

$= \left(x+2\right)^3 + 2^3$

$= \left(x+2+2\right)\left(\left(x+2\right)^2 - 2\left(x+2\right) + 2^2\right)$ $\left[\because a^3 + b^3 = \left(a+b\right)\left(a^2-ab+b^2\right)\right]$

$= \left(x+4\right)\left(x^2+4x+4-2x-4+4\right)$ $\left[\because \left(a+b\right)^2 = a^2+2ab+b^2\right]$

$= \left(x+4\right)\left(x^2+2x+4\right)$

$\therefore x^3 + 6x^2 + 12x + 16 = \left(x+4\right)\left(x^2+2x+4\right)$

20. a^3+b^3+a+b

Sol:

$=\left(a^3+b^3\right)+1\left(a+b\right)$

$=\left(a+b\right)\left(a^2-ab+b^2\right)+1\left(a+b\right)$

$=\left(a+b\right)\left(a^2-ab+b^2+1\right)$

$\therefore a^3+b^3+a+b=\left(a+b\right)\left(a^2-ab+b^2+1\right)$

21. $a^3-\frac{1}{a^3}-2a+\frac{2}{a}$

Sol:

$=\left(a^3-\frac{1}{a^3}\right)-2\left(a-\frac{1}{a}\right)$

$=\left(a^3-\left(\frac{1}{a}\right)^3\right)-2\left(a-\frac{1}{a}\right)$

$=\left(a-\frac{1}{a}\right)\left(a^2+a\times\frac{1}{a}+\left(\frac{1}{a}\right)^2\right)-2\left(a-\frac{1}{a}\right)$ $\left[\because a^3-b^3=\left(a-b\right)\left(a^2+ab+b^2\right)\right]$

$=\left(a-\frac{1}{a}\right)\left(a^2+1+\frac{1}{a^2}\right)-2\left(a-\frac{1}{a}\right)$

$=\left(a-\frac{1}{a}\right)\left(a^2+1+\frac{1}{a^2}-2\right)$

$=\left(a-\frac{1}{a}\right)\left(a^2+\frac{1}{a^2}-1\right)$

$\therefore a^3-\frac{1}{a^3}-2a+\frac{2}{a}=\left(a-\frac{1}{a}\right)\left(a^2+\frac{1}{a^2}-1\right)$

22. $a^3+3a^2b+3ab^2+b^3-8$

Sol:

$=\left(a+b\right)^3-8$ $\left[\because a^3+3a^2b+3ab^2+b^3=\left(a+b\right)^3\right]$

$=\left(a+b\right)^3-2^3$

$=\left(a+b-2\right)\left(\left(a+b\right)^2+\left(a+b\right)2+2^2\right)$

$=\left(a+b-2\right)\left(a^2+b^2+2ab+2a+2b+4\right)$

$\therefore a^3+3a^2b+3ab^2+b^3-8=\left(a+b-2\right)\left(a^2+b^2+2ab+2a+2b+4\right)$

23. $8a^3 - b^3 - 4ax + 2bx$

Sol:

$= 8a^3 - b^3 - 2x(2a - b)$

$= (2a)^3 - b^3 - 2x(2a - b)$

$= (2a - b)\left((2a)^2 + 2a \times b + b^2\right) - 2x(2a - b) \quad \left[\because a^3 - b^3 = (a - b)\left(a^2 + ab + b^2\right)\right]$

$= (2a - b)\left(4a^2 + 2ab + b^2\right) - 2x(2a - b)$

$= (2a - b)\left(4a^2 + 2ab + b^2 - 2x\right)$

$\therefore 8a^3 - b^3 - 4ax + 2bx = (2a - b)\left(4a^2 + 2ab + b^2 - 2x\right)$

24. Simplify:

Sol:

(i) $\dfrac{173 \times 173 \times 173 + 127 \times 127 \times 127}{173 \times 173 - 173 \times 127 + 127 \times 127}$

$= \dfrac{173^2 + 127^3}{173^2 - 173 \times 127 + 127^2}$

$= \dfrac{(173 + 127)\left(173^2 - 173 \times 127 + 127^2\right)}{\left(173^2 - 173 \times 127 + 127^2\right)} \quad \left[\because a^3 + b^3 = (a + b)\left(a^2 - ab + b^2\right)\right]$

$= (173 + 127) = 300$

(ii) $\dfrac{155 \times 155 \times 155 - 55 \times 55 \times 55}{155 \times 155 + 155 \times 55 + 55 \times 55}$

$= \dfrac{155^3 - 55^3}{155^2 + 155 \times 55 + 55^2}$

$= \dfrac{(155 - 55)\left(155^2 + 155 \times 55 + 55^2\right)}{\left(155^2 + 155 \times 55 + 55^2\right)} \quad \left[\because a^3 - b^3 = (a - b)\left(a^2 + ab + b^2\right)\right]$

$= (155 - 55) = 100$

Exercise – 5.3

Factorize:

1. $64a^3+125b^3+240a^2b+300ab^2$

Sol:

$64a^3+125b^3+240a^2b+300ab^2$

$=(4a)^3+(5b)^3+3\times(4a)^2\times 5b+3(4a)(5b)^2$

$=(4a+5b)^3$ $\quad\left[\because a^3+b^3+3a^2b+3ab^2=(a+b)^3\right]$

$=(4a+5b)(4a+5b)(4a+5b)$

$\therefore 64a^3+125b^3+240a^2b+300ab^2=(4a+5b)(4a+5b)(4a+5b)$

2. $125x^3-27y^3-225x^2y+135xy^2$

Sol:

$125x^3-27y^3-225x^2y+135xy^2$

$=(5x)^3-(3y)^3-3\times(5x)^2\times 3y+3\times(5x)(3y)^2$

$=(5x-3y)^3$ $\quad\left[\because a^3-b^3-3a^2b+3ab^2=(a-b)^3\right]$

$=(5x-3y)(5x-3y)(5x-3y)$

$\therefore 125x^3-27y^3-225x^2y+135xy^2=(5x-3y)(5x-3y)(5x-3y)$

3. $\frac{8}{27}x^3+1+\frac{4}{3}x^2+2x$

Sol:

$=\left(\frac{2}{3}x\right)^3+(1)^3+3\times\left(\frac{2}{3}x\right)^2\times 1+3(1)^2\times\left(\frac{2}{3}x\right)$

$=\left(\frac{2}{3}x+1\right)^3$ $\quad\left[\because a^3+b^3+3a^2b+3ab^2=(a+b)^3\right]$

$=\left(\frac{2}{3}x+1\right)\left(\frac{2}{3}x+1\right)\left(\frac{2}{3}x+1\right)$

$\therefore \frac{8}{27}x^3+1+\frac{4}{3}x^2+2x=\left(\frac{2}{3}x+1\right)\left(\frac{2}{3}x+1\right)\left(\frac{2}{3}x+1\right)$

4. $8x^3+27y^3+36x^2y+54xy^2$

Sol:

$8x^3+27y^3+36x^2y+54xy^2$

$=(2x)^3+(3y)^3+3\times(2x)^2\times3y+3\times(2x)(3y)^2$

$=(2x+3y)^3$ $\quad\left[\because a^3+b^3+3a^2b+3ab^2=(a+b)^3\right]$

$=(2x+3y)(2x+3y)(2x+3y)$

$\therefore\quad 8x^3+27y^3+36x^2y+54xy^2=(2x+3y)(2x+3y)(2x+3y)$

5. $a^3 - 3a^2b + 3ab^2 - b^3 + 8$

Sol:

$a^3-3a^2b+3ab^2-b^3+8$

$=(a-b)^3+8$ $\quad\left[\because a^3-b^3-3a^2b+3ab^2=(a-b)^3\right]$

$=(a-b)^3+2^3$

$=(a-b+2)\left((a-b)^2-(a-b)2+2^2\right)$ $\quad\left[\because a^3+b^3=(a+b)\left(a^2-ab+b^2\right)\right]$

$=(a-b+2)\left(a^2+b^2-2ab-2(a-b)+4\right)$

$=(a-b+2)\left(a^2+b^2-2ab-2a+2b+4\right)$

$\therefore a^3-3a^2b+3ab^2-b^3+8=(a-b+2)\left(a^2+b^2-2ab-2a+2b+4\right)$

6. $x^3+8y^3+6x^2y+12xy^2$

Sol:

$=(x)^3+(2y)^3+3\times x^2\times2y+3\times x\times(2y)^2$

$=(x+2y)^3$ $\quad\left[\because a^3+b^3+3a^2b+3ab^2=(a+b)^3\right]$

$=(x+2y)(x+2y)(x+2y)$

$\therefore x^3+8y^3+6x^2y+12xy^2=(x+2y)(x+2y)(x+2y)$

7. $8x^2+y^3+12x^2y+6xy^2$

Sol:

$8x^2+y^3+12x^2y+6xy^2$

$=(2x)^3+y^3+3\times(2x)^2\times y+3(2x)\times y^2$

$=(2x+y)^3$ $\quad\left[\because a^3+b^3+3a^2b+3ab^2=(a+b)^3\right]$

$=(2x+y)(2x+y)(2x+y)$

$\therefore 8x^3+y^3+12x^2y+6xy^2=(2x+y)(2x+y)(2x+y)$

8. $8a^3+27b^3+36a^2b+54ab^2$

Sol:

$=(2a)^3+(3b)^3+3\times(2a)^2\times 3b+3\times(2a)(3b)^2$

$=(2a+3b)^3$ $\quad\left[\because a^3+b^3+3a^2b+3ab^2=(a+b)^3\right]$

$=(2a+3b)(2a+3b)(2a+3b)$

$\therefore 8a^3+27b^3+36a^2b+54ab^2=(2a+3b)(2a+3b)(2a+3b)$

9. $8a^3-27b^3-36a^2b+54ab^2$

Sol:

$8a^3-27b^3-36a^2b+54ab^2$

$=(2a)^3-(3b)^3-3\times(2a)^2 3b+3(2a)(3b)^2$

$=(2a-3b)^3$ $\quad\left[\because a^3-b^3-3a^2b+3ab^2=(a-b)^3\right]$

$=(2a-3b)(2a-3b)(2a-3b)$

$\therefore 8a^3-27b^3-36a^2b+54ab^2=(2a-3b)(2a-3b)(2a-3b)$

10. $x^3-12x(x-4)-64$

Sol:

$x^3-12x(x-4)-64$

$=x^3-12x^2+48x-64$

$=(x)^3-3\times x^2\times 4+3\times 4^2\times x-4^3$

$=(x-4)^3$ $\quad\left[\because a^3-3a^2b+3ab^2-b^3=(a-b)^3\right]$

$=(x-4)(x-4)(x-4)$

$\therefore x^3-12x(x-4)-64=(x-4)(x-4)(x-4)$

11. $a^3x^3-3a^2bx^2+3ab^2x-b^3$

Sol:

$=(ax)^3-3(ax)^2\times b+3(ax)b^2-b^3$

$=(ax-b)^3$ $\quad\left[\because a^2-3a^2b+3ab^2-b^3=(a-b)^3\right]$

$=(ax-b)(ax-b)(ax-b)$

$\therefore a^3x^3-3a^2bx^2+3ab^2x-b^3=(ax-b)(ax-b)(ax-b)$

Exercise – 5.4

1. $a^3+8b^3+64c^3-24abc$

Sol:

$a^3+8b^3+64c^3-24abc$

$=(a)^3+(2b)^3+(4c)^3-3\times a\times 2b\times 4c$

$=(a+2b+4c)\left(a^2+(2b)^2+(4c)^2-a\times 2b-2b\times 4c-4c\times a\right)$

$\left[\because a^3+b^3+c^3-3abc=(a+b+c)\left(a^2+b^2+c^2-ab-bc-ca\right)\right]$

$=(a+2b+4c)\left(a^2+4b^2+16c^2-2ab-8bc-4ac\right)$

$\therefore a^3+8b^3+64c^3-24abc=(a+2b+4c)\left(a^2+4b^2+16c^2-2ab-8bc-4ac\right)$

2. $x^3-8y^3+27z^3+18xyz$

Sol:

$=x^3+(-2y)^3+(3z)^3-3\times x\times(-2y)(3z)$

$=(x+(-2y)+3z)\left(x^2+(-2y)^2+(3z)^2-x(-2y)-(-2y)(3z)-3z(x)\right)$

$\left[\because a^3+b^3+c^3-3abc=(a+b+c)\left(a^2+b^2+c^2-ab-bc-ca\right)\right]$

$=(x-2y+3z)\left(x^2+4y^2+9z^2+2xy+6yz-3zx\right)$

$\therefore x^3-8y^3+27z^3+18xyz=(x-2y+3z)\left(x^2+4y^2+9z^2+2xy+6yz-3zx\right)$

3. $\frac{1}{27}x^3-y^3+125z^3+5xyz$

Sol:

$\frac{1}{27}x^3-y^3+125z^3+5xyz$

$=\left(\frac{x}{3}\right)^3+(-y)^3+(5z)^3-3\times\frac{x}{3}(-y)(5z)$

$=\left(\frac{x}{3}+(-y)+5z\right)\left(\left(\frac{x}{3}\right)^2+(-y)^2+(5z)^2-\frac{x}{3}(-y)-(-y)5z-5z\left(\frac{x}{3}\right)\right)$

$=\left(\frac{x}{3}-y+5z\right)\left(\frac{x^2}{9}+y^2+25z^2+\frac{xy}{3}+5xyz-\frac{5}{3}zx\right)$

$\therefore \frac{1}{27}x^3-y^3+125z^3+5xyz=\left(\frac{x}{3}-y+5z\right)\left(\frac{x^2}{9}+y^2+25z^2+\frac{xy}{3}+5yz-\frac{5}{3}zx\right)$

4. $8x^3+27y^3-216z^3+108xyz$

Sol:

$=(2x)^3+(3y)^3+(-6z)^3-3(2x)(3y)(-6z)$

$=(2x+3y+(-6z))\left((2x)^2+(3y)^2+(-6z)^2-2x\times 3y-3y(-6z)-(-6z)2x\right)$

$=(2x+3y-6z)\left(4x^2+9y^2+36z^2-6xy+18yz+12zx\right)$

$\therefore 8x^3+27y^3-216z^3+108xyz=(2x+3y-6z)\left(4x^2+9y^2+36z^2-6xy+18yz+12zx\right)$

5. $125+8x^3-27y^3+90xy$

Sol:

$=5^3+(2x)^3+(-3y)^3-3\times 5\times 2x\times(-3y)$

$=(5+2x+(-3y))\left(5^2+(2x)^2+(-3y)^2-5(2x)-2x(-3y)-(-3y)5\right)$

$=(5+2x-3y)\left(25+4x^2+9y^2-10x+6xy+15y\right)$

$\therefore 125+8x^3-27y^3+90xy=(5+2x-3y)\left(25+4x^2+9y^2-10x+6xy+15y\right)$

6. $(3x-2y)^3+(2y-4z)^3+(4z-3x)^3$

Sol:

Let $(3x-2y)=a,(2y-4z)=b,(4z-3x)=c$

$\therefore a+b+c=3x-2y+2y-4z+4z-3x=0$

$\because a+b+c=0 \therefore a^3+b^3+c^3=3abc$

$\therefore (3x-2y)^3+(2y-4z)^3+(4z-3x)^3=3(3x-2y)(2y-4z)(4z-3x)$

7. $(2x-3y)^3+(4z-2x)^3+(3y-4z)^3$

Sol:

Let $2x-3y=a,4z-2x=b,3y-4z=c$

$\therefore a+b+c=2x-3y+4z-2x+3y-4z=c$

$\because a+b+c=0 \qquad \therefore a^3+b^3+c^3=3abc$

$\therefore (2x-3y)^3+(4z-2x)^3+(3y-4z)^3=3(2x-3y)(4z-2x)(3y-4z)$

8. $\left[\frac{x}{2}+y+\frac{z}{3}\right]^3+\left[\frac{x}{2}+\frac{2y}{3}+z\right]^3+\left[-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right]^3$

Sol:

Let $\left(\frac{x}{2}+y+\frac{z}{3}\right)=a,\left(\frac{x}{3}-\frac{2y}{3}+z\right)=b,\left(-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right)=c$

$a+b+c=\frac{x}{2}+y+\frac{z}{3}+\frac{x}{3}-\frac{2y}{3}+z-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}$

$a+b+c=\left(\frac{x}{2}+\frac{x}{3}-\frac{5x}{6}\right)+\left(y-\frac{2y}{3}-\frac{y}{3}\right)+\left(\frac{z}{3}+z-\frac{4z}{3}\right)$

$a+b+c=\frac{3x}{6}+\frac{2x}{6}-\frac{5x}{6}+\frac{3y}{3}-\frac{2y}{3}-\frac{y}{3}+\frac{z}{3}+\frac{3z}{3}-\frac{4z}{3}$

$a+b+c=\frac{5x-5x}{6}+\frac{3y-3y}{3}+\frac{4z-4z}{3}$

$a+b+c=0$

$\because a+b+c=0 \qquad \therefore a^3+b^3+c^3=3abc$

$\therefore\left[\frac{x}{2}+y+\frac{z}{3}\right]^3+\left[\frac{x}{3}-\frac{2y}{3}+z\right]^3+\left[-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right]^3=3\left(\frac{x}{2}+y+\frac{z}{3}\right)\left(\frac{x}{3}-\frac{2y}{3}+z\right)\left(-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right)$

9. $(a-3b)^3+(3b-c)^3+(c-a)^3$

Sol:

Let $(a-3b)=x,(3b-c)=y,(c-a)=z$

$x+y+z=a-3b+3b-c+c-a=0$

$\because x+y+z=0 \qquad \therefore x^3+y^3+z^3=3xyz$

$\therefore (a-3b)^3+(3b-c)^3+(c-a)^3=3(a-3b)(3b-c)(c-a)$

10. $2\sqrt{2}a^3+3\sqrt{3}b^3+c^3-3\sqrt{6}abc$

Sol:

$=\left(\sqrt{2}a\right)^3+\left(\sqrt{3}b\right)^3+c^3-3\times\sqrt{2}a\times\sqrt{3}b\times c$

$=\left(\sqrt{2}a+\sqrt{3}b+c\right)\left(\left(\sqrt{2}a\right)^2+\left(\sqrt{3}b\right)^2+c^2-\left(\sqrt{2}a\right)\left(\sqrt{3}b\right)-\left(\sqrt{3}b\right)c-\left(\sqrt{2}a\right)c\right)$

$=\left(\sqrt{2}a+\sqrt{3}b+c\right)\left(2a^2+3b^2+c^2-\sqrt{6}ab-\sqrt{3}bc-\sqrt{2}ac\right)$

$\therefore 2\sqrt{2}a^3+3\sqrt{3}b^3+c^3-3\sqrt{6}abc=\left(\sqrt{2}a+\sqrt{3}b+c\right)\left(2a^2+3b^2+c^2-\sqrt{6}ab-\sqrt{3}bc-\sqrt{2}ac\right)$

11. $3\sqrt{3}a^3-b^3-5\sqrt{5}c^3-3\sqrt{15}abc$

Sol:

$=\left(\sqrt{3}a\right)^3+\left(-b\right)^3+\left(-\sqrt{5}c\right)^3-3\times\left(\sqrt{3}a\right)\left(-b\right)\left(-\sqrt{5}c\right)$

$=\left(\sqrt{3}a+\left(-b\right)+\left(-\sqrt{5}c\right)\right)\left(\left(\sqrt{3}a\right)^2+\left(-b\right)^2+\left(-\sqrt{5}c\right)^2-\sqrt{3}a\left(-b\right)-\left(-b\right)\left(-\sqrt{5}c\right)-\left(-\sqrt{5}c\right)\sqrt{3}a\right)$

$=\left(\sqrt{3}a-b-\sqrt{5}c\right)\left(3a^2+b^2+5c^2+\sqrt{3}ab-\sqrt{5}bc+\sqrt{15}ac\right)$

$\therefore 3\sqrt{3}a^3-b^3-5\sqrt{5}c^3-3\sqrt{15}abc=\left(\sqrt{3}a-b-\sqrt{5}c\right)\left(3a^2+b^2+5c^2+\sqrt{3}ab-\sqrt{5}bc+\sqrt{15}ac\right)$

12. $8x^3-125y^3+180xy+216$

Sol:

$8x^3-125y^3+180xy+216$

or, $8x^3-125y^3+216+180xy$

$=\left(2x\right)^3+\left(-5y\right)^3+6^3-3\times\left(2x\right)\left(-5y\right)\left(6\right)$

$=\left(2x+\left(-5y\right)+6\right)\left(\left(2x\right)^2+\left(-5y\right)^2+6^2-2x\left(-5y\right)-\left(-5y\right)6-6\left(2x\right)\right)$

$=\left(2x-5y+6\right)\left(4x^2+25y^2+36+10xy+30y-12x\right)$

$\therefore 8x^3-125y^3+180xy+216=\left(2x-5y+6\right)\left(4x^2+25y^2+36+10xy+30y-12x\right)$

13. $2\sqrt{2}a^3+16\sqrt{2}b^3+c^3-12abc$

Sol:

$=\left(\sqrt{2}a\right)^3+\left(2\sqrt{2}b\right)^3+c^3-3\times\sqrt{2}a\times2\sqrt{2}b\times c$

$=\left(\sqrt{2}a+2\sqrt{2}b+c\right)\left(\left(\sqrt{2}a\right)^2+\left(2\sqrt{2}b\right)^2+c^2-\left(\sqrt{2}a\right)\left(2\sqrt{2}b\right)-\left(2\sqrt{2}b\right)c-\left(\sqrt{2}a\right)c\right)$

$=\left(\sqrt{2}a+2\sqrt{2}b+c\right)\left(2a^2+8b^2+c^2-4ab-2\sqrt{2}bc-\sqrt{2}ac\right)$

$2\sqrt{2}a^3+16\sqrt{2}b^3+c^3-12abc=\left(\sqrt{2}a+2\sqrt{2}b+c\right)\left(2a^2+8b^2+c^2-4ab-2\sqrt{2}bc-\sqrt{2}ac\right)$

Exercise – 6.1

1. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer:

(i) $3x^2 - 4x + 15$

(ii) $y^2 + 2\sqrt{3}$

(iii) $3\sqrt{x} + \sqrt{2}x$

(iv) $x - \frac{4}{x}$

(v) $x^{12} + y^3 + t^{50}$

Sol:

(i) $3x^2 - 4x + 15$ is a polynomial of one variable *x*.

(ii) $y^2 + 2\sqrt{3}$ is a polynomial of one variable *y*.

(iii) $3\sqrt{x} + \sqrt{2}x$ is not a polynomial as the exponents of $3\sqrt{x}$ is not a positive integer.

(iv) $x - \frac{4}{x}$ is not a polynomial as the exponent of $\frac{-4}{x}$ is not a positive integer.

(v) $x^{12} + y^3 + t^{50}$ is a polynomial of three variables x, y, t.

2. Write the coefficient of x^2 in each of the following:

(i) $17 - 2x + 7x^2$

(ii) $9 - 12x + x^3$

(iii) $\frac{\pi}{6}x^2 - 3x + 4$

(iv) $\sqrt{3}x - 7$

Sol:

Coefficient of x^2 in

(i) $17 - 2x + 7x^2$ is 7

(ii) $9 - 12x + x^3$ is 0

(iii) $\frac{\pi}{6}x^2 - 3x + 4$ is $\frac{\pi}{6}$

(iv) $\sqrt{3}x - 7$ is 0

3. Write the degrees of each of the following polynomials:

(i) $7x^3 + 4x^2 - 3x + 12$

(ii) $12 - x + 2x^3$

(iii) $5y - \sqrt{2}$

(iv) $7 = 7 \times x^\circ$

(v) 0

Sol:

Degree of polynomial

(i) $7x^2 + 4x^2 - 3x + 12$ is 3

(ii) $12 - x + 2x^3$ is 3

(iii) $5y - \sqrt{2}$ is 1

(iv) $7 = 7 \times x^\circ$ is 0

(v) 0 is un defined.

4. Classify the following polynomials as linear, quadratic, cubic and biquadratic polynomials:

(i) $x + x^2 + 4$

(ii) $3x - 2$

(iii) $2x + x^2$

(iv) 3y

(v) $t^2 + 1$

(vi) $7t^4 + 4t^3 + 3t - 2$

Sol:

Given polynomial

(i) $x + x^2 + 4$ is quadratic as degree of polynomial is 2.

(ii) $3x - 2$ is linear as degree of polynomial is 1.

(iii) $2x + x^2$ is quadratic as degree of polynomial is 2.

(iv) 3y is linear as degree of polynomial is 2.

(v) $t^2 + 1$ is quadratic as degree of polynomial is 2.

(vi) $7t^4 + 4t^3 + 3t - 2$ is bi-quadratic as degree of polynomial is 4.

5. Classify the following polynomials as polynomials in one-variable, two variables etc:

(i) $x^2 - xy + 7y^2$

(ii) $x^2 - 2tx + 7t^2 - x + t$

(iii) $t^3 - 3t^2 + 4t - 5$

(iv) $xy + yz + zx$

Sol:

(i) $x^2 - xy + 7y^2$ is a polynomial in two variables x, y.

(ii) $x^2 - 2tx + 7t^2 - x + t$ is a polynomial in 2 variables x, t.

(iii) $t^3 - 3t^2 + 4t - 5$ is a polynomial in 1 variables t.

(iv) $xy + yz + zx$ is a polynomial in 3 variables x, y, z.

6. Identify polynomials in the following:

(i) $f(x) = 4x^3 - x^2 - 3x + 7$

(ii) $g(x) = 2x^3 - 3x^2 + \sqrt{x} - 1$

(iii) $p(x) = \frac{2}{3}x^2 - \frac{7}{4}x + 9.$

(iv) $q(x) = 2x^2 - 3x + \frac{4}{x} + 2$

(v) $h(x) = x^4 - x^{\frac{3}{2}} + x - 1$

(vi) $f(x) = 2 + \frac{3}{x} + 4x$

Sol:

(i) $f(x) = 4x^3 - x^2 - 3x + 7$ is a polynomial

(ii) $g(x) = 2x^3 - 3x^2 + \sqrt{x} - 1$ is not a polynomial as exponent of x in $\sqrt{x}$ is not a positive integer.

(iii) $p(x) = \frac{2}{3}x^2 - \frac{7}{4}x + 9.$ is a polynomial as all the exponents are positive integers.

(iv) $q(x) = 2x^2 - 3x + \frac{4}{x} + 2$ is not a polynomial as exponent of x in $\frac{4}{x}$ is not a positive integer.

(v) $h(x) = x^4 - x^{\frac{3}{2}} + x - 1$ is not a polynomial as exponent of x in $-x^{\frac{3}{2}}$ is not a positive integer.

(vi) $f(x) = 2 + \frac{3}{x} + 4x$ is not a polynomial as exponent of x in $\frac{3}{x}$ is not a positive integer.

7. Identify constant, linear, quadratic and cubic polynomials from the following polynomials:

(i) $f(x)=0$

(ii) $g(x)=2x^3-7x+4$

(iii) $h(x)=-3x+\frac{1}{2}$

(iv) $p(x)=2x^2-x+4$

(v) $q(x)=4x+3$

(vi) $r(x)=3x^2+4x^2+5x-7$

Sol:

Given polynomial

(i) $f(x)=0$ is a constant polynomial as 0 is a constant

(ii) $g(x)=2x^3-7x+4$ is a cubic polynomial as degree of the polynomial is 3.

(iii) $h(x)=-3x+\frac{1}{2}$ is a linear polynomial as degree of the polynomial is 1.

(iv) $p(x)=2x^2-x+4$ is a quadratic as the degree of the polynomial is 2.

(v) $q(x)=4x+3$ is a linear polynomial as the degree of the polynomial is 1.

(vi) $r(x)=3x^2+4x^2+5x-7$ is a cubic polynomial as the degree is 3.

8. Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Sol:

Example of a binomial with degree 35 is $7x^{35}-5$

Example of a monomial with degree 100 is $2t^{100}$

Exercise – 6.2

1. If f(x) = $2x^3 - 13x^2 + 17x + 12$, find (i) f(2) (ii) f (-3) (iii) f(0)

Sol:

We have

$f(x) = 2x^3 - 13x^2 + 17x + 12$

(i) $f(2) = 2\times(2)^3 - 13\times(2)^2 + 17\times(2) + 12$

$= (2\times8) - (13\times4) + (17\times2) + 12$

$= 16 - 52 + 34 + 12 = 10$

(ii) $f(-3) = 2\times(-3)^3 - 13\times(-3)^2 + 17\times(-3) + 12$

$= 2\times(-27) - 13\times(9) + 17\times(-3) + 12$

$= -54 - 117 - 51 + 12 = -210$

(iii) $f(0) = 2\times(0)^3 - 13\times(0)^2 + 17\times(0) + 12$

$= 0 - 0 + 0 + 12 = 12$

2. Verify whether the indicated numbers are zeroes of the polynomials corresponding to them in the following cases:

(i) $f(x) = 3x + 1, x = -\frac{1}{3}$

(ii) $f(x) = x^2 - 1, x = 1, -1$

(iii) $g(x) = 3x^2 - 2, x = \frac{2}{\sqrt{3}}, -\frac{2}{\sqrt{3}}$

(iv) $p(x) = x^3 - 6x^2 + 11x - 6, x = 1, 2, 3$

(v) $f(x) = 5x - \pi, x = \frac{4}{5}$

(vi) $f(x) = x^2 \text{ and } x = 0$

(vii) $f(x) = lx + m,\ x = -\frac{m}{l}$

(viii) $f(x) = 2x + 1, x = \frac{1}{2}$

Sol:

(i) $f(x) = 3x + 1, x = -\frac{1}{3}$

We have

$f(x) = 3x + 1$

Put $x=-\frac{1}{3} \Rightarrow f\left(-\frac{1}{3}\right)=\cancel{3}\times\left(-\frac{1}{\cancel{3}}\right)+1=-1+1=0$

$\therefore x=-\frac{1}{3}$ is a root of $f\left(x\right)=3x+1$

(ii) $f\left(x\right)=x^2-1, x=1,-1$

We have $f\left(x\right), x^2-1$

Put $x=1$ and $x=-1$

$\Rightarrow f\left(1\right)=\left(1\right)^2-1$ and $f\left(-1\right)=\left(-1\right)^2-1$

$=1-1=0$ $\qquad =1-1=0$

$\therefore x=1,-1$ are the roots of $f\left(x\right)=x^2-1$

(iii) $g\left(x\right)=3x^2-2, x=\frac{2}{\sqrt{3}},-\frac{2}{\sqrt{3}}$

We have $g\left(x\right)=3x^2-2$

Put $x=\frac{2}{\sqrt{3}}$ and $x=-\frac{2}{\sqrt{3}}$

$\Rightarrow g\left(\frac{2}{\sqrt{3}}\right)=3\left(\frac{2}{\sqrt{3}}\right)^2-2$ and $g\left(\frac{-2}{\sqrt{3}}\right)=3\left(\frac{-2}{\sqrt{3}}\right)^2-2$

$=\cancel{3}\times\frac{4}{\cancel{3}}-2$ $\qquad =\cancel{3}\left(\frac{4}{\cancel{3}}\right)-2$

$=4-2=2\neq 0$ $\qquad =4-2=2\neq 0$

$\therefore x=\frac{2}{\sqrt{3}},\frac{-2}{\sqrt{3}}$ are not roots of $g\left(x\right)=3x^2-2$

(iv) $p\left(x\right)=x^3-6x^2+11x-6, x=1,2,3$

Put $x=1\Rightarrow p\left(1\right)=\left(1\right)^3-6\left(1\right)^2+n\left(1\right)-6=1-6+11-6=0$

$x=2\Rightarrow p\left(2\right)=\left(2\right)^3-6\left(2\right)^2+11\left(2\right)-6=8-24+22-6=0$

$x=3\Rightarrow p\left(3\right)=3^3-6\left(3^2\right)+11\left(3\right)-6=27-54+33-6=0$

$\therefore x=1,2,3$ are roots of $p\left(x\right)=x^3-6x^2+11x-6$

(v) We know $f\left(x\right)=5x-\pi, x=\frac{4}{5}$

Put $x=\frac{4}{5}\Rightarrow f\left(\frac{4}{5}\right)=\cancel{5}\times\frac{4}{\cancel{5}}-\pi=4-\pi\neq 0$

$\therefore x=\frac{4}{5}$ is not a root of $f\left(x\right)=5x-\pi$

(vi) We have $f(x) = x^2$ *and* $x = 0$

Put $x = 0 \Rightarrow f(0) = (0)^2 = 0$

$\therefore x = 0$ is a root of $f(x) = x^2$

(vii) $f(x) = lx + m$ and $x = -\frac{m}{l}$

Put $x = \frac{-m}{l} \Rightarrow f\left(\frac{-m}{l}\right) = l \times \left(\frac{-m}{l}\right) + m = -m + m = 0$

$\therefore x = -\frac{m}{l}$ is a root of $f(x) = lx + m$

(viii) $f(x) = 2x + 1, x = \frac{1}{2}$

Put $x = \frac{1}{2} \Rightarrow f\left(\frac{1}{x}\right) = \cancel{2} \times \left(\frac{1}{\cancel{2}}\right) + 1 = 1 + 1 = 2 \neq 0$

$\therefore x = \frac{1}{2}$ is not a root of $f(x) = 2x + 1$

3. If x = 2 is a root of the polynomial f(x) = $2x^2 - 3x + 7a$, find the value of a.

Sol:

We have $f(x) = 2x^2 - 3x + 7a$

Put $x = 2 \Rightarrow f(2) = 2(2)^2 - 3(2) + 7a$

$= 2 \times 4 - 3 \times 2 + 7a = 8 - 6 + 7a$

$= 2 + 7a$

Given $x = 2$ is a root of $f(x) = 2x^2 - 3x + 7a$

$\Rightarrow f(2) = 0$

$\therefore 2 + 7a = 0$

$\Rightarrow 7a = -2 \Rightarrow \boxed{a = -\frac{2}{7}}$

4. If x = $-\frac{1}{2}$ is a zero of the polynomial p(x) = $8x^3 - ax^2 - x + 2$, find the value of a.

Sol:

We have $p(x) = 8x^3 - ax^2 - x + 2$

Put $x = -\frac{1}{2}$

$\Rightarrow P\left(-\frac{1}{2}\right) = 8 \times \left(-\frac{1}{2}\right)^3 - ax\left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right) + 2$

$= \not{8} \times \frac{-1}{\not{8}} - a \times \frac{1}{4} + \frac{1}{2} + 2$

$= -1 - \frac{a}{4} + \frac{1}{2} + 2$

$= \frac{3}{2} - \frac{a}{4}$

Given that $x = \frac{-1}{2}$ is a root of $p(x)$

$\Rightarrow P\left(\frac{-1}{2}\right) = 0$

$\therefore \frac{3}{2} - \frac{a}{4} = 0 \Rightarrow \frac{a}{4} = \frac{3}{2} \Rightarrow a = \frac{3}{\not{2}} \times \not{4}^{2}$

$\Rightarrow \boxed{a = 6}$

5. If x = 0 and x = -1 are the roots of the polynomial f(x) = $2x^3 - 3x^2 + ax + b$, find the value of a and b.

Sol:

We have $f(x) = 2x^3 - 3x^2 + ax + b$

Put

$x = 0 \Rightarrow f(0) = 2 \times (0)^3 - 3(0)^2 + a(0) + b = 0 - 0 + 0 + b = b$

$x = -1 \Rightarrow f(-1) = 2 \times (-1)^3 - 3 \times (-1)^2 + a(-1) + b = 2 \times (-1) - 3 \times (1) - a + b$

$= -2 - 3 - a + b$

$= -5 - a + b$

Since $x = 0$ and $x = -1$ are roots of $f(x)$

$\Rightarrow f(0) = 0 \;\; and \;\; f(-1) = 0$

$\Rightarrow b = 0 \qquad \Rightarrow -5 - a + b = 0$

$\boxed{b = 0}$ and $a - b = -5$

$\Rightarrow a - 0 = -5$

$\Rightarrow \boxed{a = -5}$

$\therefore a = -5$ and $b = 0$

6. Find the integral roots of the polynomial f(x) = $x^3 + 6x^2 + 11x + 6$.

Sol:

We have

$f(x) = x^3 + 6x^2 + 11x + 6$

Clearly, $f\left(x\right)$ is a polynomial with integer coefficient and the coefficient of the highest degree term i.e., the leading coefficients is 1.

Therefore, integer roots of $f\left(x\right)$ are limited to the integer factors of 6, which are $\pm1, \pm2, \pm3, \pm6$

We observe that

$f\left(-1\right)=\left(-1\right)^3+6\left(-1\right)^2+11\left(-1\right)+6=-1+6-11+6=0$

$f\left(-2\right)=\left(-2\right)^3+6\left(-2\right)^2+11\left(-2\right)+6=-8+24-22+6=0$

$f\left(-3\right)=\left(-3\right)^2+6\left(-3\right)^2+11\left(-3\right)+6=-27+54-33+6=0$

$\therefore$ Hence, integral roots of $f\left(x\right)$ are $-1,-2,-3.$

7. Find rational roots of the polynomial f(x) = $2x^3+x^2-7x-6$

Sol:

We have

$f\left(x\right)=2x^3+x^2-7x-6$

Clearly, $f\left(x\right)$ is a cubic polynomial with integer coefficient. If $\frac{b}{c}$ is a rational roots in lowest terms, then the value of b are limited to the factors of 6 which $\pm1, \pm2, \pm3, \pm6$ and values of c are limited to the factors of 2 which are $\pm1, \pm2.$

Hence, the possible rational roots of $f\left(x\right)$ are

$\pm1, \pm2, \pm3, \pm6, \pm\frac{1}{2}, \pm\frac{3}{2}$

We observe that

$f\left(-1\right)=2\left(-1\right)^3+\left(-1\right)^2-7\left(-1\right)-6=-2+1+7-6=0$

$f\left(2\right)=2\left(2\right)^3+\left(2\right)^2-7\left(2\right)-6=16+4-14-6=0$

$f\left(-\frac{3}{2}\right)=2\left(-\frac{3}{2}\right)^3+\left(-\frac{3}{2}\right)^2-7\left(\frac{3}{2}\right)-6=-\frac{27}{4}+\frac{9}{4}+\frac{21}{2}-6=0$

$\therefore$ Hence, $-1, 2, \frac{-3}{2}$ are the rational roots of $f\left(x\right)$

Exercise– 6.3

In each of the following, using the remainder theorem, find the remainder when f(x) is divided by g(x) and verify the result by actual division: (1 – 8)

1. $f\left(x\right)=x^3+4x^2-3x+10$, $g\left(x\right)=x+4$

Sol:

We have $f\left(x\right)=x^3+4x^2-3x+10$ and $g\left(x\right)=x+4$

Therefore, by remainder theorem when $f\left(x\right)$ is divided by $g\left(x\right)=x-\left(-4\right)$, the remainder is equal to $f\left(-4\right)$

Now, $f\left(x\right)=x^3+4x^2-3x+10$

$\Rightarrow f\left(-4\right)=\left(-4\right)^3+\left(-4\right)^2-3\left(-4\right)+10$

$=-64+4\times16+12+10$

$=-64+64+12+10=22$

Hence, required remainder is 22.

2. $f\left(x\right)=4x^4-3x^3-2x^2+x-7$, $g\left(x\right)=x-1$

Sol:

We have

$f\left(x\right)=4x^4-3x^3-2x^2+x-7$ and $g\left(x\right)=x-1$

Therefore by remainder theorem when $f\left(x\right)$ is divide by $g\left(x\right)=x-1$, the remainder is equal to $f\left(+1\right)$

Now, $f\left(x\right)=4x^4-3x^3-2x^2+x-7$

$\Rightarrow f\left(1\right)=4\left(+1\right)^4-3\left(+1\right)^3-2\left(+1\right)^2+\left(+1\right)-7$

$=4\times1-3\left(+1\right)-2\left(1\right)+1-7$

$=4-3-2+1-7=-7$

Hence, required remainder is -7

3. $f\left(x\right)=2x^4-6x^3+2x^2-x+2$, $g\left(x\right)=x+2$

Sol:

We have

$f\left(x\right)=2x^4-6x^3+2x^2-x+2$ and $g\left(x\right)=x+2$

Therefore, by remainder theorem when $f(x)$ is divide by $g(x)=x-(-2)$, the remainder is equal to $f(-2)$

Now, $f(x)=2x^4-6x^3+2x^2-x+2$

$\Rightarrow f(-2)=2(-2)^4-6(-2)^3+2(-2)^2-(-2)+2$

$=2\times16-6\times(-8)+2\times4+2+2$

$=32+48+8+4=92$

Hence, required remainder is 92.

4. $f(x)=4x^3-12x^2+14x-3$, $g(x)=2x-1$

Sol:

We have

$f(x)=4x^3-12x^2+14x-3$ and $g(x)=2x-1$

Therefore, by remainder theorem when $f(x)$ is divide by $g(x)=2\left(x-\frac{1}{2}\right)$, the remainder is equal to $f\left(\frac{1}{2}\right)$

Now, $f(x)=4x^3-12x^2+14x-3$

$\Rightarrow f\left(\frac{1}{2}\right)=4\left(\frac{1}{2}\right)^3-12\left(\frac{1}{2}\right)^2+14\left(\frac{1}{2}\right)-3$

$=\left(\not{4}\times\frac{1}{\not{8}_2}\right)-\left(\not{12}^3\times\frac{1}{\not{4}}\right)+\left(\not{14}^7\times\frac{1}{\not{2}}\right)-3$

$=\frac{1}{2}-3+7-3=\frac{1}{2}+1=\frac{3}{2}$

Hence, required remainder is $\frac{3}{2}$.

5. $f(x)=x^3-6x^2+2x-4$, $g(x)=1-2x$

Sol:

We have

$f(x)=x^3-6x^2+2x-4$ and $g(x)=1-2x$

Therefore, by remainder theorem when $f(x)$ is divided by $g(x)=-2\left(x-\frac{1}{2}\right)$, the remainder is equal to $f\left(\frac{1}{2}\right)$

Now, $f\left(x\right)=x^3-6x^2+2x-4$

$$\Rightarrow f\left(\frac{1}{2}\right)=\left(\frac{1}{2}\right)^3-6\left(\frac{1}{2}\right)^2+2\left(\frac{1}{2}\right)-4$$

$$=\frac{1}{8}-\left(\overset{3}{\cancel{6}}+\frac{1}{\cancel{4}_2}\right)+\cancel{2}\times\frac{1}{\cancel{2}}-4$$

$$=\frac{1}{8}-\frac{3}{2}+1-4=-\frac{35}{8}$$

Hence, the required remainder is $-\frac{35}{8}$

6. $f\left(x\right)=x^4-3x^2+4$, $g\left(x\right)=x-2$

Sol:

We have

$f\left(x\right)=x^4-3x^2+4$ and $g\left(x\right)=x-2$

Therefore, by remainder theorem when $f\left(x\right)$ is divided by $g\left(x\right)=x-2$, the remainder is equal to $f\left(2\right)$

Now, $f\left(x\right)=x^4-3x^2+4$

$$\Rightarrow f\left(2\right)=2^4-3\left(2\right)^2+4$$

$$=16-\left(3\times4\right)+4=16-12+4=20-12=8$$

Hence, required remainder is 8.

7. $f\left(x\right)=9x^3-3x^2+x-5$, $g\left(x\right)=x-\frac{2}{3}$

Sol:

We have $f\left(x\right)=9x^3-3x^2+x-5$ and $g\left(x\right)=x-\frac{2}{3}$

Therefore, by remainder theorem when $f\left(x\right)$ is divided by $g\left(x\right)=x-\frac{2}{3}$, the remainder is equal to $f\left(\frac{2}{3}\right)$

Now, $f\left(x\right)=9x^3-3x^2+x-5$

$$\Rightarrow f\left(\frac{2}{3}\right)=9\left(\frac{2}{3}\right)^3-3\left(\frac{2}{3}\right)^2+\left(\frac{2}{3}\right)-5$$

$$= \left(\cancel{9} \times \frac{8}{\cancel{27}_{3}} \right) - \left(\cancel{3} \times \frac{4}{\cancel{9}_{3}} \right) + \frac{2}{3} - 5$$

$$= \frac{8}{3} - \frac{4}{3} + \frac{2}{3} - 5 = \frac{6}{3} - 5 = 2 \cdot 5 = -3$$

Hence, the required remainder is -3.

8. $f(x) = 3x^4 + 2x^3 - \frac{x^2}{3} - \frac{x}{9} + \frac{2}{27}$, $g(x) = x + \frac{2}{3}$

Sol:

We have

$f(x) = 3x^4 + 2x^3 - \frac{x^2}{3} - \frac{x}{9} + \frac{2}{27}$ and $g(x) = x + \frac{2}{3}$

Therefore, by remainder theorem when $f(x)$ is divided by $g(x) = x - \left(-\frac{2}{3}\right)$, the remainder is equal to $f\left(-\frac{2}{3}\right)$

Now, $f(x) = 3x^4 + 2x^3 - \frac{x^2}{3} - \frac{x}{9} + \frac{2}{27}$

$$\Rightarrow f\left(-\frac{2}{3}\right) = 3 \times \left(\frac{-2}{3}\right)^4 + 2\left(\frac{-2}{3}\right)^3 - \frac{\left(\frac{-2}{3}\right)^2}{3} - \frac{\left(\frac{-2}{3}\right)}{9} + \frac{2}{27}$$

$$= \cancel{3} \times \frac{16}{\cancel{81}_{27}} + 2 \times \frac{-8}{27} - \frac{4}{9 \times 3} - \left(\frac{-2}{3 \times 9}\right) + \frac{2}{27}$$

$$= \frac{16}{27} - \frac{16}{27} - \frac{4}{27} + \frac{2}{27} + \frac{2}{27}$$

$$= \frac{16 - 16 - 4 + 2 + 2}{27} = \frac{0}{27} = 0$$

Hence, required remainder is 0.

9. If the polynomials $2x^3 + ax^2 + 3x - 5$ and $x^3 + x^2 - 4x + a$ leave the same remainder when divided by x – 2, find the value of a.

Sol:

Let $p(x) = 2x^3 + ax^2 + 3x - 5$ and $q(x) = x^3 + x^2 - 4x + a$ be the given polynomials

The remainders when $p(x)$ and $q(x)$ are divided by $(x-2)$ are $p(2)$ and $q(2)$ respectively.

By the given condition we have

$p(2)=q(2)$

$\Rightarrow 2(2)^3+a(2)^2+3(2)-5=2^3+2^2-4(2)+a$

$\Rightarrow 16+4a+6-5=8+4-8+a$

$\Rightarrow 3a+13=0 \Rightarrow 3a=-13 \Rightarrow \boxed{a=\frac{-13}{3}}$

10. The polynomials ax^3+3x^2-3 and $2x^3-5x+a$ when divided by (x – 4) leave the remainders R_1 and R_2 respectively. Find the values of a in each of the following cases, if (i) $R_1 = R_2$ (ii) $R_1 + R_2 = 0$ (iii) $2R_1 - R_2 = 0$.

Sol:

Let $p(x)=ax^3+3x^2-3$ and $q(x)=2x^3-5x+a$ be the given polynomials.

Now,

$R_1=$ Remainder when $p(x)$ is divided by $x-4$

$\Rightarrow R_1=p(4)$

$\Rightarrow R_1=a(4)^3+3(4)^2-3 \qquad [\because p(x)=ax^3+3x^2-3]$

$\Rightarrow R_1=64a+48-3$

$\Rightarrow \boxed{R_1=64a+45}$

And,

$R_2=$ Remainder when $q(x)$ is divided by $x-4$

$\Rightarrow R_2=q(4)$

$\Rightarrow R_2=q(4)^3-5(4)+a \qquad [\because q(x)=2x^3-5x+a]$

$\Rightarrow R_2=128-20+a$

$\Rightarrow \boxed{R_2=108+a}$

(i) Given condition is $R_1=R_2$

$\Rightarrow 64a+45=108+a$

$\Rightarrow 63a-63=0 \Rightarrow 63a=63 \Rightarrow \boxed{a=1}$

(ii) Given condition is $R_1+R_2=0$

$\Rightarrow 64a+45+108+a=0$

$\Rightarrow 65a+153=0 \Rightarrow 65a=-153 \Rightarrow \boxed{a=\frac{-153}{65}}$

(iii) Given condition is $2R_1 - R_2 = 0$

$\Rightarrow 2(64a+45)-(108+a)=0$

$\Rightarrow 128a+90-108-a=0$

$\Rightarrow 127a-18=0 \Rightarrow 127a=18 \Rightarrow \boxed{a=\frac{18}{127}}$

11. If the polynomials ax^3+3x^2-13 and $2x^3-5x+a$, when divided by (x – 2) leave the same remainder, find the value of a.

Sol:

Let $p(x)=ax^3+3x^2-13$ and $q(x)=2x^3-5x+a$ be the given polynomials

The remainders when $p(x)$ and $q(x)$ are divided by $(x-2)$ are $p(2)$ and $q(2)$.

By the given condition we have

$p(2)=q(2)$

$\Rightarrow a(2)^3+3(2)^2-13=2(2)^3-5(2)+a$

$\Rightarrow 8a+12-13=16-10+a$

$\Rightarrow 7a-7=0 \Rightarrow 7a=7 \Rightarrow a=\frac{7}{7} \Rightarrow \boxed{a=1}$

(i) $x \Rightarrow x-0$

By remainder theorem, required remainder is equal to $f(0)$

Now, $f(x)=x^3+3x^2+3x+1$

$\Rightarrow f(0)=0^3+3\times 0^2+3\times 0+1=0+0+0+1=1$

Hence, required remainder is 1.

(ii) $x+\pi \Rightarrow x-(-\pi)$

By remainder theorem, required remainder is equal to $f(-\pi)$

Now, $f(x)=x^3+3x^2+3x+1$

$\Rightarrow f(-\pi)=(-\pi)^3+3(-\pi)^2+3(-\pi)+1$

$=-\pi^3+3\pi^2-3\pi+1$

Hence, required remainder is $-\pi^3+3\pi^2-3\pi+1$.

(iii) $5+2x \Rightarrow 2\left(x-\left(-\frac{5}{2}\right)\right)$

By remainder theorem, required remainder is equal to $f\left(-\frac{5}{2}\right)$.

Now, $f(x)=x^3+3x^2+3x+1$

$$\Rightarrow f\left(-\frac{5}{2}\right)=\left(-\frac{5}{2}\right)^3+3\left(-\frac{5}{2}\right)^2+3\left(-\frac{5}{2}\right)+1$$

$$=\frac{-125}{8}+\frac{3\times 25}{4}+\frac{3\times -5}{2}+1$$

$$=\frac{-128}{8}+\frac{75}{4}-\frac{15}{2}+1$$

$$=\frac{-27}{8}$$

Hence, required remainder is $\frac{-27}{8}$.

Exercise – 6.4

In each of the following, use factor theorem to find whether polynomial g(x) is a factor of polynomial f(x) or, not: (1 – 7)

1. $f(x)=x^3-6x^2+11x-6$; $g(x)=x-3$

Sol:

We have $f(x)=x^3-6x^2+11x-6$ and $g(x)=x-3$

In order to find whether polynomial $g(x)=x-3$ is a factor of $f(x)$, it is sufficient to show that $f(3)=0$

Now, $f(x)=x^3-6x^2+11x-6$

$\Rightarrow f(3)=3^3-6(3)^2+11(3)-6$

$=27-54+33-6=60-60=0$

Hence, $g(x)$ is a factor of $f(x)$

2. $f(x)=3x^4+17x^3+9x^2-7x-10$; $g(x)=x+5$

Sol:

We have $f(x)=3x^4+17x^3+9x^2-7x-10$ and $g(x)=x+5$

In order to find whether $g(x)=x-(-5)$ is a factor of $f(x)$ or not, it is sufficient to show that $f(-5)=0$

Now, $f(x)=3x^4+17x^3+9x^2-7x-10$

$\Rightarrow f(-5)=3(-5)^4+17(-5)^3+9(-5)^2-7(-5)-10$

$= 3 \times 625 + 17 \times (-125) + 9 \times 25 + 35 - 10$

$= 1875 - 2125 + 225 + 35 - 10$

$= 0$

Hence, $g(x)$ is a factor of $f(x)$

3. $f(x) = x^5 + 3x^4 - x^3 - 3x^2 + 5x + 15$, $g(x) = x + 3$

Sol:

We have $f(x) = x^5 + 3x^4 - x^3 - 3x^2 + 5x + 15$ and $g(x) = x + 3$

In order to find whether $g(x) = x - (-3)$ is a factor of $f(x)$ or not, it is sufficient to prove that $f(-3) = 0$

Now, $f(x) = x^5 + 3x^4 - x^3 - 3x^2 + 5x + 15$

$\Rightarrow f(-3) = (-3)^5 + 3(-3)^4 - 3(-3)^2 + 5(-3) + 15$

$= -243 + 243 - (-27) - 3(9) + 5(-3) + 15$

$= -243 + 243 + 27 - 27 - 15 + 15$

$= 0$

Hence, $g(x)$ is a factor of $f(x)$

4. $f(x) = x^3 - 6x^2 - 19x + 84$, $g(x) = x - 7$

Sol:

We have $f(x) = x^3 - 6x^2 - 19x + 84$ and $g(x) = x - 7$

In order to find whether $g(x) = x - 7$ is a factor of $f(x)$ or not, it is sufficient to show that $f(7) = 0$

Now, $f(x) = x^3 - 6x^2 - 19x + 84$

$\Rightarrow f(7) = 7^3 - 6(7)^2 - 19(7) + 84$

$= 343 - 294 - 133 + 84 = 427 - 427$

$= 0$

Hence $g(x)$ is a factor $f(x)$

5. $f(x) = 3x^3 + x^2 - 20x + 12$ and $g(x) = 3x - 2$

Sol:

We have

$f(x) = 3x^3 + x^2 - 20x + 12$ and $g(x) = 3x - 2$

In order to find whether $g(x)=3\left(x-\frac{2}{3}\right)$ is a factor of $f(x)$ or not, it is sufficient to prove that $f\left(\frac{2}{3}\right)=0$

Now, $f(x)=3x^3+x^2-20x+12$

$\Rightarrow f\left(\frac{2}{3}\right)=3\left(\frac{2}{3}\right)^3+\left(\frac{2}{3}\right)^2-20\left(\frac{2}{3}\right)+12$

$=3\times\frac{8}{\cancel{27}_{9}}+\frac{4}{9}=\frac{40}{3}+12$

$=\frac{12}{9}-\frac{40}{3}+12=\frac{12-20+108}{9}=\frac{120-120}{9}=0$

Hence $g(x)=3x-2$ is a factor of $f(x)$

6. $f(x)=2x^3-9x^2+x+12$, $g(x)=3-2x$

Sol:

We have $f(x)=2x^3-9x^2+x+12$ and $g(x)=3-2x$

In order to find whether $g(x)=3-2x=-2\left(x-\frac{3}{2}\right)$ is a factor of $f(x)$ or not, it is sufficient to prove that $f\left(\frac{3}{2}\right)=0$

Now, $f(x)=2x^3-9x^2+x+12$

$\Rightarrow f\left(\frac{3}{2}\right)=2\left(\frac{3}{2}\right)^3-9\left(\frac{3}{2}\right)^2+\left(\frac{3}{2}\right)+12$

$=\cancel{2}\times\frac{27}{\cancel{8}_{4}}-9\times\frac{9}{4}+\frac{3}{2}+12$

$=\frac{27}{4}-\frac{81}{4}+\frac{3}{2}+12=\frac{27-81+6+48}{4}=\frac{81-81}{4}$

$=0$

Hence $g(x)=3-2x$ is a factor of $f(x)$

7. $f(x)=x^3-6x^2+11x-6$, $g(x)=x^2-3x+2$

Sol:

We have

$f(x)=x^3-6x^2+11x-6$ and $g(x)=x^2-3x+2$

$\Rightarrow g(x) = x^2 - 3x + 2 = (x-1)(x-2)$

Clearly, $(x-1)$ and $(x-2)$ are factors of $g(x)$

In order to find whether $g(x) = (x-1)(x-2)$ is a factor of $f(x)$ or not, it is sufficient to prove that $(x-1)$ and $(x-2)$ are factors of $f(x)$.

i.e., we should prove that $f(1) = 0$ and $f(2) = 0$

Now, $f(x) = x^3 - 6x^2 + 11x - 6$

$\Rightarrow f(1) = 1^3 - 6(1)^2 + 11(1) - 6 = 1 - 6 + 11 - 6 = 12 - 12 = 0$

$\Rightarrow f(2) = 2^3 - 6(2)^2 + 11(2) - 6 = 8 - 24 + 22 - 6 = 30 - 30 = 0$

$\therefore (x-1)$ and $(x-2)$ are factors of $f(x)$

$\Rightarrow g(x) = (x-1)(x-2)$ is factor of $f(x)$

8. Show that (x – 2), (x + 3) and (x – 4) are factors of $x^3 - 3x^2 - 10x + 24$.

Sol:

Let $f(x) = x^3 - 3x^2 - 10x + 24$ be the given polynomial.

In order to prove that $(x-2), (x+3), (x-4)$ are factors of $f(x)$, it is sufficient to prove that $f(2) = 0$, $f(-3) = 0$ and $f(4) = 0$ respectively.

Now $f(x) = x^3 - 3x^2 - 10x + 24$

$\Rightarrow f(2) = 2^3 - 3(2)^2 - 10(2) + 24 = 8 - 12 - 20 + 24 = 0$

$\Rightarrow f(-3) = (-3)^3 - 3(-3)^2 - 10(-3) + 24 = -27 - 27 + 30 + 24 = 0$

$\Rightarrow f(4) = 4^3 - 3(4)^2 - 10(4) + 24 = 64 - 48 - 40 + 24 = 0$

Hence, $(x-2), (x+3)$ and $(x-4)$ are factors of the given polynomial.

9. Show that (x + 4), (x – 3) and (x – 7) are factors $x^3 - 6x^2 - 19x + 84$

Sol:

Let $f(x) = x^3 - 6x^2 - 19x + 84$ be the given polynomial

In order to prove that $(x+4), (x-3)$ and $(x-7)$ are factors of $f(x)$, it is sufficient to prove that $f(-4) = 0$, $f(3) = 0$ and $f(7) = 0$ respectively

Now, $f(x) = x^3 - 6x^2 - 19x + 84$

$\Rightarrow f(-4) = (-4)^3 - 6(-4)^2 - 19(-4) + 84 = -64 - 96 + 76 + 84 = 0$

$\Rightarrow f(3) = (3)^3 - 6(3)^2 - 19(3) + 84 = 27 - 54 - 57 + 84 = 0$

$\Rightarrow f(7) = 7^3 - 6(7)^2 - 19(7) + 84 = 343 - 294 - 133 + 84 = 0$

Hence, $(x+4), (x-3)$ and $(x-7)$ are factors of the given polynomial $x^3 - 6x^2 - 19x + 84$.

10. For what value of a is (x – 5) a factor of $x^3 - 3x^2 + ax - 10$?

Sol:

Let $f(x) = x^3 - 3x^2 + ax - 10$ be the given polynomial

From factor theorem,

If $(x-5)$ is a factor of $f(x)$ then $f(5) = 0$

Now, $f(x) = x^3 - 3x^2 + ax - 10$

$\Rightarrow f(5) = 5^3 - 3(5)^2 + a(5) - 10 = 0$

$\Rightarrow 125 - 3(25) + 5a - 10 = 0$

$\Rightarrow 5a + 40 = 0$

$\Rightarrow 5a = -40$

$\Rightarrow \boxed{a = -8}$

Hence $(x-5)$ is a factor of $f(x)$ if $a = -8$

11. Find the value of a such that (x – 4) is a factor of $5x^3 - 7x^2 - ax - 28$.

Sol:

Let $f(x) = 5x^3 - 7x^2 - ax - 28$ be the given polynomial from factor theorem, if $(x-4)$ is a factor of $f(x)$ then $f(4) = 0$

$\Rightarrow f(4) = 0$

$\Rightarrow 5(4)^3 - 7(4)^2 - a(4) - 28 = 0$

$\Rightarrow 5 \times 64 - 7 \times 16 - 4a - 28 = 0$

$\Rightarrow 320 - 112 - 4a - 28 = 0$

$\Rightarrow 180 - 4a = 0$

$\Rightarrow 4a = 180$

$\Rightarrow a = \frac{180}{4} = 45$

Hence $(x-4)$ is a factor of $f(x)$ when $a = 45$

12. For what value of a, if x + 2 is a factor of factor of $4x^4 + 2x^3 - 3x^2 + 8x + 5a$.

Sol:

Let $f(x) = 4x^4 + 2x^3 - 3x^2 + 8x + 5a$ be the given polynomial

From factor theorem if $(x+2)$ is a factor of $f(x)$ then $f(-2) = 0$

Now, $f(x) = 4x^4 + 2x^3 - 3x^2 + 8x + 5a$

$\Rightarrow f(-2) = 0$

$\Rightarrow 4(-2)^4 + 2(-2)^3 - 3(-2)^2 + 8(-2) + 5a = 0$

$\Rightarrow 64 - 16 - 12 - 16 + 5a = 0 \Rightarrow 5a + 20 = 0$

$\Rightarrow 5a = 20$

$\Rightarrow \boxed{a = -4}$

Hence $(x+2)$ is a factor of $f(x)$ when $a = -4$

13. Find the value of k if x – 3 is a factor of $k^2x^3 - kx^2 + 3kx - k$.

Sol:

Let $f(x) = k^2x^3 - kx^2 + 3kx - k$ be the given polynomial from factor theorem if $(x-3)$ is a factor of $f(x)$ then $f(3) = 0$

$\Rightarrow k^2(3)^3 - k(3)^2 + 3k(3) - k = 0$

$\Rightarrow 27k^2 - 9k + 9k - k = 0$

$\Rightarrow 27k^2 - k = 0 \Rightarrow k(27k - 1) = 0$

$\Rightarrow k = 0$ and $27k - 1 = 0 \Rightarrow k = \frac{1}{27}$

Hence, $(x-3)$ is a factor of $f(x)$ when $k = 0$ or $k = \frac{1}{27}$

14. Find the values of a and b, if $x^2 - 4$ is a factor of $ax^4 + 2x^3 - 3x^2 + bx - 4$.

Sol:

Let $f(x) = ax^4 + 2x^3 - 3x^2 + bx - 4$ and $g(x) = x^2 - 4$

We have $g(x) = x^2 - 4 = (x-2)(x+2)$

Given $g(x)$ is a factor of $f(x)$.

$\Rightarrow (x-2)$ and $(x+2)$ are factors of $f(x)$

From factor theorem,

If $(x-2)$ and $(x+2)$ are factors of $f(x)$ then $f(2) = 0$ and $f(-2) = 0$ respectively

$\Rightarrow f(2) = 0 \Rightarrow a(2)^4 + 2(2)^3 - 3(2)^2 + b(2) - 4 = 0$

$\Rightarrow 16a+16-12+2b-4=0$

$\Rightarrow 16a+2b=0 \Rightarrow 2(8a+b)=0$

$\Rightarrow \boxed{8a+b=0}$ ....(1)

Similarly $f(-2)=0 \Rightarrow a(-2)^4+2(-2)^3-3(-2)^2+b(-2)-4=0$

$\Rightarrow 16a-16-12-2b-4=0$

$\Rightarrow 16a-2b-32=0 \Rightarrow 2(8a+b)=32$

$\Rightarrow \boxed{8a-b=16}$ ....(2)

Adding equation (1) and (2)

$8a+b+8a-b=16 \Rightarrow 16a=16 \Rightarrow \boxed{a=1}$

Put $a=1$ in equation (1)

$\Rightarrow 8\times1+b=0 \Rightarrow \boxed{b=-8}$

Hence, $a=1$ and $b=-8$

15. Find α and β, if x + 1 and x + 2 are factors of $x^3+3x^2-2\alpha x+\beta$.

Sol:

Let $f(x)=x^3+3x^2-2\alpha x+\beta$ be the given polynomial from factor theorem, if $(x+1)$ and $(x+2)$ are factors of $f(x)$ then $f(-1)=0$ and $f(-2)=0$

$\Rightarrow f(-1)=0 \Rightarrow (-1)^3+3(-1)^2-2\alpha(-1)+\beta=0$

$\Rightarrow -1+3+2\alpha+\beta=0 \Rightarrow 2\alpha+\beta+2=0$ ....(1)

Similarly,

$f(-2)=0 \Rightarrow (-2)^3+3(-2)^2-2\alpha(-2)+\beta=0$

$\Rightarrow -8+12+4\alpha+\beta=0 \Rightarrow 4\alpha+\beta+4=0$ ....(2)

Subtract equation (1) from (2)

$\Rightarrow 4\alpha+\beta+4-(2\alpha+\beta+2)=0-0$

$\Rightarrow 4\alpha+\beta+4-2\alpha-\beta-2=0$

$\Rightarrow 2\alpha+2=0 \Rightarrow 2\alpha=-2 \Rightarrow \boxed{\alpha=-1}$

Put $\alpha=-1$ in equation (1)

$\Rightarrow 2(-1)+\beta+2=0 \Rightarrow -2+\beta+2=0 \Rightarrow \boxed{\beta=0}$

Hence, $\alpha=-1$ and $\beta=0$

16. Find the values of p and q so that $x^4 + px^3 + 2x^2 - 3x + q$ is divisible by $(x^2 - 1)$

Sol:

Let $f(x) = x^4 + px^3 + 2x^2 - 3x + q$ be the given polynomial.

and let $g(x) = x^2 - 1 = (x-1)(x+1)$

Clearly, $(x-1)$ and $(x+1)$ are factors of $g(x)$

Given $g(x)$ is a factor of $f(x)$

$\Rightarrow (x-1)$ and $(x+1)$ are factors of $f(x)$

From factor theorem,

If $(x-1)$ and $(x+1)$ are factors of $f(x)$ then $f(1) = 0$ and $f(-1) = 0$ respectively

$\Rightarrow f(1) = 0 \Rightarrow 1^4 + p(1)^3 + 2(1)^2 - 3(1) + q = 0$

$\Rightarrow 1 + p + 2 - 3 + q = 0 \Rightarrow p + q = 0$ ....(1)

$\Rightarrow f(-1) = 0 \Rightarrow (-1)^4 + p(-1)^3 + 2(-1)^2 - 3(-1) + q = 0$

$\Rightarrow 1 + (-p) + 2 + 3 + q = 0 \Rightarrow q - p + 6 = 0$ ....(2)

Adding equation (1) and (2)

$\Rightarrow p + q + q - p + 6 = 0 \Rightarrow 2q + 6 = 0 \Rightarrow 2q = -6 \Rightarrow \boxed{q = -3}$

Put $q = -3$ in equation (1)

$\Rightarrow p - 3 = 0 \Rightarrow \boxed{p = 3}$

Hence $x^2 - 1$ is divisible by $f(x)$ when $p = 3, q = -3$

17. Find the values of a and b so that (x + 1) and (x – 1) are factors of $x^4 + ax^3 - 3x^2 + 2x + b$.

Sol:

Let $f(x) = x^4 + ax^3 - 3x^2 + 2x + b$ be the given polynomial.

From factor theorem; if $(x+1)$ and $(x-1)$ are factors of $f(x)$ then $f(-1) = 0$ and $f(1) = 0$ respectively.

$\Rightarrow f(-1) = 0 \Rightarrow (-1)^4 + a(-1)^3 - 3(-1)^2 + 2(-1) + b = 0$

$\Rightarrow 1 - a - 3 - 2 + b = 0 \Rightarrow b - a - 4 = 0$ ....(1)

$\Rightarrow f(1) = 0 \Rightarrow (1)^4 + a(1)^3 - 3(1)^2 + 2(1) + b = 0$

$\Rightarrow 1 + a - 3 + 2 + b = 0 \Rightarrow a + b = 0$ ....(2)

Adding equation (1) and (2)

$\Rightarrow b-a-4+a+b=0+0$

$\Rightarrow 2b-4=0 \Rightarrow 2b=4 \Rightarrow b=\frac{\cancel{4}^{2}}{\cancel{2}} \Rightarrow \boxed{b=2}$

Substitute $b=2$ in equation (2)

$\Rightarrow a+2=0 \Rightarrow \boxed{a=-2}$

Hence, $a=-2$ and $b=2$

18. If $x^3+ax^2-bx+10$ *is divisible by* x^2-3x+2, find the values of a and b.

Sol:

Let $f(x)=x^3+ax^2-bx+10$ and $g(x)=x^2-3x+2$ be the given polynomials.

We have $g(x)=x^2-3x+2=(x-2)(x-1)$

$\Rightarrow$ Clearly, $(x-1)$ and $(x-2)$ are factors of $g(x)$

Given that $f(x)$, is divisible by $g(x)$

$\Rightarrow g(x)$ is a factor of $f(x)$

$\Rightarrow (x-2)$ and $(x-1)$ are factors of $f(x)$

From factor theorem,

If $(x-1)$ and $(x-2)$ are factors of $f(x)$ then $f(1)=0$ and $f(2)=0$ respectively.

$\Rightarrow f(1)=0 \Rightarrow (1)^3+a(1)^2-b(1)+10=0$

$\Rightarrow 1+a-b+10=0 \Rightarrow a-b+11=0$ ....(1)

$\Rightarrow f(2)=0 \Rightarrow (2)^3+a(2)^2-b(2)+10=0$

$\Rightarrow 8+4a-2b+10=0$

$\Rightarrow 4a-2b+18=0$

$\Rightarrow 2(2a-b+9)=0$

$\Rightarrow 2a-b+9=0$ ....(2)

Subtract equation (1) from (2)

$\Rightarrow 2a-b+9-(a-b+11)=0-0$

$\Rightarrow 2a-b+9-a+b-11=0 \Rightarrow a-2=0 \Rightarrow \boxed{a=2}$

Put $a=2$ in equation (1)

$\Rightarrow a-b+11=0 \Rightarrow 2-b+11=0 \Rightarrow 13-b=0 \Rightarrow \boxed{b=13}$

Hence, $a=2$ and $b=13$

$\therefore x^3+ax^2-bx+10$ is divisible by x^2-3x+2 when $a=2$ and $b=13$

19. If both x + 1 and x – 1 are factors of $ax^3 + x^2 - 2x + b$, find the values of a and b.

Sol:

Let $f(x) = ax^3 + x^2 - 2x + b$ be the given polynomial.

Given $(x+1)$ and $(x-1)$ are factor of $f(x)$.

From factor theorem,

If $(x+1)$ and $(x-1)$ are factors of $f(x)$ then $f(-1) = 0$ and $f(1) = 0$ respectively.

$\Rightarrow f(-1) = 0 \Rightarrow a(-1)^3 + (-1)^2 - 2(-1) + b = 0$

$\Rightarrow -a + 1 + 2 + b = 0 \Rightarrow b - a + 3 = 0$ ....(1)

$\Rightarrow f(1) = 0 \Rightarrow a(1)^3 + (1)^2 - 2(1) + b = 0$

$\Rightarrow a + 1 - 2 + b = 0 \Rightarrow b + a - 1 = 0$ ....(2)

Adding equation (1) and (2)

$\Rightarrow b - a + 3 + b + a - 1 = 0 + 0$

$\Rightarrow 2b + 2 = 0 \Rightarrow 2b = -2 \Rightarrow \boxed{b = -1}$

Put $b = -1$ in equation (1)

$\Rightarrow -1 - a + 3 = 0 \Rightarrow 2 - a = 0 \Rightarrow \boxed{a = 2}$

Hence the values of a, b are 2, −1 respectively.

20. What must be added to $x^3 - 3x^2 - 12x + 19$ so that the result is exactly divisibly by $x^2 + x - 6$?

Sol:

Let $p(x) = x^3 - 3x^2 - 12x + 19$ and $q(x) = x^2 + x - 6$.

By division algorithm, when $p(x)$ is divided by $q(x)$, the remainder is a linear expression in x.

So, let $r(x) = ax + b$ is added to $p(x)$ so that $p(x) + r(x)$ is divisible by $q(x)$.

Let $f(x) = p(x) + r(x)$

$\Rightarrow f(x) = x^3 - 3x^2 - 12x + 19 + ax + b$

$\Rightarrow \boxed{f(x) = x^3 - 3x^2 + x(a - 12) + b + 19}$

We have,

$q(x) = x^2 + x - 6 = (x+3)(x-3)$

Clearly, $q(x)$ is divisible by $(x-2)$ and $(x+3)$

i.e., $(x-2)$ and $(x+3)$ are factors of $q(x)$

We have,

$f(x)$ is divisible by $q(x)$

$\Rightarrow (x-2)$ and $(x+3)$ are factors of $f(x)$

From factors theorem,

If $(x-2)$ and $(x+3)$ are factors of $f(x)$ then $f(2)=0$ and $f(-3)=0$ respectively.

$\Rightarrow f(2)=0 \Rightarrow 2^3-3(2)^2+2(a-12)+b+19=0$

$\Rightarrow 8-12+2a-24+b+19=0$

$\Rightarrow 2a+b-9=0$ ....(1)

Similarly

$f(-3)=0 \Rightarrow (-3)^3-3(-3)^2+(-3)(a-12)+b+19=0$

$\Rightarrow -27-27-3a+36+b+19=0$

$\Rightarrow b-3a+1=0$ ....(2)

Subtract equation (1) from (2)

$b-3a+1-(2a+b-9)=0-0$

$\Rightarrow b-3a+1-2a-6+9=0$

$\Rightarrow -5a+10=0 \Rightarrow 5a=10 \Rightarrow \boxed{a=2}$

Put $a=2$ in equation (2)

$\Rightarrow b-3\times 2+1=0 \Rightarrow b-6+1=0 \Rightarrow b-5=0 \Rightarrow \boxed{b=5}$

$\therefore r(x)=ax+b \Rightarrow \boxed{r(x)=2x+5}$

Hence, $x^3-3x^2-12x+19$ is divisible by x^2+x-6 when $2x+5$ is added to it.

21. What must be subtracted from $x^3-6x^2-15x+80$ so that the result is exactly divisible by x^2+x-12 ?

Sol:

Let $p(x)=x^3-6x^2-15x+80$ and $q(x)=x^2+x-12$

By division algorithm, when $p(x)$ is divided by $q(x)$ the remainder is a linear expression in *x*.

So, let $r(x)=ax+b$ is subtracted from $p(x)$, So that $p(x)-r(x)$ is divisible by $q(x)$

Let $f(x)=p(x)-r(x)$

Clearly, $(3x-2)$ and $(x+3)$ are factors of $q(x)$

Therefore, $f(x)$ will be divisible by $q(x)$ if $(3x-2)$ and $(x+3)$ are factors of $f(x)$

i.e., from factor theorem,

$f\left(\frac{2}{3}\right)=0$ and $f(-3)=0$ $\left[\because 3x-2=0 \Rightarrow x=\frac{2}{3} \text{ and } x+3=0 \Rightarrow x=-3\right]$

$\Rightarrow f\left(\frac{2}{3}\right)=3\left(\frac{2}{3}\right)^3+\left(\frac{2}{3}\right)^2+\frac{2}{3}(a-22)+b+9=0$

$\Rightarrow 3\times\frac{8}{27}+\frac{4}{9}+\frac{2}{3}a-\frac{44}{3}+b+9=0$

$\Rightarrow \frac{12}{9}+\frac{2}{3}a-\frac{44}{3}+b+9=0$

$\Rightarrow \frac{12+6a-132+9b+81}{9}=0$

$\Rightarrow 6a+9b-39=0$

$\Rightarrow 3(2a+3b-13)=0 \Rightarrow 2a+3b-13=0$ ....(1)

Similarly,

$f(-3)=0 \Rightarrow 3(-3)^3+(-3)^2+(-3)(a-22)+b+9=0$

$\Rightarrow -81+9-3a+66+b+9=0$

$\Rightarrow b-3a+3=0$

$\Rightarrow 3(b-3a+3)=0 \Rightarrow 3b-9a+9=0$ ....(2)

Subtract equation (1) from (2)

$\Rightarrow 3b-9a+9-(2a+3b-13)=0-0$

$\Rightarrow 3b-9a+9-2a-3b+13=0$

Put $a=4$ in equation (2)

$\Rightarrow 4\times4-b-20=0$

$\Rightarrow 16-b-20=0 \Rightarrow -b-4=0 \Rightarrow \boxed{b=-4}$

Putting the value of a and b in $r(x)=ax+b$,

We get $\boxed{r(x)=4x-4}$

Hence, $p(x)$ is divisible by $q(x)$, if $r(x)=4x-4$ is subtracted from it.

22. What must be added to $3x^3+x^2-22x+9$ so that the result is exactly divisible by $3x^2+7x-6$?

Sol:

Let $p(x)=3x^3+x^2-22x+9$ and $q(x)=3x^2+7x-6$

By division algorithm,

When $p(x)$ is divided by $q(x)$, the remainder is a linear equation in *x*.

So, let $r(x)=ax+b$ is added to $p(x)$, so that $p(x)+r(x)$ is divisible by $q(x)$

Let $f(x)=p(x)+r(x)$

$\Rightarrow f(x)=3x^3+x^2-22x+9+(ax+b)$

$\Rightarrow \boxed{f(x)=3x^3+x^2+x(a-22)+b+9}$

We have,

$q(x)=3x^2+7x-6=3x^2+9x-2x-6=3x(x+3)-2(x+3)$

$=(3x-2)(x+3)$

$=f(x)=x^3-6x^2-15x+80-(ax+b)$

$\Rightarrow \boxed{f(x)=x^3-6x^2-x(a+15)+80-b}$

We have,

$q(x)=x^2+x-12=x^2+4x-3x-18=x(x+4)-3(x+4)$

$=(x-3)(x+4)$

Clearly, $(x-3)$ and $(x+4)$ are factors of $q(x)$.

Therefore, $f(x)$ will be divisible by $q(x)$ if $(x-3)$ and $(x+4)$ are factor of $f(x)$.

i.e., from factors theorem,

$f(3)=0$ and $f(-4)=0$ $\quad [\because x-3=0\Rightarrow x=3 \text{ and } x+4=0\Rightarrow x=-4]$

$\Rightarrow f(3)=0\Rightarrow (3)^3-6(3)^2-3(a+15)+80-b=0$

$\Rightarrow 27-54-3a-45+80-b=0$

$\Rightarrow 8-3a-b=0$ ....(1)

$\Rightarrow f(-4)=0\Rightarrow (-4)^3-6(-4)^2-(4)(a+15)+80-b=0$

$\Rightarrow -64-96-4a+60+80-b=0$

$\Rightarrow 4a-b-20=0$ ....(2)

Subtract equation (1) from (2)

$\Rightarrow 4a-b-20-(8-3a-b)=0-0$

$\Rightarrow 4a-b-20-8+3a+b=0$

$\Rightarrow 7a-28=0\Rightarrow 7a=28\Rightarrow \boxed{a=4}$

$\Rightarrow -11a+22=0\Rightarrow 11a=22\Rightarrow \boxed{a=2}$

Put $a=2$ in equation (1)

$\Rightarrow 2\times 2+3b-13=0$

$\Rightarrow 4+3b-13=0\Rightarrow 3b-9=0\Rightarrow 3b=9\Rightarrow \boxed{b=3}$

Putting the value of a and b in $r(x)=ax+b$,

We get, $\boxed{r(x)=2x+3}$

Hence, $3x^3+x^2-22x+9$ will be divisible by $3x^2+7x-6$, if $2x+3$ is added to it.

23. If x – 2 is a factor of each of the following two polynomials, find the values of a in each case:

(i) $x^3 - 2ax^2 + ax - 1$

(ii) $x^5 - 3x^4 - ax^3 + 3ax^2 + 2ax + 4$

Sol:

(i) Let $f(x) = x^3 - 2ax^2 + ax - 1$ be the given polynomial.

From factor theorem,

If $(x-2)$ is a factor of $f(x)$ then $f(2) = 0$ $[\because x - 2 = 0 \Rightarrow x = 2]$

$\Rightarrow f(2) = 0 \Rightarrow 2^3 - 2a(2)^2 + a(2) - 1 = 0$

$\Rightarrow 8 - 8a + 2a - 1 = 0$

$\Rightarrow 7 - 6a = 0 \Rightarrow 6a = 7 \Rightarrow \boxed{a = \frac{7}{6}}$

Hence, $(x-2)$ is a factor of $f(x)$ when $a = \frac{7}{6}$

(ii) Let $f(x) = x^5 - 3x^4 - ax^3 + 3ax^2 + 2ax + 4$ be the given polynomial.

From factor theorem

If $x - 2$ is a factor of $f(x)$ then $f(2) = 0$ $[\because x - 2 = 0 \Rightarrow x = 2]$

$\Rightarrow f(2) = 0 \Rightarrow 2^5 - 3(2)^4 - a(2)^3 + 3a(2)^2 + 2a(2) + 4 = 0$

$\Rightarrow 32 - 48 - 8a + 12a + 4a + 4 = 0$

$\Rightarrow 8a - 12 = 0 \Rightarrow 8a = 12 \Rightarrow \boxed{a = \frac{3}{2}}$

Hence, $(x-2)$ is a factor of $f(x)$ when $a = \frac{3}{2}$

24. In each of the following two polynomials, find the value of a, if x – a is a factor:

(i) $x^6 - ax^5 + x^4 - ax^3 + 3x - a + 2$

(ii) $x^5 - a^2x^3 + 2x + a + 1$

Sol:

(i) Let $f(x) = x^6 - ax^5 + x^4 - ax^3 + 3x - a + 2$ be the given polynomial.

From factor theorem,

If $(x-a)$ is a factor of $f(x)$ then $f(a) = 0$ $[\because x - a = 0 \Rightarrow x = a]$

$\Rightarrow f(a) = 0 \Rightarrow a^6 - a(a)^5 + a^4 - a(a)^3 + 3(a) - a + 2 = 0$

$\Rightarrow a^6 - a^6 + a^4 - a^4 + 3a - a + 2 = 0$

$\Rightarrow 2a + 2 = 0 \Rightarrow 2a = -2 \Rightarrow \boxed{a = -1}$

Hence, $(x-a)$ is a factor of $f(x)$, if $\boxed{a = -1}$

(ii) Let $f(x) = x^5 - a^2x^3 + 2x + a + 1$ be the given polynomial.

From factor theorem,

If $(x-a)$ is a factor of $f(x)$ then $f(a) = 0$ $[\because x - a = 0 \Rightarrow x = a]$

$\Rightarrow f(a) = 0 \Rightarrow a^5 - a^2(a)^3 + 2(a) + a + 1 = 0$

$\Rightarrow a^5 - a^5 + 2a + a + 1 = 0$

$\Rightarrow 3a + 1 = 0 \Rightarrow 3a = -1 \Rightarrow \boxed{a = -\frac{1}{3}}$

Hence, $(x-a)$ is a factor of $f(x)$, if $a = -\frac{1}{3}$

25. In each of the following two polynomials, find the value of a, if x + a is a factor:

(i) $x^3 + ax^2 - 2x + a + 4$

(ii) $x^4 - a^2x^2 + 3x - a$

Sol:

(i) Let $f(x) = x^3 + ax^2 - 2x + a + 4$ be the given polynomial.

From factor theorem,

If $(x+a)$ is a factor of $f(x)$ then $f(-a) = 0$ $[\because x + a = 0 \Rightarrow x = -a]$

$\Rightarrow f(-a) = 0 \Rightarrow (-a)^3 + a(-a)^2 - 2(-a) + a + 4 = 0$

$\Rightarrow -a^3 + a^3 + 2a + a + 4 = 0$

$\Rightarrow 3a + 4 = 0 \Rightarrow 3a = -4 \Rightarrow \boxed{a = -\frac{4}{3}}$

Hence, $(x+a)$ is a factor of $f(x)$, if $a = -\frac{4}{3}$

(ii) Let $f(x) = x^4 - a^2x^2 + 3x - a$ be the given polynomial

From factor theorem,

If $(x+a)$ is a factor of $f(x)$ then $f(-a) = 0$ $[\because x + a = 0 \Rightarrow x = -a]$

$\Rightarrow f(-a) = 0 \Rightarrow (-a)^4 - a^2(-a)^2 + 3(-a) - a = 0$

$\Rightarrow a^4 - a^4 - 3a - a = 0$

$\Rightarrow -3a - a = 0 \Rightarrow -4a = 0 \Rightarrow \boxed{a = 0}$

Hence, $(x+a)$ is a factor of $f(x)$, if $a = 0$

Exercise – 6.5

Using factor theorem, factorize each of the following polynomials:

1. $x^3+6x^2+11x+6$

Sol:

Let $f(x)=x^3+6x^2+11x+6$ be the given polynomial.

The constant term in $f(x)$ is 6 and factors of 6 are $\pm1,\pm2,\pm3$ and ±6

Putting $x=-1$ in $f(x)$, we have

$f(-1)=(-1)^3+6(-1)^2+11(-1)+6=-1+6-11+6=0$

$\therefore (x+1)$ is a factor of $f(x)$

Similarly, $(x+2)$ and $(x+3)$ are factors of $f(x)$.

Since $f(x)$ is a polynomial of degree 3. So, it cannot have more than three linear factors.

$\therefore f(x)=k(x+1)(x+2)(x+3)$

$\Rightarrow x^3+6x^2+11x+6=k(x+1)(x+2)(x+3)$

Putting $x=0$ on both sides, we get

$0+0+0+6=k(0+1)(0+2)(0+3)$

$6=k(1)(2)(3)$

$\Rightarrow 6=6k \Rightarrow \boxed{k=1}$

Putting $k=1$ in $f(x)=k(x+1)(x+2)(x+3)$, we get

$\boxed{f(x)=(x+1)(x+2)(x+3)}$

Hence, $x^3+6x^2+11x+6=(x+1)(x+2)(x+3)$

2. x^3+2x^2-x-2

Sol:

Let $f(x)=x^3+2x^2-x-2$

The constant term in $f(x)$ is equal to -2 and factors of -2 and $\pm1,\pm2$.

Putting $x=1$ in $f(x)$, we have

$f(1)=1^3+2(1)^2-1-2=0$

$\therefore (x-1)$ is a factor of $f(x)$

Similarly, $(x+1),(x+2)$ are factors of $f(x)$.

Since $f(x)$ is a polynomial of degree 3. So, it cannot have more than three linear factors.

$\therefore f(x)=k(x-1)(x+1)(x+2)$

$\Rightarrow x^3+2x^2-x-2=k(x-1)(x+1)(x+2)$

Putting $x=0$ on both sides, we get $0+0-0-2=k(-1)(+1)(+2)$

$-2=-2k \Rightarrow \boxed{k=1}$

Putting $k=1$ in $f(x)=k(x-1)(x+1)(x+2)$, we get

$\boxed{f(x)=(x-1)(x+1)(x+2)}$

Hence, $x^3+2x^2-x-2=(x-1)(x+1)(x+2)$

3. $x^3-6x^2+3x+10$

Sol:

Let $f(x)=x^3-6x^2+3x+10$

The constant term in $f(x)$ is equal to 10 and factors of 10 are $\pm1, \pm2, \pm5$ and ±10

Putting $x=-1$ in $f(x)$, we have

$f(-1)=(-1)^3-6(-1)^2+3(-1)+10=-1-6-3+10=0$

$\therefore (x+1)$ is a factor of $f(x)$

Similarly, $(x-2)$ and $(x-5)$ are factors of $f(x)$

Since $f(x)$ is a polynomial of degree 3. So, it cannot have more than three linear factors.

$\therefore f(x)=k(x+1)(x-2)(x-5)$

Putting $x=0$ on both sides, we get

$\Rightarrow x^3-6x^2+3x+10=k(x+1)(x-2)(x-5)$

$0-0+0+10=k(1)(-2)(-5)$

$\Rightarrow 10=10k \Rightarrow \boxed{k=1}$

Putting $k=1$ in $f(x)=k(x+1)(x-2)(x-5)$, we get

$\boxed{f(x)=(x+1)(x-2)(x-5)}$

Hence, $x^3-6x^2+3x+10=(x+1)(x+2)(x-5)$

4. $x^4-7x^3+9x^2+7x-10$

Sol:

Let $f(x)=x^4-7x^3+9x^2+7x-10$

The constant term in $f(x)$ is equal to -10 and factors of -10 are $\pm1,\pm2,\pm5$ and ±10

Putting $x=1$ in $f(x)$, we have

$f(1)=1^4-7(1)^3+9(1)^2+7(1)-10=1-7+9+7-10=0$

$\therefore (x-1)$ is a factor of $f(x)$

Similarly $(x+1),(x-2),(x-5)$ are also factors of $f(x)$

Since $f(x)$ is a polynomial of degree 4. So, it cannot have more than four linear factors

$\therefore f(x)=k(x-1)(x+1)(x-2)(x-5)$

$\Rightarrow x^4-7x^3+9x^2+7x-10=k(x-1)(x+1)(x-2)(x-5)$

Putting $x=0$ on both sides, we get

$\Rightarrow 0-0+0+0-10=k(-1)(1)(-2)(-5)$

$\Rightarrow -10=k(-10)$

$\Rightarrow \boxed{k=1}$

Putting $k=1$ in $f(x)=k(x-1)(x+1)(x-2)(x-5)$, we get

$\boxed{f(x)=(x+1)(x-1)(x-2)(x-5)}$

Hence, $x^4-7x^3+9x^2+7x-10=(x+1)(x-1)(x-2)(x-5)$

5. $x^4-2x^3-7x^2+8x+12$

Sol:

Let $f(x)=x^4-2x^3-7x^2+8x+12$

The constant term in $f(x)$ is equal to $+12$ and factors of $+12$ are $\pm1,\pm2,\pm3,\pm4,\pm6$ and ±12

Putting $x=-1$ in $f(x)$, we have

$f(-1)=(-1)^4-2(-1)^3-7(-1)^2+8(-1)+12$

$=1+2-7-8+12=0$

$\therefore (x+1)$ is a factor of $f(x)$

Similarly $(x+2),(x-2),(x-3)$ are also factors of $f(x)$

Since $f(x)$ is a polynomial of degree 4. So, it cannot have more than four linear factors.

$\therefore f(x)=k(x+1)(x+2)(x-2)(x-3)$

$\Rightarrow x^4-2x^3-7x^2+8x+12=k(x+1)(x+2)(x-2)(x-3)$

Putting $x=0$ on both sides, we get

$\Rightarrow 0-0-0+0+12=k(1)(2)(-2)(-3)$

$\Rightarrow \cancel{12}=k(\cancel{12})$

$=\boxed{k=1}$

Putting $k=1$ in $f(x)=k(x+1)(x+2)(x-2)(x-3)$, we get

$\boxed{f(x)=(x+1)(x+2)(x-2)(x-3)}$

Hence, $x^4-2x^3-7x^2+8x+12=(x+1)(x+2)(x-2)(x-3)$

6. $x^4+10x^3+35x^2+50x+24$

Sol:

Let $f(x)=x^4+10x^3+35x^2+50x+24$

The constant term in $f(x)$ is equal to $+24$ and factors of $+24$ are

$\pm1,\pm2,\pm3,\pm4,\pm6,\pm8,\pm12,\pm24$

Putting $x=-1$ in $f(x)$, we have

$f(-1)=(-1)^4+10(-1)^3+35(-1)^2+50(-1)+24$

$=1-10+35-50+24=0$

$\therefore (x+1)$ is a factor of $f(x)$

Similarly, $(x+2),(x+3)$ and $(x+4)$ are also factors of $f(x)$.

Since $f(x)$ is polynomial of degree 4. So, it cannot have more than four linear factors.

$\therefore f(x)=k(x+1)(x+2)(x+3)(x+4)$

$\Rightarrow x^4+10x^3+35x^2+50x+24=k(x+1)(x+2)(x+3)(x+4)$

Putting $x=0$ on both sides, we get

$\Rightarrow 0+0+0+0+24=k(1)(2)(3)(4)$

$\Rightarrow 24=24k\Rightarrow \boxed{k=1}$

Putting $k=1$ in $f(x)=k(x+1)(x+2)(x+3)(x+4)$, we get

$\boxed{f(x)=(x+1)(x+2)(x+3)(x+4)}$

Hence, $x^4+10x^3+35x^2+50x+24=(x+1)(x+2)(x+3)(x+4)$

7. $2x^4-7x^3-13x^2+63x-45$

Sol:

Let $f(x)=2x^4-7x^3-13x^2+63x-45$

The factors of the constant term -45 are $\pm1,\pm3,\pm5,\pm9,\pm15$ and ±45

The factor of the coefficient of x^4 is 2. Hence possible rational roots of $f(x)$ are

$\pm1,\pm3,\pm5,\pm9,\pm15,\pm45,\pm\frac{1}{2},\pm\frac{3}{2},\pm\frac{5}{2},\pm\frac{9}{2},\pm\frac{15}{2},\pm\frac{45}{2}$

We have,

$f(1)=2(1)^4-7(1)^3-13(1)^2+63(1)-45$

$=2-7-13+63-45=0$

And $f(3)=2(3)^4-7(3)^3-13(3)^2+63(3)-45$

$=162-189-117+189-45=0$

And $f(3)=2(3)^4-7(3)^3-13(3)^2+63(3)-45$

$=162-189-117+189-45=0$

So, $(x-1)$ and $(x-3)$ are factors of $f(x)$

$\Rightarrow(x-1)(x-3)$ is also a factor of $f(x)$

$\Rightarrow(x^2-4x+3)$ is a factor of $f(x)$

Let us now divide

$f(x)=2x^4-7x^3-13x^2+63x-45$ by (x^2-4x+3) to get thee other factors of $f(x)$.

By long division, we have

$$
\begin{array}{l}
x^2-4x+3\overline{)2x^4-7x^3-13x^2+63x-45}(2x^2+x-15 \\
\qquad\qquad 2x^4-8x^3+6x \\
\qquad\qquad -\quad +\quad - \\
\hline
\qquad\qquad x^3-19x^2+63x \\
\qquad\qquad x^3-4x^2+3x \\
\qquad\qquad -\quad +\quad - \\
\hline
\qquad\qquad -15x^2+60x-45 \\
\qquad\qquad -15x^2+60x-45 \\
\qquad\qquad +\quad -\quad + \\
\hline
\qquad\qquad 0
\end{array}
$$

$\therefore 2x^4-7x^3-13x^2+63x-45=(x^2-4x+3)(2x^2+x-15)$

$\Rightarrow 2x^4-7x^3-13x^2+63x-45=(x-1)(x-3)(2x^2+x-15)$

Now,

$2x^2 + x - 15 = 2x^2 + 6x - 5x - 15 = 2x(x+3) - 5(x+3)$

$= (2x-5)(x+3)$

Hence $2x^4 - 7x^3 - 13x^2 + 63x - 45 = (x-1)(x-3)(x+3)(2x-5)$

8. $3x^3 - x^2 - 3x + 1$

Sol:

Let $f(x) = 3x^3 - x^2 - 3x + 1$

The factors of the constant term $+1$ is ± 1.

The factors of the coefficient of x^3 is 3.

Hence possible rational roots of $f(x)$ are $\pm 1, \pm \frac{1}{3}$

We have,

$f(1) + 3(1)^3 - (1)^2 - 3(1) + 1 = 3 - 1 - 3 + 1 = 0$

So, $(x-1)$ is a factor of $f(x)$.

Let us now divide $f(x) = 3x^3 - x^2 - 3x + 1$ by $(x-1)$ to get the other factors.

By long division method, we have

$$x-1\overline{)3x^3 - x^2 - 3x + 1}(3x^2 + 2x - 1$$

$$\begin{array}{l} 3x^3 - 3x^2 \\ - \quad + \\ \hline \quad 2x^2 - 3x \\ 2x^2 - 2x \\ - \quad + \\ \hline -x + 1 \\ -x + 1 \\ + \quad - \\ \hline \quad 0 \end{array}$$

$\therefore 3x^4 - x^2 - 3x + 1 = (x-1)(3x^2 + 2x - 1)$

Now,

$3x^2 + 2x - 1 = 3x^2 + 3x - x - 1 = 3x(x+1) - 1(x+1) = (3x-1)(x+1)$

Hence, $3x^3 - x^2 - 3x + 1 = (x-1)(x+1)(3x-1)$

9. $x^3 - 23x^2 + 142x - 120$

Sol:

Let $f(x) = x^3 - 23x^2 + 142x - 120$

The constant term in $f(x)$ is equal to –120 and factors of –120 are

$\pm1, \pm2, \pm3, \pm4, \pm5, \pm6, \pm8, \pm10, \pm12, \pm15, \pm20, \pm24, \pm30, \pm40, \pm60, \pm120$.

Putting $x = 1$ we have

$f(1) = 1^3 - 23(1)^2 + 142(1) - 120 = 1 - 23 + 142 - 120 = 0$

So, $(x-1)$ is a factor of $f(x)$.

Let us now divide $f(x) = x^3 - 23x^2 + 142x - 120$ by $(x-1)$ to get the other factors.

By long division, we have

$x-1\overline{)x^3 - 23x^2 + 142x - 120}(x^2 - 22x + 120$

$x^3 - x^2$

$- \quad +$

$-22x^2 + 142x$

$-22x^2 + 22x$

$+ \quad -$

$120x - 120$

$120x - 120$

$- \quad +$

0

$\therefore x^3 - 23x^2 + 142x - 120 = (x-1)(x^2 - 22x + 120)$

Now,

$x^2 - 22x + 120 = x^2 - 10x - 12x + 120 = x(x-10) - 12(x-10)$

$= (x-12)(x-10)$

Hence, $x^3 - 23x^2 + 142x - 120 = (x-1)(x-10)(x-12)$

10. $y^3 - 7y + 6$

Sol:

Let $f(y) = y^3 - 7y + 6$

The constant term in $f(y)$ is $+6$ and factors of $+6$ are $\pm1, \pm2, \pm3$ and ±6

Putting $y = 1$ we have

$f(1) = 1^3 - 7(1) + 6 = 1 - 7 + 6 = 0$

$\therefore (y-1)$ is a factor of $f(y)$

Similarly it can be verified that $(y-2)$ and $(y+3)$ are also factors of $f(y)$

Since $f(y)$ is a polynomial of degree 3. So, it cannot have more than 3 linear factors.

$\therefore f(y) = k(y-1)(y-2)(y+3)$

$\Rightarrow y^3 + 7y + 6 = k(y-1)(y-2)(y+3)$

Putting $y = 0$ on both sides, we get

$\Rightarrow 0 - 0 + 6 = k(-1)(-2)(3)$

$\Rightarrow 6 = 6k \Rightarrow \boxed{k = 1}$

Putting $k = 1$ in $f(y) = k(y-1)(y-2)(y+3)$, we get

$\boxed{f(y) = (y-1)(y-2)(y+3)}$

Hence, $y^3 - 7y + 6 = (y-1)(y-2)(y+3)$

11. $x^3 - 10x^2 - 53x - 42$

Sol:

Let $f(x) = x^3 - 10x^2 - 53x - 42$

The constant term in $f(x)$ is -42 and factors of -42 are $\pm1, \pm2, \pm3, \pm6, \pm7, \pm14, \pm21, \pm42$

Putting $x = -1$, we get

$f(-1) = (-1)^3 - 10(1)^2 - 53(-1) - 42 = -1 - 10 + 53 - 42 = 0$

So, $(x+1)$ is a factor of $f(x)$

Let us now divide $f(x) = x^3 - 10x^2 - 53x - 42$ by $(x+1)$ to get the other factors.

By long division, we have

$$
\begin{array}{l}
x+1\overline{)\,x^3 - 10x^2 - 53x - 42\,}(\,x^2 - 11x - 42 \\
\quad\; x^3 + x^2 \\
\quad\; - \;\; - \\
\hline
\quad -11x^2 - 53x \\
\quad -11x^2 - 11x \\
\quad + \qquad + \\
\hline
\qquad -42x - 42 \\
\qquad -42x - 42 \\
\qquad + \qquad + \\
\hline
\qquad\quad 0
\end{array}
$$

$\therefore x^3 - 10x^2 - 53x - 42 = (x+1)(x^2 - 11x - 42)$

Now, $x^2-11x-42=x^2-14x+3x-42=x(x-14)+3(x-14)$

$=(x+3)(x-14)$

Hence, $x^3-10x^2-53x-42=(x+1)(x+3)(x-14)$

12. $y^3-2y^2-29y-42$

Sol:

Let $f(y)=y^3-2y^2-29y-42$

The constant term in $f(y)$ is -42 and factors of -42 are $\pm1,\pm2,\pm3,\pm6,\pm7,\pm14,\pm21,\pm42$

Putting $y=-2$ we get

$f(-2)=(-2)^3-2(-2)^2-29(-2)-42$

$=-8-8+58-42=0$

So, $(y+2)$ is a factor of $f(y)$

Let us now divide $f(y)=y^3-2y^2-29y-42$ by $(y+2)$ to get the other factors

By long division, we get

$$
\begin{array}{r|l}
y+2 & y^3-2y^2-29y-42\ (y^2-4y-21 \\
 & y^3+2y^2 \\
 & -\quad + \\
\hline
 & -4y^2-29y \\
 & -4y^2-8y \\
 & +\quad + \\
\hline
 & -21y-42 \\
 & -21y-42 \\
 & +\quad + \\
\hline
 & 0
\end{array}
$$

$\therefore y^3-2y^2-29y-42=(y+2)(y^2-4y-21)$

Now,

$y^2-4y-21=y^2-7y+3y-21=y(y-7)+3(y-7)$

$=(y+3)(y-7)$

Hence, $y^3-2y^2-29y-42=(y+2)(y+3)(y-7)$

$(y-2)$ to get the other factors.

By long division, we have

$$y-2\overline{)2y^3-5y^2-19y+42}\left(2y^2-y-21\right.$$
$$2y^3-4y^2$$
$$-\quad+$$
$$-y^2-19y$$
$$-y^2-2y$$
$$+\quad-$$
$$-21y+42$$
$$-21y+42$$
$$+\quad-$$
$$0$$

$$\therefore 2y^3-5y^2-19y+42=(y-2)(2y^2-y-21)$$
$$=(y-2)(y+3)(2y-7)$$

13. $2y^3 - 5y^2 - 19y + 42$

Sol:

(y – 2) (y + 3) (2y – 7)

14. $x^3+13x^2+32x+20$

Sol:

Let $f(x)=x^3+13x^2+32x+20$

The constant term in $f(x)$ is 20 and factors of $+20$ are $\pm1, \pm2, \pm4, \pm5, \pm10, \pm20$.

Putting $x=-1$, we get

$$f(-1)=(-1)^3+13(-1)^2+32(-1)+20$$
$$=-1+13-32+20=0$$

So, $(x+1)$ is a factor of $f(x)$.

Let us now divide $f(x)=x^3+13x^2+32x+20$ by $(x+1)$ to get the remaining factors.

By long division, we have

$$x+1\overline{)x^3+13x^2+32x+20}\left(x^2+12x+20\right.$$
$$x^3+x^2$$
$$-\quad-$$
$$12x^2+32x$$
$$12x^2+32x$$
$$-\quad-$$
$$20x+20$$
$$20x+20$$
$$-\quad-$$
$$0$$

$\therefore x^3+13x^2+32x+20=(x+1)(x^2+12x+20)$

Now,

$x^2+12x+20=x^2+10x+2x+20=x(x+10)+2(x+10)$

$=(x+2)(x+10)$

Hence, $x^3+13x^2+32x+20=(x+1)(x+2)(x+10)$

15. x^3-3x^2-9x-5

Sol:

Let $f(x)=x^3-3x^2-9x-5$

The constant term in $f(x)$ is -5 and factors of -5 are $\pm1,\pm5.$

Putting $x=-1,$ we get

$f(-1)=(-1)^3-3(-1)^2-9(-1)-5=-1-3+9-5=0$

So, $(x+1)$ is a factor of $f(x)$.

Let us now divide $f(x)=x^3-3x^2-9x-5$ by $(x+1)$ to get the other factors.

By long division, we have

$$
\begin{array}{l}
x+1\overline{)\,x^3-3x^2-9x-5\,}(x^2-4x-5 \\
\quad x^3+x^2 \\
\quad -\quad - \\
\hline
\quad\quad -4x^2-9x \\
\quad\quad -4x^2-4x \\
\quad\quad +\quad\quad + \\
\hline
\quad\quad\quad -5x-5 \\
\quad\quad\quad -5x-5 \\
\quad\quad\quad +\quad\ + \\
\hline
\quad\quad\quad 0
\end{array}
$$

$\therefore x^3-3x^2-9x-5=(x+1)(x^2-4x-5)$

Now,

$(x^2-4x-5)=x^2-5x+x-5=x(x-5)+1(x-5)$

$=(x+1)(x-5)$

Hence, $x^3-3x^2-9x-5=(x+1)(x+1)(x-5)$

$=(x+1)^2(x-5)$

16. $2y^3+y^2-2y-1$

Sol:

Let $f(y)=2y^3+y^2-2y-1$

The factors of the constant term of y^3 is 2. Hence possible rational roots are $\pm1, \pm\frac{1}{2}$.

We have,

$f(1)=2(1)^3+(1)^2-2(1)-1=2+1-2-1=0$

So, $(y-1)$ is a factor of $f(y)$

Let us now divide $f(y)=2y^3+y^2-2y-1$ by $(y-1)$ to get the other factors.

By long division, we have

$$
\begin{array}{l}
y-1\overline{)2y^3+y^2-2y-1}(2y^2+3y+1 \\
\quad 2y^3-2y^2 \\
\quad - \quad + \\
\hline
\quad\quad 3y^2-2y \\
\quad\quad 3y^2-2y \\
\quad\quad - \quad + \\
\hline
\quad\quad\quad y-1 \\
\quad\quad\quad y-1 \\
\quad\quad\quad - \; + \\
\hline
\quad\quad\quad 0
\end{array}
$$

$\therefore 2y^3-y^2-2y-1=(y-1)(2y^2+3y+1)$

Now,

$2y^2+3y+1=2y^2+2y+y+1=2y(y+1)+1(y+1)$

$=(2y+1)(y+1)$

Hence, $2y^3+y^2-2y-1=(y-1)(y+1)(2y+1)$

17. x^3-2x^2-x+2

Sol:

Let $f(x)=x^3-2x^2-x+2$

The constant term in $f(x)$ is 2 and factors of 2 are $\pm1, \pm2$.

Putting $x=1$, we have

$f(1)=1^3-2(1)^2-1+2=1-2-1+2=0$

So, $(x-1)$ is a factor of $f(x)$

Let us now divide $f(x)=x^3-2x^2-x+2$ by $(x-1)$ to get the remaining factors.

By long division, we have

$$x-1\overline{\big)x^3-2x^2-x+2}\big(x^2-x-2$$
$$x^3-x^2$$
$$-\quad+$$
$$\overline{\qquad -x^2-x}$$
$$-x^2+x$$
$$+\quad-$$
$$\overline{\qquad -2x+2}$$
$$-2x+2$$
$$+\quad-$$
$$\overline{\qquad 0}$$

$\therefore x^3-2x^2-x+2=(x-1)(x^2-x-2)$

Now,

$x^2-x-2=x^2-2x+x-2=x(x-2)+1(x-2)$

$=(x+1)(x-2)$

Hence $x^3-2x^2-x+2=(x-1)(x+2)(x-2)$

18. Factorize each of the following polynomials:

(i) $x^3+13x^2+31x-45$ given that $x+9$ is a factor

(ii) $4x^3+20x^2+33x+18$ given that $2x+3$ is a factor

Sol:

(i) Let $f(x)=x^3+13x^2+31x-45$

Given that $(x+9)$ is a factor of $f(x)$

Let us divide $f(x)$ by $(x+9)$ to get the other factors. By long division, we have

$$x+9\overline{\big)x^3+12x^2+31x-45}\big(x^2+4x-5$$
$$x^3+9x^2$$
$$-\quad-$$
$$\overline{\qquad 4x^2+31x}$$
$$4x^2+36x$$
$$-\quad-$$
$$\overline{\qquad -5x-45}$$
$$-5x-45$$
$$+\quad+$$
$$\overline{\qquad 0}$$

$\therefore f(x)=x^3+13x^2+31x-45$

$\Rightarrow f(x)=(x+9)(x^2+4x-5)$

Now,

$x^2+4x-5=x^2+5x-x-5=x(x+5)-1(x+5)$

$=(x-1)(x+5)$

$\Rightarrow f(x)=(x+9)(x+5)(x-1)$

$\therefore x^3+13x^2+31x-45=(x-1)(x+5)(x+9)$

(ii) Let $f(x)=4x^3+20x^2+33x+18$

Given that $2x+3$ is a factor of $f(x)$

Let us divide $f(x)$ by $(2x+3)$ to get the other factors. By long division, we have

$$
\begin{array}{r|l}
2x+3 & 4x^3+20x^2+33x+18 \;(\; 2x^2+7x+6 \\
 & 4x^3+6x^2 \\
 & - \quad - \\
\hline
 & 14x^2+33x \\
 & 14x^2+21x \\
 & + \quad - \\
\hline
 & 12x+18 \\
 & 12x+18 \\
 & - \quad - \\
\hline
 & 0
\end{array}
$$

Now,

$4x^3+20x^2+33x+18=(2x+3)(2x^2+7x+6)$

We have,

$2x^2+7x+6=2x^2+4x^2+3x+6=2x(x+2)+3(x+2)$

$=(2x+3)(x+2)$

$\Rightarrow 4x^2+20x^2+33x+18=(2x+3)(2x+3)(x+2)$

$=(2x+3)^2(x+2)$

Hence, $4x^3+20x^2+33x+18=(x+2)(2x+3)^2$

We have,

$2x^2+7x+6=2x^2+4x+3x+6=2x(x+2)+3(x+2)$

$=(2x+3)(x+2)$

$\Rightarrow 4x^3+20x^2+33x+18=(2x+3)(2x+3)(x+2)$

$=(2x+3)^2(x+2)$

Hence, $4x^3+20x^2+33x+18=(x+2)(2x+3)^2$

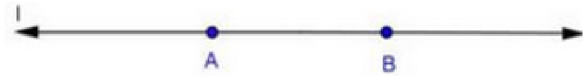
Exercise – 7.1

1. Define the following terms:

(i) Line segment
(ii) Collinear points
(iii) Parallel lines
(iv) Intersecting lines
(v) Concurrent lines
(vi) Ray
(vii) Half-line

Sol:

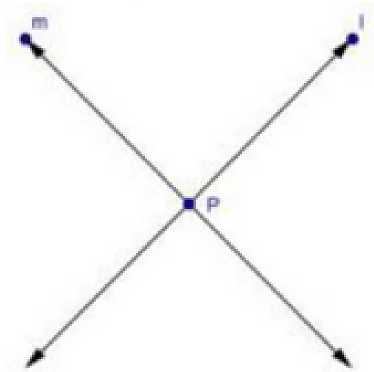
(i) Line-segment- Give two points A and B on a line *l*, the connected part (segment) of the line with end points at A and B is called the line segment AB.



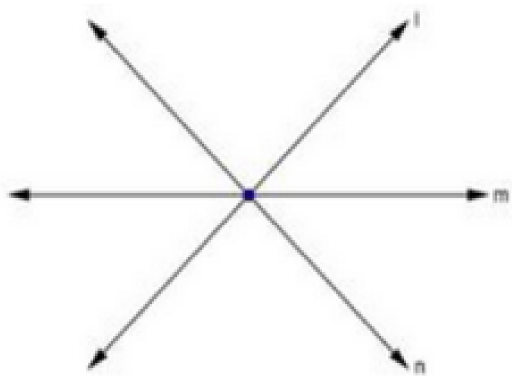
(ii) Collinear points – Three or more points are said to be collinear if there is a line which contains all of them.

(iii) Parallel lines – Two lines *l* and m in a plane are said to be parallel lines if they do not intersect each other.

(iv) Intersecting lines – Two lines are intersecting if they have a common point. The common point is called point of intersection.



(v) Concurrent lines – Three or more lines are said to be concurrent if there is a point which lies on all of them.



(vi) Ray – A line in which one end point is fixed and the other part can be extended endlessly.

(vii) Half-line – If A, B, C be the points on a line *l*, such that A lies between B and C, and we delete the point A from line *l*, the two parts of l that remain are each called half-line.

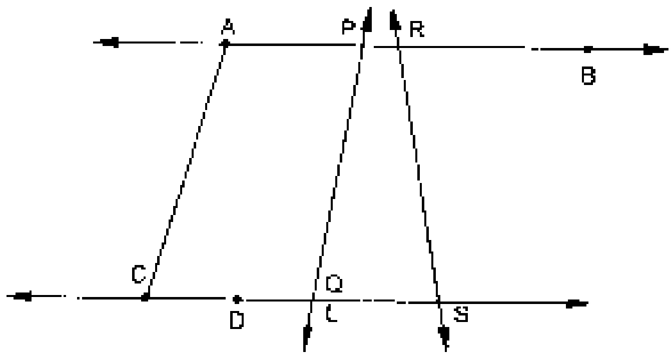
2. (i) How many lines can pass through a given point?
(ii) In how many points can two distinct lines at the most intersect?
Sol:
(i) Infinitely many
(ii) one

3. (i) Given two points P and Q, find how many line segments do they deter-mine.
(ii) Name the line segments determined by the three collinear points P, Q and R.
Sol:
(i) One
(ii) PQ, QR, PR

4. Write the truth value (T/F) of each of the following statements:
(i) Two lines intersect in a point.
(ii) Two lines may intersect in two points.
(iii) A segment has no length.
(iv) Two distinct points always determine a line.
(v) Every ray has a finite length.
(vi) A ray has one end-point only.
(vii) A segment has one end-point only.
(viii) The ray AB is same as ray BA.
(ix) Only a single line may pass through a given point.
(x) Two lines are coincident if they have only one point in common.
Sol:
(i) False
(ii) False
(iii) False
(iv) True
(v) False
(vi) True
(vii) False

(viii) False
(ix) False
(x) False

5. In the below fig., name the following:



Sol:

(i) Five line segments
AB, CD, AC, PQ, DS

(ii) Five rays $\overrightarrow{PA}, \overrightarrow{RB}, \overrightarrow{DC}, \overrightarrow{QS}, \overrightarrow{DS}$

(iii) Four collinear points. C, D, Q, S

(iv) Two pairs of non–intersecting line segments AB and CD
AB and LS

6. Fill in the blanks so as to make the following statements true:

(i) Two distinct points in a plane determine a ________ line.

(ii) Two distinct ________ in a plane cannot have more than one point in common.

(iii) Given a line and a point, not on the line, there is one and only ________ line which passes through the given point and is ________ to the given line.

(iv) A line separates a plane into ________ parts namely the and the ________ itself.

Sol:

(i) Unique

(ii) Lines

(iii) Perpendicular, perpendicular

(iv) Three, two half planes, line.

Exercise – 8.1

1. Write the complement of each of the following angles:
(i) 20° (ii) 35° (iii) 90° (iv) 77° (v) 30°

Sol:

(i) Given angle is $20°$
Since, the sum of an angle and its complement is $90°$.
$\therefore$ its, complement will be $(90-20=70°)$

(ii) Given angle is $35°$
Since, the sum of an angle and its complement is $90°$.
$\therefore$ its, complements will be $(90-35°=55°)$

(iii) The given angle is $90°$
Since, the sum of an angle and its complement is $90°$.
$\therefore$ [its, complement will be $(90-90°=0°)$]

(iv) The given angle is $77°$
Since, the sum of an angle and its complement is $90°$.
$\therefore$ its, complement will be $(90-77°=13°)$

(v) The given angle is $30°$.
Since, the sum of an angle and its complement is $90°$.
$\therefore$ its, complement will be $(90-30°=60°)$

2. Write the supplement of each of the following angles:
(i) 54° (ii) 132° (iii) 138°

Sol:

(i) The given angle is $54°$
Since, the sum of an angle and its supplement is $180°$.
$\therefore$ its, supplement will be $180°-54°=126°$

(ii) The given angle is $132°$
Since, the sum of an angle and its supplement is $180°$.
$\therefore$ its, supplement will be $180°-132°=48°$

(iii) The given angle is $138°$
Since, the sum of an angle and its supplement is $180°$.
$\therefore$ its, supplement will be $180°-138°=42°$

3. If an angle is 28° less than its complement, find its measure.
Sol:
Angle measured will be 'x' say
$\therefore$ its complement will be $(90-x)^\circ$
It is given that
Angle = Complement –28°
$\Rightarrow x=(90-x)^\circ-28^\circ$
$\Rightarrow x^\circ=90^\circ-28^\circ-x^\circ$
$\Rightarrow 2x^\circ=62^\circ$
$\Rightarrow x=31^\circ$
$\therefore$ Angle measured is 31°

4. If an angle is 30° more than one half of its complement, find the measure of the angle.
Sol:
Angle measured will be 'x' say.
$\therefore$ its complement will be $(90-x)^\circ$
It is given that
Angle $=30^\circ+\frac{1}{2}$ Complement
$\Rightarrow x^\circ=30^\circ+\frac{1}{2}(90-x)$
$\Rightarrow 3\frac{x}{2}=30^\circ+45^\circ$
$\Rightarrow 3x=150^\circ$
$\Rightarrow x=\frac{150}{3}$
$\Rightarrow x=50^\circ$
$\therefore$ Angle is 50°

5. Two supplementary angles are in the ratio 4 : 5. Find the angles.
Sol:
Supplementary angles are in the ratio $4:5$
Let the angles be 4x and 5x
It is given that they are supplementary angles
$\therefore 4x+5x=180^\circ x$
$\Rightarrow 9x=180^\circ$
$\Rightarrow x=20^\circ$
Hence, $4x=4(20)=80^\circ$
$5(x)=5(20)=100^\circ$
$\therefore$ Angles are 80° and 100°

6. Two supplementary angles differ by 48°. Find the angles.

Sol:

Given that two supplementary angles are differ by $48°$

Let the angle measured is $x°$

$\therefore$ Its supplementary angle will be $(180-x)°$

It is given that

$(180-x)-x=98°$

$\Rightarrow 180-48°=2x$

$\Rightarrow 132=2x$

$\Rightarrow x=\frac{132}{2}$

$\Rightarrow x=66°$

Hence, $180-x=114°$

Therefore, angles are $66°$ and $114°$

7. An angle is equal to 8 times its complement. Determine its measure.

Sol:

It is given that angle = 8 times its complement

Let 'x' be measured angle

$\Rightarrow$ angle = 8 complements

$\Rightarrow$ angle = $8(90-x)°$ [$\because$ complement $=(90-x)°$]

$\Rightarrow x°=8(90)-8x°$

$\Rightarrow 9x°=720°$

$\Rightarrow x=\frac{720}{9}=80$

$\therefore$ The measured angle is $80°$

8. If the angles (2x −10)° and (x − 5)° are complementary angles, find x.

Sol:

Given that,

$(2x-10)°$ and $(x-5)°$ are complementary angles.

Let x be the measured angle.

Since the angles are complementary

$\therefore$ Their sum will be $90°$

$\Rightarrow (2x-10)+(x-5)=90°$

$\Rightarrow 3x-15=90$

$\Rightarrow 3x=90°+15°$

$\Rightarrow x = \frac{105°}{3} = \frac{105°}{3} = 35°$

$\Rightarrow x = 35°$

9. If the complement of an angle is equal to the supplement of the thrice of it. Find the measure of the angle.

Sol:

The angle measured will be $'x'$ say.

Its complementary angle is $(90° - x°)$ and

Its supplementary angle is $(180° - 3x°)$

Given that,

Supplementary of thrice of the angle $= (180° - 3x°)$

According to the given information

$(90 - x)° = (180 - 3x)°$

$\Rightarrow 3x° - x° = 180° - 90°$

$\Rightarrow 2x° = 90°$

$\Rightarrow x = 45°$

The angle measured is $45°$

10. If an angle differs from its complement by 10°, find the angle.

Sol:

The measured angle will be ‘x’ say

Given that,

The angles measured will be differed by $10°$

$x° - (90 - x)° = 10°$

$\Rightarrow x - 90 + x = 10$

$\Rightarrow 2x = 100$

$\Rightarrow x = 50°$

$\therefore$ The measure of the angle will be $= 50°$

11. If the supplement of an angle is three times its complement, find the angle.

Sol:

Given that,

Supplementary of an angle = 3 times its complementary angle.

The angles measured will be $x°$

Supplementary angle of x will be $180° - x°$ and

The complementary angle of x will be $(90° - x°)$.

It's given that
Supplementary of angle = 3 times its complementary angle
$180° - x° = 3(90° - x°)$
$\Rightarrow 180° - x° = 270° - 3x°$
$\Rightarrow 3x° - x° = 270° - 180°$
$\Rightarrow 2x° = 90°$
$\Rightarrow x = 45°$
$\therefore$ Angle measured is $45°$.

12. If the supplement of an angle is two-third of itself. Determine the angle and its supplement.
Sol:
Given that
Supplementary of an angle $= \frac{2}{3}$ of angle itself.
The angle measured be 'x' say.
Supplementary angle of x will be $(180 - x)°$
It is given that
$(180 - x)° = \frac{2}{3}x°$
$\Rightarrow 180° - x° = \frac{2}{3}x°$
$\Rightarrow \frac{2}{3}x° + x° = 180°$
$\Rightarrow 2x° + 3x° = 3 \times 180°$
$\Rightarrow 5x° = 540°$
$\Rightarrow x = 108°$
Hence, supplement $= 180 - 108 = 72°$
$\therefore$ Angle will be $108°$ and its supplement will be $72°$.

13. An angle is 14° more than its complementary angle. What is its measure?
Sol:
Given that,
An angle is $14°$ more than its complementary angle
The angle measured is 'x' say
The complementary angle of 'x' is $(90 - x)$
It is given that
$x - (90 - x) = 14$

$\Rightarrow x-90+x=14$

$\Rightarrow 2x^\circ=90^\circ+14^\circ$

$\Rightarrow x^\circ=\frac{104^\circ}{2}$

$\Rightarrow x=52^\circ.$

$\therefore$ The angle measured is 52°

14. The measure of an angle is twice the measure of its supplementary angle. Find its measure.

Sol:

Given that

The angle measure of an angle is twice of the measure of the supplementary angle.

Let the angle measured will be 'x' say

$\therefore$ The supplementary angle of x is $180-x$ as per question

$x^\circ=2(180-x)^\circ$

$x^\circ=2(180^\circ)-2x^\circ$

$\Rightarrow 3x^\circ=360^\circ$

$\Rightarrow x^\circ=120^\circ$

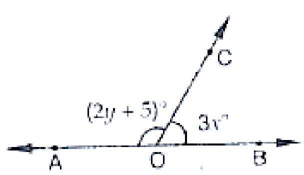
$\therefore$ The angle measured is 120°.

Exercise – 8.2

1. In the below Fig, OA and OB are opposite rays:

(i) If x = 25°, what is the value of y?

(ii) If y = 35°, what is the value of x?



Sol:

(i) Given that $x=25^\circ$

Since $\angle AOC$ and $\angle BOC$ form a linear pair

$\angle AOC+\angle BOC=180^\circ$

Given that

$\angle AOC=2y+5$ and $\angle BOC=3x$

$\therefore \angle AOC+\angle BOC=180^\circ$

$(2y+5)^\circ+3x=180^\circ$

$(2y+5)^\circ+3(25^\circ)=180^\circ$

$2y° + 5° + 75° = 180°$

$2y° + 80° = 180°$

$2y° = 180° - 80° = 100°$

$y° = \frac{100°}{2} = 50°$

$\Rightarrow \boxed{y = 50°}$

(ii) Given that if $y = 35°$

$\angle AOC + \angle BOC = 180°$

$(2y+5) + 3x = 180°$

$(2(35)+5) + 3x = 180°$

$(70+5) + 3x = 180°$

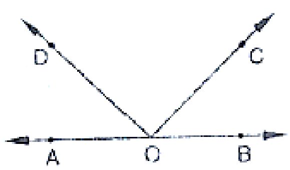
$3x = 180° - 75°$

$3x = 105°$

$x = 35°$

$\boxed{x = 35°}$

2. In the below fig, write all pairs of adjacent angles and all the linear pairs.



Sol:

Adjacent angles are

(i) $\angle AOC, \angle COB$

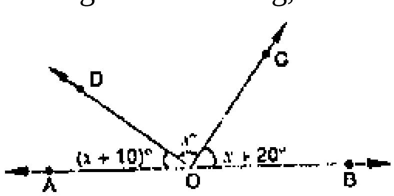
(ii) $\angle AOD, \angle BOD$

(iii) $\angle AOD, \angle COD$

(iv) $\angle BOC, \angle COD$

Linear pairs : $\angle AOD, \angle BOD; \angle AOC, \angle BOC.$

3. In the given below Fig, find x. Further find ∠BOC, ∠COD and ∠AOD.



Sol:

Since $\angle AOD$ and $\angle BOD$ are form a line pair

$\angle AOD + \angle BOD = 180°$

$\angle AOD + \angle COD + \angle BOC = 180°$

Given that

$\angle AOD = (x+10)°, \angle COD = x°, \angle BOC = (x+20)°$

$\Rightarrow (x+10)° + x° + (x+20)° = 180°$

$\Rightarrow 3x + 30° = 180°$

$\Rightarrow 3x = 180° - 30°$

$\Rightarrow 3x = 150°$

$\Rightarrow x = 50°$

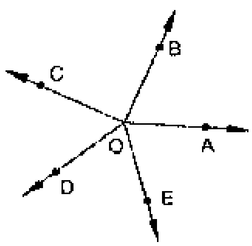
$\therefore \angle AOD = x + 10°$

$= 50° + 10° = 60°$

$\angle COD = x° = 50°$

$\angle BOC = x + 20° = 50 + 20 = 70°$

4. In the given below fig, rays OA, OB, OC, OP and 0E have the common end point O. Show that ∠AOB + ∠BOC + ∠COD + ∠DOE + ∠EOA = 360°.



Sol:

Given that

Rays OA, OB, OD and OE have the common end point O.

A ray of opposite to OA is drawn

Since $\angle AOB, \angle BOF$ are linear pairs

$\angle AOB + \angle BOF = 180°$

$\angle AOB + \angle BOC + \angle COF = 180°$ ....(1)

Also

$\angle AOE, \angle EOF$ are linear pairs

$\angle AOE + \angle EOF = 180°$

$\angle AOE + \angle DOF + \angle DOE = 180°$ ....(2)

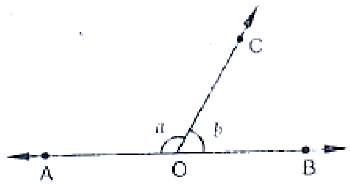
By adding (1) and (2) equations we get

$\angle AOB + \angle BOC + \angle COF + \angle AOE + \angle DOF + \angle DOE = 360°$

$\angle AOB + \angle BOC + \angle COD + \angle DOE + \angle EOA = 360°$

Hence proved.

5. In the below Fig, ∠AOC and ∠BOC form a linear pair. if a − 2b = 30°, find a and b.



Sol:

Given that,

$\angle AOC$ and $\angle BOC$ form a linear pair

If $a-2b=30°$

$\angle AOC=a°, \angle BOC=b°$

$\therefore a+b=180°$ ....(i)

Given $a-2b=30°$ ....(ii)

By subtracting (i) and (ii)

$a+b-a+2b=180°-30°$

$\Rightarrow 3b=150°$

$\Rightarrow b=\dfrac{150°}{3}$

$\Rightarrow b=50°$

Hence $a-2b=30°$

$a-2(50)°=30°$ $[\because b=50°]$

$a=30°+100°$

$a=130°$

$\therefore a=130°, b=50°.$

6. How many pairs of adjacent angles are formed when two lines intersect in a point?

Sol:

Four pairs of adjacent angle formed when two lines intersect in a point they are

$\angle AOD, \angle DOB$

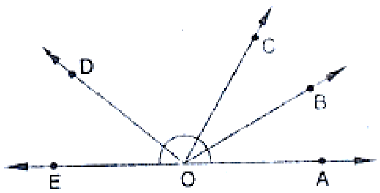
$\angle DOB, \angle BOC$

$\angle COA, \angle AOD$

$\angle BOC, \angle COA$

Hence 4 pairs

7. How many pairs of adjacent angles, in all, can you name in below fig.?



Sol:

Pairs of adjacent angles are

$\angle EOC, \angle DOC$

$\angle EOD, \angle DOB$

$\angle DOC, \angle COB$

$\angle EOD, \angle DOA$

$\angle DOC, \angle COA$

$\angle BOC, \angle BOA$

$\angle BOA, \angle BOD$

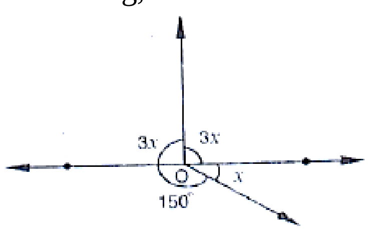
$\angle BOA, \angle BOE$

$\angle EOC, \angle COA$

$\angle EOC, \angle COB$

$\therefore$ Hence 10 pairs of adjacent angles

8. In below fig, determine the value of x.



Sol:

Since sum of all the angles round a point is equal to $360°$. Therefore

$\Rightarrow 3x + 3x + 150 + x = 360°$

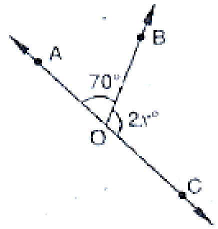
$\Rightarrow 7x° = 360° - 150°$

$\Rightarrow 7x = 210°$

$\Rightarrow x = \frac{210}{7}$

$\Rightarrow x = 30°$

9. In the below fig, AOC is a line, find x.



Sol:

Since $\angle AOB$ and $\angle BOC$ are linear pairs

$\angle AOB + \angle BOC = 180°$

$\Rightarrow 70° + 2x° = 180°$

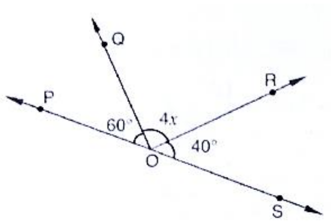
$\Rightarrow 2x° = 180° - 70°$

$\Rightarrow 2x = 110°$

$\Rightarrow x\frac{110}{2}$

$\Rightarrow x = 55°$

10. In the below fig, POS is a line, find x.



Sol:

Since $\angle POQ$ and $\angle QOS$ are linear pairs

$\angle POQ + \angle QOS = 180°$

$\Rightarrow \angle POQ + \angle QOR + \angle SOR = 180°$

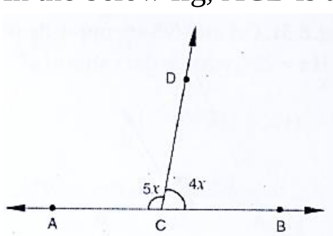
$\Rightarrow 60° + 4x° + 40° = 180°$

$\Rightarrow 4x° = 180° - 100°$

$\Rightarrow 4x° = 80°$

$\Rightarrow \boxed{x = 20°}$

11. In the below fig, ACB is a line such that ∠DCA = 5x and ∠DCB = 4x. Find the value of x.



Sol:

Here, $\angle ACD + \angle BCD = 180°$

[Since $\angle ACD, \angle BCD$ are linear pairs]

$\angle ACD = 5x, \angle BCD = 4x$

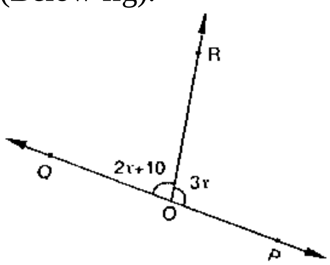
$\Rightarrow 5x + 4x = 180°$

$\Rightarrow 9x = 180°$

$\Rightarrow x = 20°$

$\therefore x = 20°$

12. Given ∠POR = 3x and ∠QOR = 2x + 10, find the value of x for which POQ will be a line. (Below fig).



Sol:

Since $\angle QOR, \angle POP$ are linear pairs

$\angle QOR + \angle POR = 180°$

$\Rightarrow 2x + 10 + 3x = 180°$ $\quad [\because \angle QOR = 2x + 10, \angle POR = 3x]$

$\Rightarrow 5x + 10 = 180°$

$\Rightarrow 5x = 180° - 10$

$\Rightarrow 5x = 170°$

$\Rightarrow x = 34°$

13. In Fig. 8.42, a is greater than b by one third of a right-angle. Find the values of a and b.

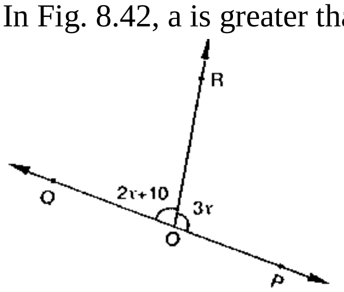


Fig. 8.41

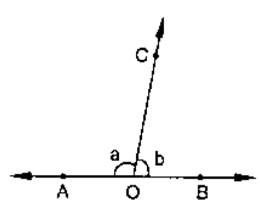


Fig. 8.42

Sol:

Since a, b are linear pair

$\Rightarrow a + b = 180°$

$\Rightarrow a = 180 - b$ ....(1)

Now,

$\Rightarrow a = b + \frac{1}{3} \times 90°$ [given]

$\Rightarrow a = b + 30°$ ....(2)

$\Rightarrow a - b = 30°$

Equating (1) and (2) equations

$180 - b = b + 30°$

$\Rightarrow 180° - 30° = b + b$

$\Rightarrow 150° = 2b$

$\Rightarrow b = \frac{150°}{2}$

$\Rightarrow b = 75°$

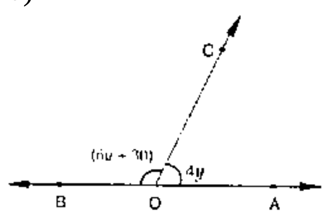
Hence $a = 180 - b$

$= 180 - 75°$ $[\because b = 75°]$

$a = 105°$

$\therefore a = 105°, b = 75°$

14. What value of y would make AOB a line in below fig, if ∠AOC = 4y and ∠BOC = (6y + 30)



Sol:

Since $\angle AOC, \angle BOC$ are linear pair

$\Rightarrow \angle AOC + \angle BOC = 180°$

$\Rightarrow 6y + 30 + 4y = 180°$

$\Rightarrow 10y + 30 = 180°$

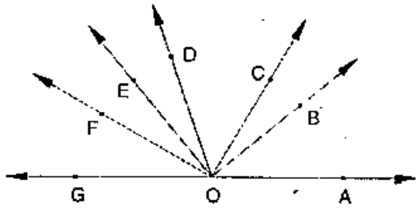
$\Rightarrow 10y = 180° - 30°$

$\Rightarrow 10y = 150°$

$\Rightarrow y = \frac{150°}{10}$

$\Rightarrow y = 15°$

15. If below fig, ∠AOF and ∠FOG form a linear pair.



∠EOB = ∠FOC = 90° and ∠DOC = ∠FOG = ∠AOB = 30°

(i) Find the measures of ∠FOE, ∠COB and ∠DOE.

(ii) Name all the right angles.

(iii) Name three pairs of adjacent complementary angles.

(iv) Name three pairs of adjacent supplementary angles.

(v) Name three pairs of adjacent angles.

Sol:

(i) $\angle FOE = x, \angle DOE = y$ and $\angle BOC = z$ sat

Since $\angle AOF, \angle FOG$ is Linear pair

$\Rightarrow \angle AOF + 30° = 180°$ $\quad [\angle AOF + \angle FOG = 180°$ and $\angle FOG = 30°]$

$\Rightarrow \angle AOF = 180° - 30°$

$\Rightarrow \angle AOF = 150°$

$\Rightarrow \angle AOB + \angle BOC + \angle COD + \angle DOE + \angle EOF = 150°$

$\Rightarrow 30° + z + 30° + y + x = 150°$

$\Rightarrow x + y + z = 150° - 30° - 30°$

$\Rightarrow x + y + z = 90°$ .....(1)

Now $\angle FOC = 90°$

$\Rightarrow \angle FOE + \angle EOD + \angle DOC = 90°$

$\Rightarrow x + y + 30° = 90°$

$\Rightarrow x + y = 90° - 30°$

$\Rightarrow x + y = 60°$ ....(2)

Substituting (2) in (1)

$x + y + z = 90°$

$\Rightarrow 60 + z = 90° \Rightarrow z = 90° - 60° = 30°$

i.e., $\angle BOC = 30°$

Given $\angle BOE = 90°$

$\Rightarrow \angle BOC + \angle COD + \angle DOE = 90°$

$\Rightarrow 30° + 30° + \angle DOE = 90°$

$\Rightarrow \angle DOE = 90° - 60° = 30°$

$\therefore \angle DOE = x = 30°$

Now, also we have

$x + y = 60°$

$\Rightarrow y = 60° - x = 60° - 30° = 30°$

$\angle FOE = 30°$

(ii) Right angles are

$\angle DOG, \angle COF, \angle BOF, \angle AOD$

(iii) Three pairs of adjacent complementary angles are

$\angle AOB, \angle BOD$;

$\angle AOC, \angle COD$;

$\angle BOC, \angle COE$.

(iv) Three pairs of adjacent supplementary angles are

$\angle AOB, \angle BOG$;

$\angle AOC, \angle COG$;

$\angle AOD, \angle DOG$.

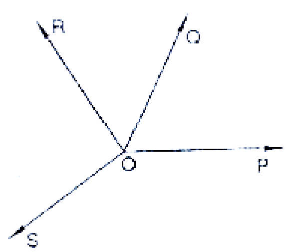
(v) Three pairs of adjacent angles

$\angle BOC, \angle COD$;

$\angle COD, \angle DOE$;

$\angle DOE, \angle EOF$,

16. In below fig, OP, OQ, OR and OS arc four rays. Prove that:
∠POQ + ∠QOR + ∠SOR + ∠POS = 360°



Sol:

Given that

OP, OQ, OR and OS are four rays

You need to produce any of the ray OP, OQ, OR and OS backwards to a point in the figure.

Let us produce ray OQ backwards to a point

T so that TOQ is a line

Ray OP stands on the TOQ

Since $\angle TOP, \angle POQ$ is linear pair

$\angle TOP + \angle POQ = 180°$ ....(1)

Similarly, ray OS stands on the line TOQ

$\therefore \angle TOS + \angle SOQ = 180°$ ....(2)

But $\angle SOQ = \angle SOR + \angle QOR$

So, (2), becomes

$\angle TOS + \angle SOR + \angle OQR = 180°$

Now, adding (1) and (3) you get

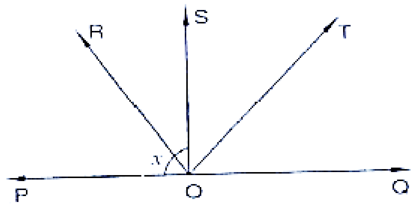
$\angle TOP + \angle POQ + \angle TOS + \angle SOR + \angle QOR = 360°$

$\Rightarrow \angle TOP + \angle TOS = \angle POS$

$\therefore (4)$ becomes

$\angle POQ + \angle QOR + \angle SOR + \angle POS = 360°$

17. In below fig, ray OS stand on a line POQ. Ray OR and ray OT are angle bisectors of ∠POS and ∠SOQ respectively. If ∠POS = x, find ∠ROT.



Sol:

Given,

Ray OS stand on a line POQ

Ray OR and Ray OT are angle bisectors of $\angle POS$ and $\angle SOQ$ respectively

$\angle POS = x$

$\angle POS$ and $\angle QOS$ is linear pair

$\angle POS + \angle QOS = 180°$

$x + \angle QOS = 180°$

$\angle QOS = 180 - x$

Now, ray or bisector $\angle POS$

$\therefore \angle ROS = \frac{1}{2}\angle POS$

$= \frac{1}{2} \times x \qquad [\because \angle POS = x]$

$\angle ROS = \frac{x}{2}$

Similarly ray OT bisector $\angle QOS$

$\therefore \angle TOS = \frac{1}{2}\angle QOS$

$= \frac{180 - x}{2}$ $[\because \angle QOS = 180 - x]$

$= 90 - \frac{x}{2}$

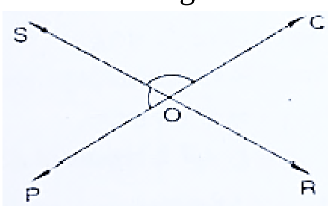
$\therefore \angle ROT = \angle ROS + \angle ROT$

$= \frac{x}{2} + 90 - \frac{x}{2}$

$= 90°$

$\therefore \angle ROT = 90°$

18. In the below fig, lines PQ and RS intersect each other at point O. If ∠POR: ∠ROQ − 5 : 7, find all the angles.



Sol:

Given $\angle POR$ and $\angle ROP$ is linear pair

$\angle POR + \angle ROP = 180°$

Given that

$\angle POR : \angle ROP = 5 : 7$

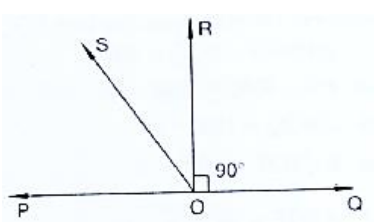
$\therefore \angle POR = \frac{5}{12} \times 180 = 75°$

Similarly $\angle ROQ = \frac{7}{5+7} \times 180° = 105°$

Now, $\angle POS = \angle ROQ = 105°$ [∵ Vertically opposite angles]

$\therefore \angle SOQ = \angle POR = 75°$ [∵ Vertically opposite angles]

19. In the below fig, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that ∠ROS = $\frac{1}{2}$ (∠QOS − ∠POS).



Sol:

Given that, OR perpendicular

$\therefore \angle POR = 90°$

$\angle POS + \angle SOR = 90°$ $[\because \angle POR = \angle POS + \angle SOR]$

$\angle ROS = 90° - \angle POS$ ....(1)

$\angle QOR = 90°$ $(\because OR \perp PQ)$

$\angle QOS - \angle ROS = 90°$

$\angle ROS = \angle QOS - 90°$ ....(2)

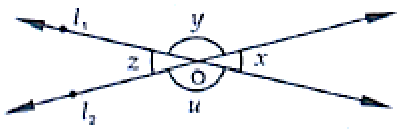
By adding (1) and (2) equations, we get

$2\angle ROS = \angle QOS - \angle POS$

$\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$

Exercise – 8.3

1. In the below fig, lines l_1 and l_2 intersect at O, forming angles as shown in the figure. If x = 45, Find the values of x, y, z and u.



Sol:

Given that

$x = 45°, y = ?, z = ?, u = ?$

Vertically opposite sides are equal

$\therefore z = x = 45°$

z and u angles are linear pair of angles

$\therefore z + u = 180°$

$z = 180° - 4$

$\Rightarrow u = 180° - x$

$\Rightarrow u = 180° - 45°$ $[\because x = 45°]$

$\Rightarrow u = 135°$

x and y angles are linear pair of angles

$\therefore x + y = 180°$

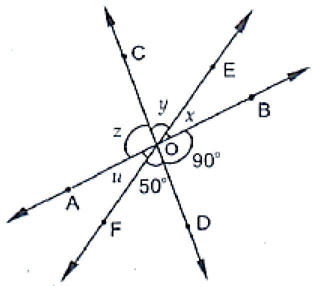
$y = 180° - x$

$y = 180° - 45°$

$y = 135°$

$\therefore x = 45°, y = 135°, z = 135°$ and $u = 45°$

2. In the below fig, three coplanar lines intersect at a point O, forming angles as shown in the figure. Find the values of x, y, z and u.



Sol:

Vertically opposite angles are equal

So $\angle BOD = z = 90°$

$\angle DOF = y = 50°$

Now, $x + y + z = 180°$ [Linear pair]

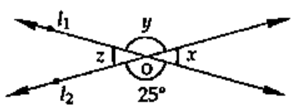
$\Rightarrow x + y + z = 180°$

$\Rightarrow 90° + 50° + x = 180°$

$\Rightarrow x = 180° - 140°$

$\Rightarrow x = 40°$

3. In the given fig, find the values of x, y and z.



Sol:

From the given figure

$\angle y = 25°$ [$\because$ Vertically opposite angles are equal]

Now

$\angle x + \angle y = 180°$ [Linear pair of angles are *x* and *y*]

$\Rightarrow \angle x = 180° - 25°$

$\Rightarrow \angle x = 155°$

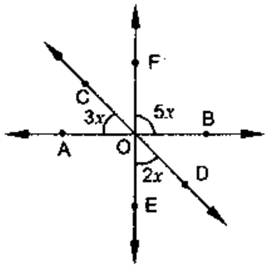
Also

$\angle z = \angle x = 155°$ [Vertically opposite angle]

$\angle y = 25°$

$\angle z = \angle z = 155°$

4. In the below fig, find the value of x.



Sol:

Vertically opposite angles are equal

$\angle AOE = \angle BOF = 5x$

Linear pair

$\angle COA + \angle AOE + \angle EOD = 180°$

$\Rightarrow 3x + 5x + 2x = 180°$

$\Rightarrow 10x = 180°$

$\Rightarrow x = 18°$

5. Prove that the bisectors of a pair of vertically opposite angles are in the same straight line.

Sol:

Given,

Lines AOB and COD intersect at point O such that

$\angle AOC = \angle BOD$

Also OE is the bisector $\angle ADC$ and OF is the bisector $\angle BOD$

To prove: EOF is a straight line vertically opposite angles is equal

$\angle AOD = \angle BOC = 5x$ ....(1)

Also $\angle AOC + \angle BOD$

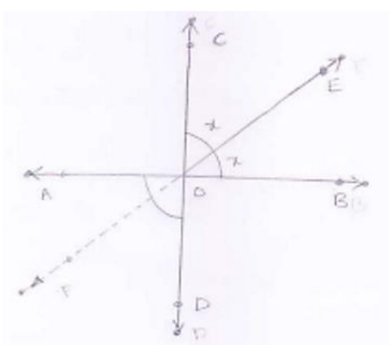
$\Rightarrow 2\angle AOE = 2\angle DOF$ ....(2)

Sum of the angles around a point is $360°$

$\Rightarrow 2\angle AOD + 2\angle AOE + 2\angle DOF = 360°$

$\Rightarrow \angle AOD + \angle AOF + \angle DOF = 180°$

From this we conclude that EOF is a straight line.



Given that :- AB and CD intersect each other at O

OE bisects $\angle COB$

To prove: $\angle AOF = \angle DOF$

Proof: OE bisects $\angle COB$

$\angle COE = \angle EOB = x$

Vertically opposite angles are equal

$\angle BOE = \angle AOF = x$ ....(1)

$\angle COE = \angle DOF = x$ ....(2)

From (1) and (2)

$\angle AOF = \angle DOF = x$

6. If one of the four angles formed by two intersecting lines is a right angle, then show that each of the four angles is a right angle.

Sol:

Given,

AB and CD are two lines intersecting at O such that

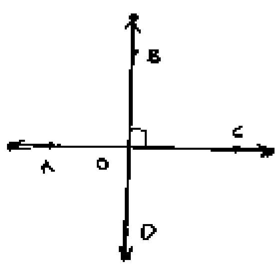
$\angle BOC = 90°$

$\angle AOC = 90°, \angle AOD = 90°$ and $\angle BOD = 90°$

Proof:

Given that $\angle BOC = 90°$

Vertically opposite angles are equal



$\angle BOC = \angle AOD = 90°$

$\angle AOC, \angle BOC$ are Linear pair of angles

$\angle AOC + \angle BOC = 180°$ [LinearPair]

$\Rightarrow \angle AOC + 90° = 180°$

$\Rightarrow \angle AOC = 90°$

Vertically opposite angles

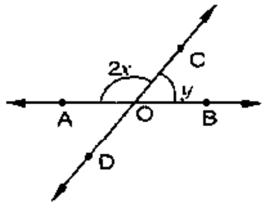
$\therefore \angle AOC = \angle BOD = 90°$

Hence, $\angle AOC = \angle BOC = \angle BOD = \angle AOD = 90°$

7. In the below fig, rays AB and CD intersect at O.

(i) Determine y when x = 60°

(ii) Determine x when y =40



Sol:

(i) Given $x = 60°$

$y = ?$

$\angle AOC, \angle BOC$ are linear pair of angles

$\angle AOC + \angle BOC = 180°$

$\Rightarrow 2x + y = 180°$

$\Rightarrow 2 \times 60 + y = 180°$ $[\because x = 60°]$

$\Rightarrow y = 180° - 120°$

$\Rightarrow \boxed{y = 60°}$

(ii) Given $y = 40°, x = ?$

$\angle AOC$ and $\angle BOC$ are linear pair of angles

$\angle AOC + \angle BOC = 180°$

$\Rightarrow 2x + y = 180°$

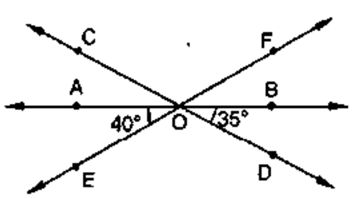
$\Rightarrow 2x + 40 = 180°$

$\Rightarrow 2x = 140°$

$\Rightarrow x = \frac{140°}{2}$

$\Rightarrow \boxed{y = 70°}$

8. In the below fig, lines AB, CD and EF intersect at O. Find the measures of ∠AOC, ∠COF, ∠DOE and ∠BOF.



Sol:

$\angle AOE$ and $\angle EOB$ are linear pair of angles

$\angle AOE + \angle EOB = 180°$

$\angle AOE + \angle DOE + \angle BOD = 180°$

$\Rightarrow \angle DOE = 180° - 40° - 35° = 105°$

Vertically opposite side angles are equal

$\angle DOE = \angle COF = 105°$

Now, $\angle AOE + \angle AOF = 180°$ [$\because$ Linear pair]

$\Rightarrow \angle AOE + \angle AOC + \angle COF = 180°$

$\Rightarrow 40° + \angle AOC + 105° = 180°$

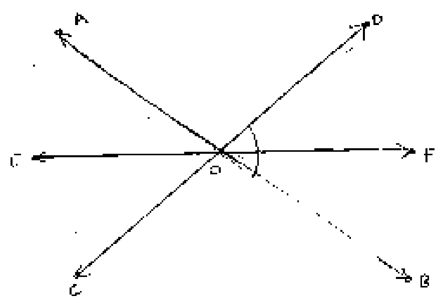
$\Rightarrow \angle AOC = 180° - 145°$

$\Rightarrow \angle AOC = 35°$

Also, $\angle BOF = \angle AOE = 40°$ [$\because$ Vertically opposite angle are equal]

9. AB, CD and EF are three concurrent lines passing through the point O such that OF bisects ∠BOD. If ∠BOF = 35°, find ∠BOC and ∠AOD.

Sol:



Given

OF bisects $\angle BOD$

OF bisects $\angle BOD$

$\angle BOF = 35°$

$\angle BOC = ?$

$\angle AOD = ?$

$\therefore \angle BOD = 2\angle BOF = 70°$ [$\because$ of bisects $\angle BOD$]

$\angle BOD = \angle AOC = 70°$ [$\angle BOD$ and $\angle AOC$ are vertically opposite angles]

Now,

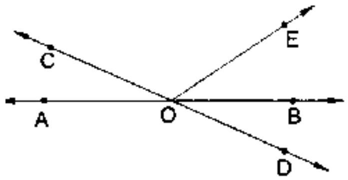
$\angle BOC + \angle AOC = 180°$

$\Rightarrow \angle BOC + 70° = 180°$

$\Rightarrow \angle BOC = 110°$

$\therefore \angle AOD = \angle BOC = 110°$ [Vertically opposite angles]

10. In below figure, lines AB and CD intersect at O. If ∠AOC + ∠BOE = 70° and ∠BOD = 40°, find ∠BOE and reflex ∠COE.



Sol:

Given that

$\angle AOC + \angle BOE = 70°$ and $\angle BOD = 40°$

$\angle BOE = ?$

Here, $\angle BOD$ and $\angle AOC$ are vertically opposite angles

$\angle BOD = \angle AOC = 40°$

Given $\angle AOC + \angle BOE = 70°$

$40° + \angle BOF = 70°$

$\angle BOF = 70° - 40°$

$\angle BOE = 30°$

$\angle AOC$ and $\angle BOC$ are linear pair of angles

$\Rightarrow \angle AOC + \angle COF + \angle BOE = 180°$

$\Rightarrow \angle COE = 180° - 30° - 40°$

$\Rightarrow \angle COE = 110°$

$\therefore$ Reflex $\angle COE = 360° - 110° = 250°$.

11. Which of the following statements are true (T) and which are false (F)?

(i) Angles forming a linear pair are supplementary.
(ii) If two adjacent angles are equal, and then each angle measures 90°.
(iii) Angles forming a linear pair can both the acute angles.
(iv) If angles forming a linear pair are equal, then each of these angles is of measure 90°.

Sol:

(i) True
(ii) False
(iii) False
(iv) true

12. Fill in the blanks so as to make the following statements true:

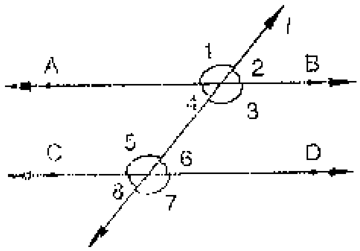
(i) If one angle of a linear pair is acute, then its other angle will be _____
(ii) A ray stands on a line, then the sum of the two adjacent angles so formed is ______
(iii) If the sum of two adjacent angles is 180°, then the ______ arms of the two angles are opposite rays.

Sol:

(i) Obtuse angle
(ii) 180°
(iii) uncommon

Exercise – 8.4

1. In below fig, AB CD and $\angle 1$ and $\angle 2$ are in the ratio 3 : 2. Determine all angles from 1 to 8.



Sol:

Let $\angle 1 = 3x$ and $\angle 2 = 2x$

$\angle 1$ and $\angle 2$ are linear pair of angle

Now, $\angle 1 + \angle 2 = 180°$

$\Rightarrow 3x + 2x = 180°$

$\Rightarrow 5x = 180°$

$\Rightarrow x = \frac{180°}{5}$

$\Rightarrow x = 36°$

$\therefore \angle 1 = 3x = 108°, \angle 2 = 2x = 72°$

Vertically opposite angles are equal

$\angle 1 = \angle 3 = 108°$

$\angle 2 = \angle 4 = 72°$

$\angle 6 = \angle 7 = 108°$

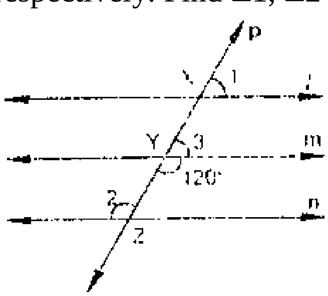
$\angle 5 = \angle 8 = 72°$

Corresponding angles

$\angle 1 = \angle 5 = 108°$

$\angle 2 = \angle 6 = 72°$

2. In the below fig, *l*, m and n are parallel lines intersected by transversal p at X, Y and Z respectively. Find $\angle 1$, $\angle 2$ and $\angle 3$.



Sol:

From the given figure:

$\angle 3 + \angle m\ YZ = 180°$ [Linear pair]

$\Rightarrow \angle 3 = 180° - 120°$

$\Rightarrow \angle 3 = 60°$

Now line l parallel to m

$\angle 1 = \angle 3$ [Corresponding angles]

$\Rightarrow \angle 1 = 60°$

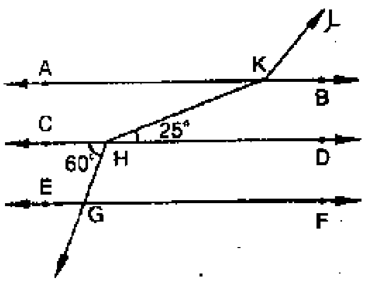
Also $m \parallel n$

$\Rightarrow \angle 2 = 120°$ [Alternative interior angle]

$\therefore \angle 1 = \angle 3 = 60°$

$\angle 2 = 120°$

3. In the below fig, AB || CD || EF and GH || KL. Find ∠HKL



Sol:

Produce LK to meet GF at N.

Now, alternative angles are equal

$\angle CHG = \angle HGN = 60°$

$\angle HGN = \angle KNF = 60°$ [Corresponding angles]

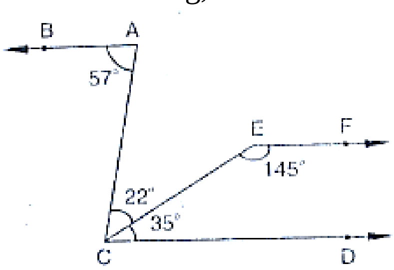
$\therefore \angle KNG = 180° - 60° = 120°$

$\angle GNK = \angle AKL = 120°$ [Corresponding angles]

$\angle AKH = \angle KHD = 25°$ [Alternative angles]

$\therefore \angle HKL = \angle AKH + \angle AKL = 25° + 120° = 145°$.

4. In the below fig, show that AB || EF.



Sol:

Produce EF to intersect AC at K.

Now, $\angle DCE + \angle CEF = 35° + 145° = 180°$

$\therefore EF \parallel CD$ [∵ Sum of Co-interior angles is 180°] ……(1)

Now, $\angle BAC = \angle ACD = 57°$

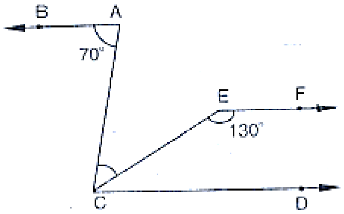
$\Rightarrow BA \| CD$ [$\because$ Alternative angles are equal] ……(2)

From (1) and (2)

$AB \| EF$ [Lines parallel to the same line are parallel to each other]

Hence proved.

5. If below fig, if AB || CD and CD || EF, find ∠ACE.



Sol:

Since $EF \| CD$

$\therefore EFC + \angle ECD = 180°$ [co-interior angles are supplementary]

$\Rightarrow \angle ECD = 180° - 130° = 50°$

Also $BA \| CD$

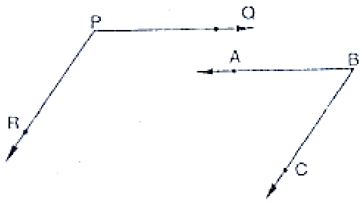
$\Rightarrow \angle BAC = \angle ACD = 70°$ [alternative angles]

But

$\angle ACE + \angle ECD = 70°$

$\Rightarrow \angle ACE = 70° - 50° = 20°$

6. In the below fig, PQ || AB and PR || BC. If ∠QPR = 102°, determine ∠ABC. Give reasons.



Sol:

AB is produce to meet PR at K

Since $PQ \| AB$

$\angle QPR = \angle BKR = 102°$ [corresponding angles]

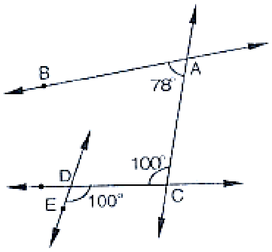
Since $PR \| BC$

$\therefore \angle RKB + \angle CBK = 180°$ [$\because$ Corresponding angles are supplementary]

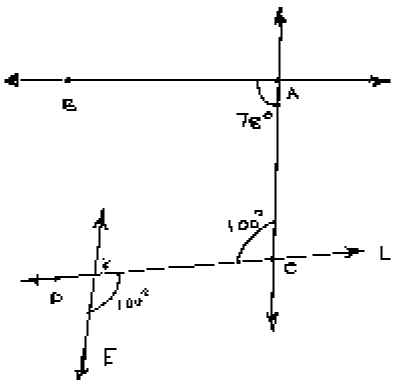
$\Rightarrow \angle CKB = 108 - 102 = 78°$

$\therefore \angle CKB = 78°$.

7. In the below fig, state which lines are parallel and why?



Sol:



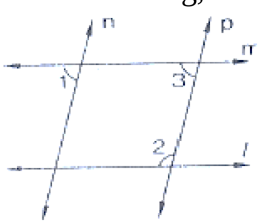
Vertically opposite angles are equal

$\angle EOC = \angle DOK = 100°$

Angle $\angle DOK = \angle ACO = 100°$

Here two lines EK and CA cut by a third line '*l*' and the corresponding angles to it are equal $\therefore EK \parallel AC$.

8. In the below fig, if *l* || m, n || p and ∠1 = 85°, find ∠2.



Sol:

Corresponding angles are equal

$\angle 1 = \angle 3 = 85°$

By using the property of co-interior angles are supplementary

$\angle 2 + \angle 3 = 180°$

$\angle 2 + 55° = 180°$

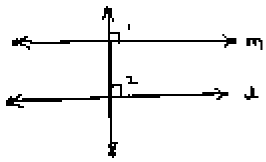
$\angle 2 = 180° - 55°$

$\angle 2 = 95°$

$\therefore \angle 2 = 95°$

9. If two straight lines are perpendicular to the same line, prove that they are parallel to each other.

Sol:



Given m perpendicular t and $l \perp t$

$\angle 1 = \angle 2 = 90°$

$\because l$ and m are two lines and it is transversal and the corresponding angles are equal

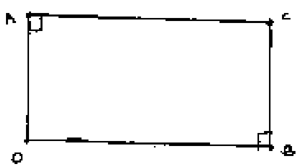
$\therefore l \parallel m$

Hence proved

10. Prove that if the two arms of an angle are perpendicular to the two arms of another angle, then the angles are either equal or supplementary.

Sol:

Consider be angles AOB and ACB



Given $OA \perp AO, OB \perp BO$

To prove: $\angle AOB = \angle ACB$ (or)

$\angle AOB + \angle ACB = 180°$

Proof:- In a quadrilateral [Sum of angles of quadrilateral]

$\Rightarrow \angle A + \angle O + \angle B + \angle C = 360°$

$\Rightarrow 180 + \angle O + \angle C = 360°$

$\Rightarrow \angle O + \angle C = 360 - 180 = 180°$

Hence, $\angle AOB + \angle ACB = 180°$ ......(i)

Also,

$\angle B + \angle ACB = 180°$ ......(i)

Also,

$\angle B + \angle ACB = 180°$ ......(i)

Also,

$\angle B + \angle ACB = 180°$

$\Rightarrow \angle ACB = 180° - 90°$

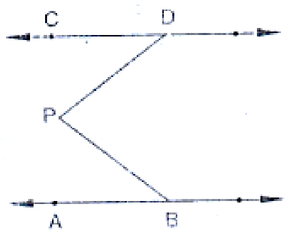
$\Rightarrow \angle ACB = 90°$ .....(ii)

From (i) and (ii)

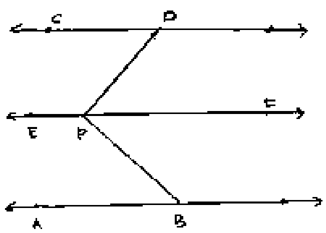
$\therefore \angle ACB = \angle AOB = 90°$

Hence, the angles are equal as well as supplementary.

11. In the below fig, lines AB and CD are parallel and P is any point as shown in the figure. Show that ∠ABP + ∠CDP = ∠DPB.



Sol:



Given that $AB \parallel CD$

Let EF be the parallel line to AB and CD which passes through P.

It can be seen from the figure

Alternative angles are equal

$\angle ABP = \angle BPF$

Alternative angles are equal

$\angle CDP = \angle DPF$

$\Rightarrow \angle ABP + \angle CDP = \angle BPF + \angle DPF$

$\Rightarrow \angle ABP + \angle CDP = \angle DPB$

Hence proved

AB parallel to CD, P is any point

To prove: $\angle ABP + \angle BPD + \angle CDP = 360°$

Construction: Draw $EF \parallel AB$ passing through P

Proof:

Since $AB \parallel EF$ and $AB \parallel CD$

$\therefore EF \parallel CD$ [Lines parallel to the same line are parallel to each other]

$\angle ABP + \angle EPB = 180°$ [Sum of co-interior angles is $180°$ $AB \parallel EF$ and BP is the transversal]

$\angle EPD + \angle COP = 180°$

[Sum of co-interior angles is $180°$ $EF \parallel CD$ and DP is transversal] …....(1)

$\angle EPD + \angle CDP = 180°$

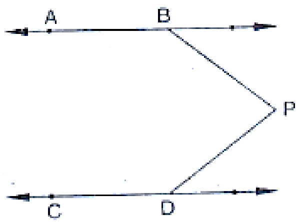
[Sum of Co-interior angles is $180°$ $EF \parallel CD$ and DP is the transversal] …(2)

By adding (1) and (2)

$\angle ABP + \angle EPB + \angle EPD + \angle CDP = 180° + 180°$

$\angle ABP + \angle EPB + \angle COP = 360°$

12. In the below fig, AB || CD and P is any point shown in the figure. Prove that: ∠ABP+∠BPD+∠CDP = 36O°



Sol:

Through P, draw a line PM parallel to AB or CD.

Now,

$AB \parallel PM \Rightarrow \angle ABP + \angle BPM = 180°$

And

$CD \parallel PM \Rightarrow \angle MPD + \angle CDP = 180°$

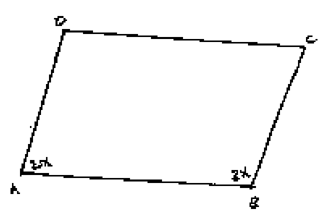
Adding (i) and (ii), we get

$\angle ABP + (\angle BPM + \angle MPD) \angle CDP = 360°$

$\Rightarrow \angle ABP + \angle BPD + \angle CDP = 360°$

13. Two unequal angles of a parallelogram are in the ratio 2 : 3. Find all its angles in degrees.

Sol:



Let $\angle A = 2x$ and $\angle B = 3x$

Now,

$\angle A + \angle B = 180°$ [Co-interior angles are supplementary]

$2x + 3x - 180°$ [$AD \parallel BC$ and AB is the transversal]

$\Rightarrow 5x = 180°$

$\Rightarrow x = \dfrac{180°}{5} = 36°$

$\therefore \angle A = 2 \times 36° = 72°$

$\angle B = 3 \times 36° = 108°$

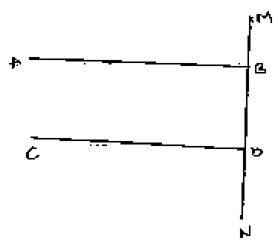
Now,

$\angle A = \angle C = 72°$ [Opposite side angles of a parallelogram are equal]

$\angle B = \angle D = 108°$

14. If each of the two lines is perpendicular to the same line, what kind of lines are they to each other?

Sol:



Let AB and CD be perpendicular to MN

$\angle ABD = 90°$ $[AB \perp MN]$ $....(i)$

$\angle CON = 90°$ $[CD \perp MN]$ $....(ii)$

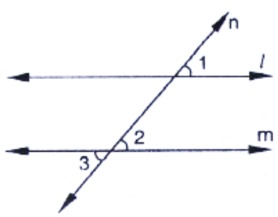
Now,

$\angle ABD = \angle CDN = 90°$ [From (i) and (ii)]

$\therefore AB$ parallel to CD,

Since corresponding angle are equal

15. In the below fig, ∠1 = 60° and ∠2 = $\left(\frac{2}{3}\right)^{rd}$ of a right angle. Prove that *l* || m.



Sol:

Given:

$\angle 1 = 60°, \angle 2 = \left(\dfrac{2}{3}\right)^{\text{rd}}$ to right angle

To prove: parallel to m

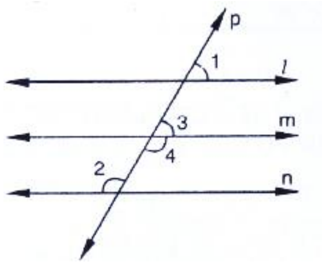
Proof $\angle 1 = 60°$

$\angle 2 = \frac{2}{3} \times 90° = 60°$

Since, $\angle 1 = \angle 2 = 60°$

$\therefore$ Parallel to m as pair of corresponding angles are equal

16. In the below fig, if $l \parallel m \parallel n$ and $\angle 1 = 60°$, find $\angle 2$.



Sol:

Since l parallel to m and p is the transversal

$\therefore$ Given: $l \parallel m \parallel n, \angle 1 = 60°$

To find $\angle 2$

$\angle 1 = \angle 3 = 60°$ [Corresponding angles]

Now,

$\angle 3$ and $\angle 4$ are linear pair of angles

$\angle 3 + \angle 4 = 180°$

$60° + \angle 4 = 180°$

$\angle 4 = 180° - 60°$

$\angle 4 = 120°$

Also, $m \parallel n$ and P is the transversal

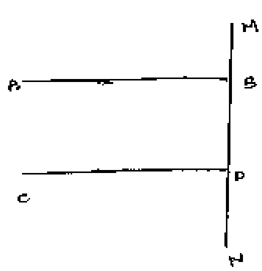
$\therefore \angle 4 = \angle 2 = 120°$ [Alternative interior angle]

Hence $\angle 2 = 120°$

17. Prove that the straight lines perpendicular to the same straight line are parallel to one another.

Sol:

Let AB and CD perpendicular to the Line MN



$\angle ABD = 90° \quad [\because AB \perp MN]$ ......(i)

$\angle CON = 90°$ $[\because CD \perp MN]$ ......(ii)

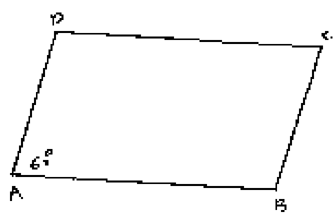
Now,

$\angle ABD = \angle CDN = 90°$ [From (i) and (ii)]

$\therefore AB \parallel CD,$ Since corresponding angles are equal.

18. The opposite sides of a quadrilateral are parallel. If one angle of the quadrilateral is 60°, find the other angles.

Sol:



Given $AB \parallel CD$

$AD \parallel BC$

Since $AB \parallel CD$ and AD is the transversal

$\therefore \angle A + \angle D = 180°$ [Co-interior angles are supplementary]

$60° + \angle D = 180°$

$\angle D = 180° - 60°$

$\angle D = 120°$

Now, $AD \parallel BC$ and AB is the transversal

$\angle A + \angle B = 180°$ [Co-interior angles are supplementary]

$60° + \angle B = 180°$

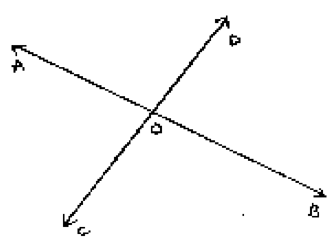
$\angle B = 180° - 60° = 120°$

Hence $\angle A = \angle C = 60°$

$\angle B = \angle D = 120°$

19. Two lines AB and CD intersect at O. If ∠AOC + ∠COB + ∠BOD = 270°, find the measures of ∠AOC, ∠COB, ∠BOD and ∠DOA.

Sol:

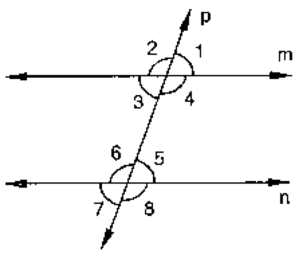


Given: $\angle AOC + \angle COB + \angle BOP = 270°$

To find: $\angle AOC, \angle COB, \angle BOD$ and $\angle DOA$

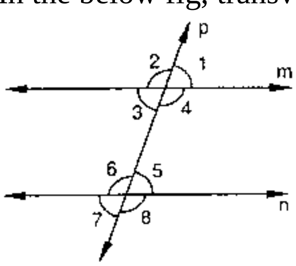
Here, $\angle AOC + \angle COB + \angle BOD + \angle AOD = 360°$ [Complete angle]
$\Rightarrow 270 + \angle AOD = 360°$
$\Rightarrow \angle AOD = 360° - 270°$
$\Rightarrow \angle AOD = 90°$
Now,
$\angle AOD + \angle BOD = 180°$ [Linear pair]
$90 + \angle BOD = 180°$
$\Rightarrow \angle BOD = 180° - 90°$
$\therefore \angle BOD = 90°$
$\angle AOD = \angle BOC = 90°$ [Vertically opposite angles]
$\angle BOD = \angle AOC = 90°$ [Vertically opposite angles]

20. In the below fig, p is a transversal to lines m and n, ∠2 = 120° and ∠5 = 60°. Prove that m || n.



Sol:
Given that
$\angle 2 = 120°, \angle 5 = 60°$
To prove
$\angle 2 + \angle 1 = 180°$ [$\because$ Linear pair]
$120° + \angle 1 = 180°$
$\angle 1 = 180° - 120°$
$\angle 1 = 60°$
Since $\angle 1 = \angle 5 = 60°$
$\therefore m \parallel n$ [As pair of corresponding angles are equal]

21. In the below fig, transversal *l* intersects two lines m and n, ∠4 = 110° and ∠7 = 65°. Is m || n?



Sol:
Given:

$\angle 4 = 110°, \angle 7 = 65°$

To find: Is $m \parallel n$

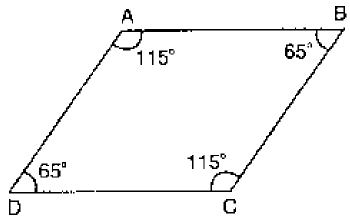
Here, $\angle 7 = \angle 5 = 65°$ [Vertically opposite angle]

Now,

$\angle 4 + \angle 5 = 110 + 65° = 175°$

$\therefore m$ is not parallel to n as the pair of co-interior angles is not supplementary.

22. Which pair of lines in the below fig, is parallel? Given reasons.



Sol:

$\angle A + \angle B = 115 + 65 = 180°$

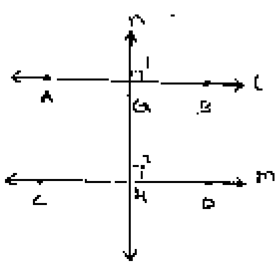
$\therefore AB \parallel BC$ [As sum of co-interior angles we supplementary]

$\angle B + \angle C = 65 + 115 = 180°$

$\therefore AB \parallel CD$ [As sum of interior angles are supplementary]

23. If *l*, m, n are three lines such that *l* || m and n ⊥ *l*, prove that n ⊥ m.

Sol:



Given $l \parallel m, n$ perpendicular l

To prove: $n \perp m$

Since $l \parallel m$ and n intersects them at G and H respectively

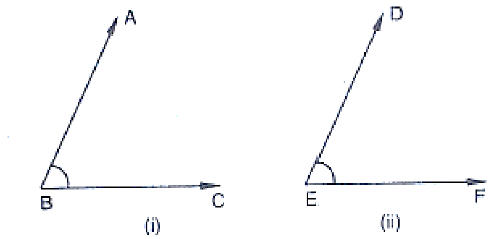
$\therefore \angle 1 = \angle 2$ [Corresponding angles]

But, $U = 90°$ $[n \perp l]$

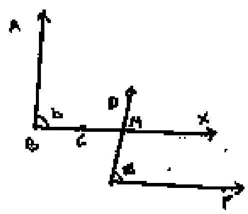
$\Rightarrow \angle 2 = 90°$

Hence n perpendicular m

24. In the below fig, arms BA and BC of ∠ABC are respectively parallel to arms ED and EF of ∠DEF. Prove that ∠ABC = ∠DEF.



Sol:



Given $AB \parallel DE$ and $BC \parallel^{lry} EF$

To prove: $\angle ABC = \angle DEF$

Construction: Produce BC to *x* such that it intersects DE at M.

Proof: Since $AB \parallel DE$ and BX is the transversal

$\therefore \angle ABC = \angle DMX$ [Corresponding angle] ……..(i)

Also,

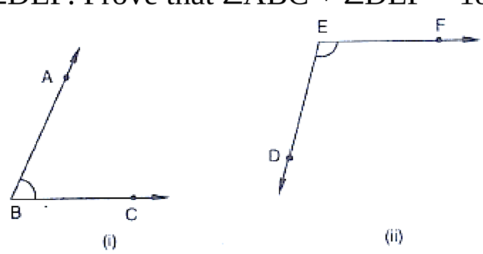
$BX \parallel EF$ and DE is the transversal

$\therefore \angle DMX = \angle DEF$ [Corresponding angles] ……..(ii)

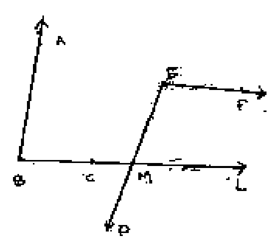
From (i) and (ii)

$\therefore \angle ABC = \angle DEF$

25. In the below fig, arms BA and BC of ∠ABC are respectively parallel to arms ED and EF of ∠DEF. Prove that ∠ABC + ∠DEF = 180°.



Sol:



Given $AB \parallel DE, BC \parallel EF$

To prove: $\angle ABC + \angle DEF = 180°$

Construction: produce BC to intersect DE at M

Proof: Since $AB \parallel EM$ and BL is the transversal

$\angle ABC = \angle EML$ [Corresponding angle] ……(1)

Also,

$EF \parallel ML$ and EM is the transversal

By the property of co-interior angles are supplementary

$\angle DEF + \angle EML = 180°$

From (i) and (ii) we have

$\therefore \angle DEF + \angle ABC = 180°$

26. Which of the following statements are true (T) and which are false (F)? Give reasons.

(i) If two lines are intersected by a transversal, then corresponding angles are equal.

(ii) If two parallel lines are intersected by a transversal, then alternate interior angles are equal.

(iii) Two lines perpendicular to the same line are perpendicular to each other.

(iv) Two lines parallel to the same line are parallel to each other.

(v) If two parallel lines are intersected by a transversal, then the interior angles on the same side of the transversal are equal.

Sol:

(i) False
(ii) True
(iii) False
(iv) True
(v) False

27. Fill in the blanks in each of the following to make the statement true:

(i) If two parallel lines are intersected by a transversal, then each pair of corresponding angles are ________

(ii) If two parallel lines are intersected by a transversal, then interior angles on the same side of the transversal are ________

(iii) Two lines perpendicular to the same line are ________ to each other.

(iv) Two lines parallel to the same line are ________ to each other.

(v) If a transversal intersects a pair of lines in such a way that a pair of alternate angles are equal, then the lines are ________

(vi) If a transversal intersects a pair of lines in such a way that the sum of interior angles on the same side of transversal is 180°, then the lines are ________.

Sol:

(i) Equal
(ii) Supplementary
(iii) Parallel
(iv) Parallel
(v) Parallel
(vi) Parallel

Exercise – 9.1

1. In a ΔABC, if ∠A = 55°, ∠B = 40°, find ∠C.

Sol:

Given $\angle A = 55°, \angle B = 40°$ then $\angle C = ?$

We know that

In $a\Delta ABC$ sum of all angles of triangle is $180°$

$i.e., \angle A + \angle B + \angle C = 180°$

$\Rightarrow 55° + 40° \angle C = 180°$

$\Rightarrow 95° + \angle C = 180°$

$\Rightarrow \angle C = 180° - 95°$

$\Rightarrow \angle C = 85°$

2. If the angles of a triangle are in the ratio 1: 2 : 3, determine three angles.

Sol:

Given that the angles of a triangle are in the ratio $1:2:3$

Let the angles be $a, 2a, 3a$

$\therefore$ We know that

Sum of all angles of triangles is $180°$

$a + 2a + 3a = 180°$

$\Rightarrow 6a = 180°$

$\Rightarrow a = \dfrac{180°}{6}$

$\Rightarrow a = 30°$

Since $a = 30°$

$2a = 2(30)° = 60°$

$3a = 3(30)° = 90°$

$\therefore$ angles are $a = 30°, 2a = 60°, 3a = 90°$

$\therefore$ Hence angles are $30°, 60°$ and $90°$

3. The angles of a triangle are (x − 40)°, (x − 20)° and ($\frac{1}{2}x - 10$)°. Find the value of x.

Sol:

Given that

The angles of a triangle are

$(x - 40°), (x - 20)°$ and $\left(\dfrac{x}{2} - 10\right)^{\circ}$

We know that

Sum of all angles of triangle is 180°

$\therefore x-40^\circ+x-20^\circ+\frac{x}{2}-10^\circ=180^\circ$

$2x+\frac{x}{2}-70^\circ=180^\circ$

$\frac{5x}{2}=180+70^\circ$

$5x=250^\circ(2)$

$x=50^\circ(2)$

$x=100^\circ$

$\therefore x=100^\circ$

4. The angles of a triangle are arranged in ascending order of magnitude. If the difference between two consecutive angles is 10°, find the three angles.

Sol:

Given that,

The difference between two consecutive angles is 10°

Let $x, x+10, x+20$ be the consecutive angles differ by 10°

$W \cdot K \cdot T$ sum of all angles of triangle is 180°

$x+x+10+x+20=180^\circ$

$3x+30=180^\circ$

$\Rightarrow 3x=180-30^\circ \Rightarrow 3x=150^\circ$

$\Rightarrow x=50^\circ$

$\therefore x=50^\circ$

$\therefore$ The required angles are

$x, x+10$ and $x+20$

$x=50$

$x+10=50+10=60$

$x+20=50+10+10=70$

The difference between two consecutive angles is 10° then three angles are $50^\circ, 60^\circ$ and 70°.

5. Two angles of a triangle are equal and the third angle is greater than each of those angles by 30°. Determine all the angles of the triangle.

Sol:

Given that,

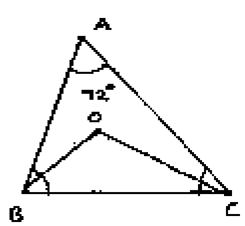
Two angles are equal and the third angle is greater than each of those angles by 30°.

Let $x, x, x+30$ be the angles of a triangle

We know that
Sum of all angles of a triangle is $180°$
$x+x+x+30=180°$
$3x+30=180°$
$\Rightarrow 3x=180°-30°$
$\Rightarrow 3x=150°$
$\Rightarrow x=\frac{150°}{3}$
$\Rightarrow x=50°$
$\therefore$ The angles are $x, x, x+30$
$x=50°$
$x+30=80°$
$\therefore$ The required angles are $50°, 50°, 80°$

6. If one angle of a triangle is equal to the sum of the other two, show that the triangle is a right triangle.
Sol:
If one angle of a triangle is equal to the sum of other two
i.e., $\angle B=\angle A+\angle C$
Now, in ΔABC
(Sum of all angles of triangle $180°$)
$\angle A+\angle B+\angle C=180°$
$\angle B+\angle B=180°$ $\quad [\because \angle B=\angle A+\angle C]$
$2\angle B=180°$
$\angle B=\frac{180°}{2}$
$\angle B=90°$
$\therefore ABC$ is a right angled a triangle.

7. ABC is a triangle in which ∠A — 72°, the internal bisectors of angles B and C meet in O. Find the magnitude of ∠ROC.
Sol:



Given,

ABC is a triangle

$\angle A = 72°$ and internal bisector of angles B and C meeting O

In $\Delta ABC = \angle A + \angle B + \angle C = 180°$

$\Rightarrow 72° + \angle B + \angle C = 180°$

$\Rightarrow \angle B + \angle C = 180° - 72°$ divide both sides by '2'

$\Rightarrow \frac{\angle B}{2} + \frac{\angle C}{2} = \frac{108°}{2}$ ....(1)

$\Rightarrow \angle OBC + \angle OCB = 54°$ ....(1)

Now in $\Delta BOC \Rightarrow \angle OBC + \angle OCB + \angle BOC = 180°$

$\Rightarrow 54° + \angle BOC = 180°$

$\Rightarrow \angle BOC = 180° - 54° = 126°$

$\therefore \angle BOC = 126°$

8. The bisectors of base angles of a triangle cannot enclose a right angle in any case.

Sol:

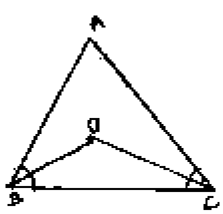
In a ΔABC

Sum of all angles of triangles is $180°$

i.e., $\angle A + \angle B + \angle C = 180°$ divide both sides by '2'

$\Rightarrow \frac{1}{2}\angle A + \frac{1}{2}\angle B + \frac{1}{2}\angle C = 180°$

$\Rightarrow \frac{1}{2}\angle A + \angle OBC + \angle OBC = 90°$ [$\because OB, OC$ insects $\angle B$ and $\angle C$]



$\Rightarrow \angle OBC + \angle OCB = 90° - \frac{1}{2} A$

Now in ΔBOC

$\therefore \angle BOC + \angle OBC + \angle OCB = 180°$

$\Rightarrow \angle BOC + 90° - \frac{1}{2}\angle A = 180°$

$\Rightarrow \angle BOC = 90° - \frac{1}{2}\angle A$

Hence, bisectors of a base angle cannot enclose right angle.

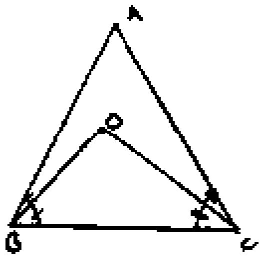
9. If the bisectors of the base angles of a triangle enclose an angle of 135°, prove that the triangle is a right triangle.

Sol:

Given the bisectors the base angles of an triangle enclose an angle of $135°$

i.e., $\angle BOC = 135°$

But, W.K.T



$\angle BOC = 90° + \frac{1}{2}\angle B$

$\Rightarrow 135° = 90° + \frac{1}{2}\angle A$

$\Rightarrow \frac{1}{2}\angle A = 135° - 90°$

$\Rightarrow \angle A = 45°(2)$

$\Rightarrow \angle A = 90°$

$\therefore \Delta ABC$ is right angled triangle right angled at A.

10. In a ΔABC, ∠ABC = ∠ACB and the bisectors of ∠ABC and ∠ACB intersect at O such that ∠BOC = 120°. Show that ∠A = ∠B = ∠C = 60°.

Sol:

Given,

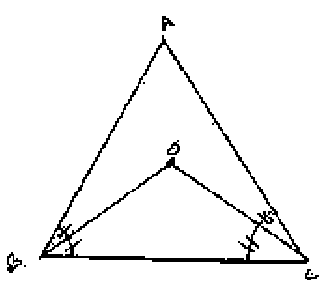
In ΔABC

$\angle ABC = \angle ACB$

Divide both sides by '2'

$\frac{1}{2}\angle ABC = \frac{1}{2}\angle ACB$

$\Rightarrow \angle OBC = \angle OCB$ [$\because OB, OC$ bisects $\angle B$ and $\angle C$]



Now

$\angle BOC = 90° + \frac{1}{2}\angle A$

$\Rightarrow 120° - 90° = \frac{1}{2}\angle A$

$\Rightarrow 30° \times (2) = \angle A$

$\Rightarrow \angle A = 60°$

Now in ΔABC

$\angle A + \angle ABC + \angle ACB = 180°$ (Sum of all angles of a triangle)

$\Rightarrow 60° + 2\angle ABC = 180°$ $[\because \angle ABC = \angle ACB]$

$\Rightarrow 2\angle ABC = 180° - 60°$

$\Rightarrow \angle ABC = \frac{120°}{2} = 90°$

$\Rightarrow \angle ABC = \angle ACB$

$\therefore \angle ACB = 60°$

Hence proved.

11. Can a triangle have:

(i) Two right angles?

(ii) Two obtuse angles?

(iii) Two acute angles?

(iv) All angles more than 60°?

(v) All angles less than 60°?

(vi) All angles equal to 60°?

Justify your answer in each case.

Sol:

(i) No,

Two right angles would up to $180°$, So the third angle becomes zero. This is not possible, so a triangle cannot have two right angles.

[Since sum of angles in a triangle is $180°$]

(ii) No,

A triangle can't have 2 obtuse angles. Obtuse angle means more than $90°$ So that the sum of the two sides will exceed $180°$ which is not possible. As the sum of all three angles of a triangle is $180°$.

(iii) Yes

A triangle can have 2 acute angle. Acute angle means less the $90°$ angle

(iv) No,

Having angles-more than $60°$ make that sum more than $18°$. Which is not possible

[$\because$ The sum of all the internal angles of a triangle is $180°$]

(v) No,
Having all angles less than $60°$ will make that sum less than $180°$ which is not possible.
[$\because$ The sum of all the internal angles of a triangle is $180°$]

(vi) Yes
A triangle can have three angles are equal to $60°$. Then the sum of three angles equal to the $180°$. Which is possible such triangles are called as equilateral triangle.
[$\because$ The sum of all the internal angles of a triangle is $180°$]

12. If each angle of a triangle is less than the sum of the other two, show that the triangle is acute angled.

Sol:

Given each angle of a triangle less than the sum of the other two

$\therefore \angle A + \angle B + \angle C$

$\Rightarrow \angle A + \angle A < \angle A + \angle B + \angle C$

$\Rightarrow 2\angle A < 180°$ [Sum of all angles of a triangle]

$\Rightarrow \angle A < 90°$

Similarly $\angle B < 90°$ and $\angle C < 90°$

Hence, the triangles acute angled.

Exercise -10.1

1. In Fig. 10.22, the sides BA and CA have been produced such that: BA = AD and CA = AE. Prove that segment DE || BC.

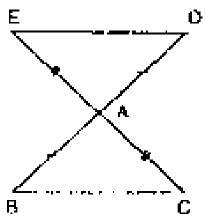


Fig. 10.22

Sol:

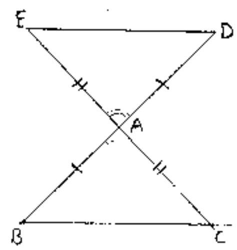
Given that, the sides BA and CA have been produced such that $BA = AD$ and $CA = AE$ and given to prove $DE \parallel BC$

Consider triangle BAC and DAE,

We have

$BA = AD$ and $CA = AE$ [$\because$ given in the data]

And also $\angle BAC = \angle DAE$ [$\because$ vertically opposite angles]



So, by SAS congruence criterion, we have $\Delta BAC \cong \Delta DAE$

$\Rightarrow BC = DE$ and $\angle DEA = \angle BCA, \angle EDA = \angle CBA$

[Corresponding parts of congruent triangles are equal]

Now, DE and BC are two lines intersected by a transversal DB such that $\angle DEA = \angle BCA,$ i.e., alternate angles are equal

Therefore, $DE \parallel BC$

2. In a ΔPQR, if PQ = QR and L, M and N are the mid-points of the sides PQ, QR and RP respectively. Prove that: LN = MN.

Sol:

Given that, in ΔPQR, $PQ = QR$ and L,M,N are midpoints of the sides PQ, QP and RP respectively and given to prove that $LN = MN$

Here we can observe that PQR is and isosceles triangle

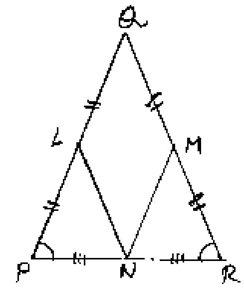
$\Rightarrow PQ = QR$ and $\angle QPR = \angle QRP$ ……..(1)

And also, L and M are midpoints of PQ and QR respectively

$\Rightarrow PL = LQ = \frac{PQ}{2}, QM = MR = \frac{QR}{2}$

And also, $PQ = QR$

$\Rightarrow PL = LQ = QM = MR = \frac{PQ}{2} = \frac{QR}{2}$ ....(2)



Now, consider ΔLPN and ΔMRN,

$LP = MR$ [From – (2)]

$\angle LPN = \angle MRN$ [From – (1)]

$\because \angle QPR$ and $\angle LPN$ and $\angle QRP$ and $\angle MRN$ are same

$PN = NR$ [$\because N$ is midpoint of PR]

So, by SAS congruence criterion, we have $\Delta LPN \cong \Delta MRN$

$\Rightarrow LN = MN$

[$\because$ Corresponding parts of congruent triangles are equal]

3. In Fig. 10.23, PQRS is a square and SRT is an equilateral triangle. Prove that
(i) PT = QT (ii) ∠TQR = 15°

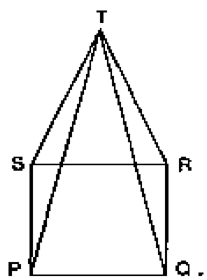


Fig. 10.23

Sol:

Given that PQRS is a square and SRT is an equilateral triangle. And given to prove that (i) $PT = QT$ and (ii) $\angle TQR = 15°$

Now, PQRS is a square

$\Rightarrow PQ = QR = RS = SP$ ....(1)

And $\angle SPQ = \angle PQR = \angle QRS = \angle RSP = 90° =$ right angle

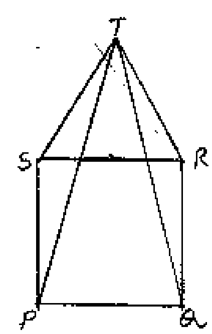
And also, SRT is an equilateral triangle.

$\Rightarrow SR = RT = TS$ ....(2)

And $\angle TSR = \angle SRT = \angle RTS = 60°$

From (1) and (2)

$PQ = QR = SP = SR = RT = TS$ ....(3)



And also,

$\angle TSR = \angle TSR + \angle RSP = 60° + 90° + 150°$

$\angle TRQ = \angle TRS + \angle SRQ = 60° + 90° + 150°$

$\Rightarrow \angle TSR = \angle TRQ = 150°$ ....(4)

Now, in ΔTSR and ΔTRQ

$TS = TR$ [From (3)]

$\angle TSP = \angle TRQ$ [From (4)]

$SP = RQ$ [From (3)]

So, by SAS congruence criterion we have

$\Delta TSP \cong \Delta TRQ$

$\Rightarrow \boxed{PT = QT}$ [Corresponding parts of congruent triangles are equal]

Consider ΔTQR,

$QR = TR$ [From (3)]

$\Rightarrow \Delta TQR$ is a isosceles triangle

$\Rightarrow \angle QTR = \angle TQR$ [angles opposite to equal sides]

Now,

Sum of angles in a triangle is equal to $180°$

$\Rightarrow \angle QTR + \angle TQR + \angle TRQ = 180°$

$\Rightarrow 2\angle TQR + 150° = 180°$ [From (4)]

$\Rightarrow 2\angle TQR = 180° - 150°$

$\Rightarrow 2\angle TQR = 30° \Rightarrow \boxed{\angle TQR = 15°}$

$\therefore$ Hence proved

4. Prove that the medians of an equilateral triangle are equal.

Sol:

Given to prove that the medians of an equilateral triangle are equal

Median: The line joining the vertex and midpoint of opposite side.

Now, consider an equilateral triangle ABC

Let D,E,F are midpoints of BC, CA and AB.

Then, AD, BE and CF are medians of ΔABC.

Now,

D is midpoint of BC $\Rightarrow BD = DC = \frac{BC}{2}$

Similarly, $CE = EA = \frac{AC}{2}$

$AF = FB = \frac{AB}{2}$

Since ΔABC is an equilateral triangle $\Rightarrow AB = BC = CA$ …….(1)

$\Rightarrow BD = DC = CE = EA = AF = FB = \frac{BC}{2} = \frac{AC}{2} = \frac{AB}{2}$ …….(2)

And also, $\angle ABC = \angle BCA = \angle CAB = 60°$ …….(3)

Now, consider ΔABD and ΔBCE

$AB = BC$ [From (1)]

$BD = CE$ [From (2)]

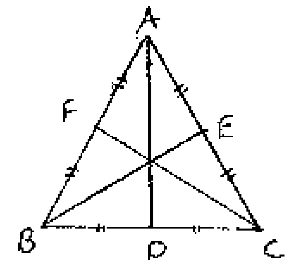
$\angle ABD = \angle BCE$ [From (3)] [$\angle ABD$ and $\angle ABC$ and $\angle BCE$ and $\angle BCA$ are same]

So, from SAS congruence criterion, we have

$\boxed{\Delta ABD \cong \Delta BCE}$

$\Rightarrow \boxed{AD = BE}$ ………(4)

[Corresponding parts of congruent triangles are equal]



Now, consider ΔBCE and ΔCAF,

BC = CA [From (1)]

$\angle BCE = \angle CAF$ [From (3)]

[$\angle BCE$ and $\angle BCA$ and $\angle CAF$ and $\angle CAB$ are same]

CE = AF [From (2)]

So, from SAS congruence criterion, we have $\boxed{\Delta BCE \cong \Delta CAF}$

$\Rightarrow \boxed{BE = CF}$ ……….(5)

[Corresponding parts of congruent triangles are equal]

From (4) and (5), we have

$AD = BE = CF$

$\Rightarrow$ Median $AD =$ Median $BE =$ Median CF

$\therefore$ The medians of an equilateral triangle are equal

$\therefore$ Hence proved

5. In a ΔABC, if ∠A=l20° and AB = AC. Find ∠B and ∠C.

Sol:

Consider a ΔABC

Given that $\angle A = 120°$ and $AB = AC$ and given to find $\angle B$ and $\angle C$

We can observe that ΔABC is an isosceles triangle since $AB = AC$

$\Rightarrow \boxed{\angle B = \angle C}$ ………(1) [Angles opposite to equal sides are equal]

We know that sum of angles in a triangle is equal to $180°$

$\Rightarrow \angle A + \angle B + \angle C = 180°$

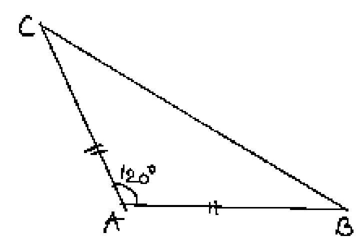
$\Rightarrow \angle A + \angle B + \angle B = 180°$ [From (1)]

$\Rightarrow 120° + 2\angle B = 180°$

$\Rightarrow 2\angle B = 180° - 120°$

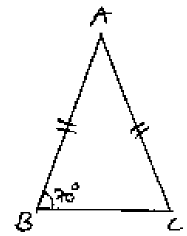
$\Rightarrow 2\angle B = 60° \Rightarrow \boxed{\angle B = 30°}$

$\Rightarrow \angle C = \angle B = 30°$



6. In a ΔABC, if AB = AC and ∠B = 70°, find ∠A.

Sol:



Consider ΔABC, we have $\angle B = 70°$ and $AB = AC$

Since, $AB = AC$ ΔABC is an isosceles triangle

$\Rightarrow \angle B = \angle C$ [Angles opposite to equal sides are equal]

$\Rightarrow \angle B = \angle C = 70°$

And also,

Sum of angles in a triangle $= 180°$

$\Rightarrow \angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle A + 70° + 70° = 180°$

$\Rightarrow \angle A + 140° = 180°$

$\Rightarrow \angle A = 180° - 140° \Rightarrow \boxed{\angle A = 40°}$

7. The vertical angle of an isosceles triangle is 100°. Find its base angles.

Sol:

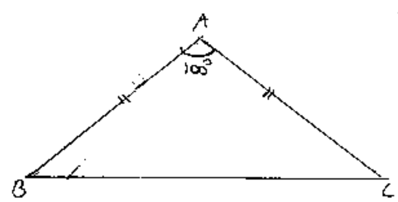
Consider an isosceles ΔABC such that $AB = AC$

Given that vertical angle A is $100°$. Given to find the base angles

Since ΔABC is isosceles

$\angle B = \angle C$ [Angles opposite to equal angles are equal]

And also,



Sum of the interior angles of a triangle $= 180°$

$\Rightarrow \angle A + \angle B + \angle C = 180°$

$\Rightarrow 100° + \angle B + \angle B = 180°$

$\Rightarrow 2\angle B = 180° - 100° \Rightarrow 2\angle B = 80°$

$\Rightarrow \boxed{\angle B = 40°}$

$\therefore \angle B = \angle C = 40°$

8. In Fig. 10.24, AB = AC and ∠ACD =105°, find ∠BAC.

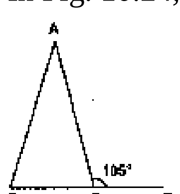


Fig. 10.24

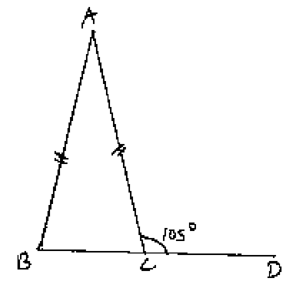
Sol:

Consider the given figure

We have,

$AB = AC$ and $\angle ACD = 105°$

Since,



$\angle BCD = 180° =$ Straight angle

$\Rightarrow \angle BCA + \angle ACD = 180°$

$\Rightarrow \angle BCA + 105° = 180°$

$\Rightarrow \angle BCA = 180° - 105° \Rightarrow \boxed{\angle BCA = 75°}$ ......(1)

And also,

ΔABC is an isosceles triangle $[\because AB = AC]$

$\Rightarrow \angle ABC = \angle ACB$ [Angles opposite to equal sides are equal]

From (1), we have

$\angle ACB = 75° \Rightarrow \angle ABC = \angle ACB = 75°$

And also,

Sum of interior angles of a triangle $= 180°$

$\Rightarrow \angle ABC = \angle BCA + \angle CAB = 180°$

$\Rightarrow 75° + 75° + \angle CAB = 180°$

$\Rightarrow 150° + \angle BAC = 180° \Rightarrow \angle BAC = 180° - 150° = 30°$

$\therefore \boxed{\angle BAC = 30°}$

9. Find the measure of each exterior angle of an equilateral triangle.

Sol:

Given to find the measure of each exterior angle of an equilateral triangle consider an equilateral triangle ABC.

We know that for an equilateral triangle

$AB = BC = CA$ and $\angle ABC = \angle BCA = \angle CAB = \frac{180°}{3} = 60°$ ……(1)

Now,

Extend side BC to D, CA to E and AB to F.

Here

BCD is a straight line segment

$\Rightarrow \angle BCD =$ Straight angle $= 180°$

$\angle BCA + \angle ACD = 180°$

$\Rightarrow 60° + \angle ACD = 180°$ $[From\ (1)]$

$\Rightarrow \boxed{\angle ACD = 120°}$

Similarly, we can find $\angle FAB$ and $\angle FBC$ also as $120°$ because ABC is an equilateral triangle

$\therefore \angle ACD = \angle EAB = \angle FBC = 120°$

Hence, the median of each exterior angle of an equilateral triangle is $120°$

10. If the base of an isosceles triangle is produced on both sides, prove that the exterior angles so formed are equal to each other.

Sol:

ED is a straight line segment and B and C are points on it.

$\Rightarrow \angle EBC = \angle BCD =$ straight angle $= 180°$

$\Rightarrow \angle EBA + \angle ABC = \angle ACB + \angle ACD$

$\Rightarrow \angle EBA = \angle ACD + \angle ACB - \angle ABC$

$\Rightarrow \angle EBA = \angle ACD$ $\qquad \left[From\ (1)\ \angle ABC = \angle ACD\right]$

$\Rightarrow \boxed{\angle ABE = \angle ACD}$

$\therefore$ Hence proved

11. In Fig. 10.25, AB = AC and DB = DC, find the ratio ∠ABD : ∠ACD.

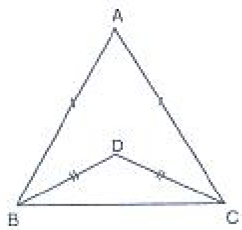
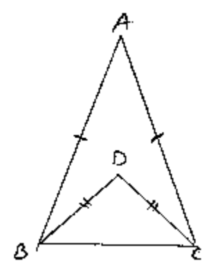


Fig. 10.25

Sol:

Consider the figure



Given

$AB = AC, DB = DC$ and given to find the ratio

$\angle ABD = \angle ACD$

Now, ΔABC and ΔDBC are isosceles triangles since $AB = AC$ and $DB = DC$ respectively

$\Rightarrow \angle ABC = \angle ACB$ and $\angle DBC = \angle DCB$ $\qquad$ [$\because$ angles opposite to equal sides are equal]

Now consider,

$\angle ABD : \angle ACD$

$\Rightarrow (\angle ABC - \angle DBC) : (\angle ACB - \angle DCB)$

$\Rightarrow (\angle ABC - \angle DBC) : (\angle ABC - \angle DBC)$ $\qquad \left[\because \angle ABC - \angle ACB\ and\ \angle DBC = \angle DCB\right]$

$\Rightarrow 1:1$

$\therefore \angle ABD : \angle ACD = 1:1$

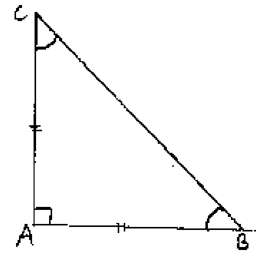
12. Determine the measure of each of the equal angles of a right-angled isosceles triangle.

OR

ABC is a right-angled triangle in which ∠A = 90° and AB = AC. Find ∠B and ∠C.

Sol:

Given to determine the measure of each of the equal angles of right – angled isosceles triangle



Consider on a right – angled isosceles triangle ABC such that

$\angle A = 90°$ and $AB = AC$

Since, $AB = AC \Rightarrow \angle C = \angle B$ ……….(1)

[Angles opposite to equal sides are equal]

Now,

Sum of angles in a triangle $= 180°$

$\Rightarrow \angle A + \angle B + \angle C = 180°$

$\Rightarrow 90° + \angle B + \angle B = 180°$ $[\because \angle A = 90° \text{ and } \angle B = \angle C]$

$\Rightarrow 2\angle B = 90°$

$\Rightarrow \boxed{\angle B = 45°} \Rightarrow \angle C = 45°$

$\therefore \angle B = \angle C = 45°$

Hence, the measure of each of the equal angles of a right-angled isosceles triangle is $45°$.

13. AB is a line segment. P and Q are points on opposite sides of AB such that each of them is equidistant from the points A and B (See Fig. 10.26). Show that the line PQ is perpendicular bisector of AB.

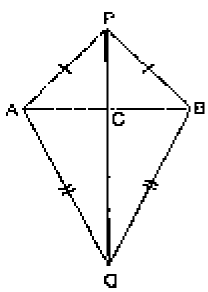


Fig. 10.26

Sol:

Consider the figure,

We have

AB is a line segment and P,Q are points on opposite sides of AB such that

$AP = BP$ ....(1)

$AQ = BQ$ ....(2)

We have to prove that PQ is perpendicular bisector of AB.

Now consider ΔPAQ and ΔPBQ,

We have $AP = BP$ [$\because$ From (1)]

$AQ = BQ$ [$\because$ From (2)]

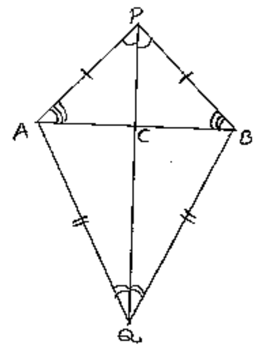
And $PQ = PQ$ [Common site]

$\Rightarrow \boxed{\Delta PAQ \cong \Delta PBQ}$ …….(3) [From SSS congruence]

Now, we can observe that ΔAPB and ΔABQ are isosceles triangles.(From 1 and 2)

$\Rightarrow \angle PAB = \angle PBA$ and $\angle QAB = \angle QBA$

Now consider ΔPAC and ΔPBC,



C is the point of intersection of AB and PQ.

$PA = PB$ [From (1)]

$\angle APC = \angle BPC$ [From (3)]

$PC = PC$ [Common side]

So, from SAS congruency of triangle $\Delta PAC \cong \Delta PBC$

$\Rightarrow AC = CB$ and $\angle PCA = \angle PCB$ …….(4)

[$\because$ Corresponding parts of congruent triangles are equal]

And also, ACB is line segment

$\Rightarrow \angle ACP + \angle BCP = 180°$

But $\angle ACP = \angle PCB$

$\Rightarrow$ $\boxed{\angle ACP = \angle PCB = 90°}$ ………(5)

We have $AC = CB \Rightarrow C$ is the midpoint of AB

From (4) and (5)

We can conclude that PC is the perpendicular bisector of AB

Since C is a point on the line PQ, we can say that PQ is the perpendicular bisector of AB.

Exercise -10.2

1. In Fig. 10.40, it is given that RT = TS, ∠1 = 2∠2 and ∠4 = 2∠3. Prove that ΔRBT ≅ ΔSAT.

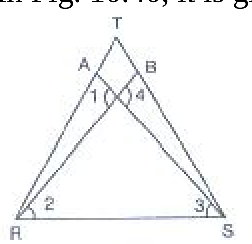


Fig. 10.40

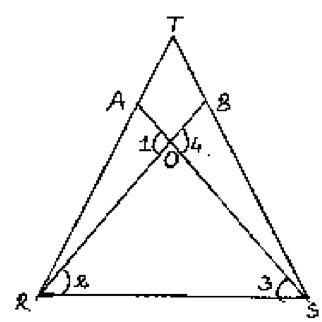
Sol:

In the figure given that

$RT = TS$ ……..(1)

$\angle 1 = 2\angle 2$ ……..(2)

And $\angle 4 = 2\sqrt{3}$ ……..(3)



And given to prove $\Delta RBT \cong \Delta SAT$

Let the point of intersection of RB and SA be denoted by O

Since RB and SA intersect at O.

$\therefore \angle AOR = \angle BOS$ [Vertically opposite angles]

$\Rightarrow \angle 1 = \angle 4$

$\Rightarrow 2\angle 2 = 2\angle 3$ [From (2) and (3)]

$\Rightarrow \boxed{\angle 2 = \angle 3}$ ……..(4)

Now we have $RT = TS$ in ΔTRS

$\Rightarrow \Delta TRS$ is an isosceles triangle

$\therefore \boxed{\angle TRS = \angle TSR}$ ……..(5) [Angles opposite to equal sides are equal]

But we have

$\angle TRS = \angle TRB + \angle 2$ ………(6)

And $\angle TSR = \angle TSA + \angle 3$ ……….(7)

Putting (6) and (7) in (5) we get

$\angle TRB + \angle 2 = \angle TSA + \angle B$

$\Rightarrow \boxed{\angle TRB = \angle TSA}$ $[\because From\ (4)]$

Now consider ΔRBT and ΔSAT

$RT = ST$ [From (1)]

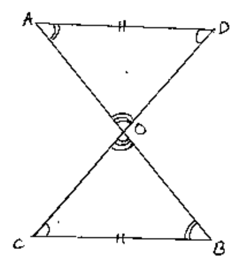
$\angle TRB = \angle TSA$ [From (4)]

$\angle RTB = \angle STA$ [Common angle]

From ASA criterion of congruence, we have $\boxed{\Delta RBT \cong \Delta SAT}$

2. Two lines AB and CD intersect at O such that BC is equal and parallel to AD. Prove that the lines AB and CD bisect at O.

Sol:



Given that lines AB and CD intersect at O

Such that $BC \parallel AD$ and $BC = AD$ ……..(1)

We have to prove that AB and CD bisect at O.

To prove this first we have to prove that $\Delta AOD \cong \Delta BOC$

3. BD and CE are bisectors of ∠B and ∠C of an isosceles ΔABC with AB = AC. Prove that BD = CE.

Sol:

Given that ΔABC is isosceles with $AB = AC$ and BD and CE are bisectors of $\angle B$ and $\angle C$

We have to prove $BD = CE$

Since $AB = AC \Rightarrow \angle ABC = \angle ACB$ …….(1)

[$\because$ Angles opposite to equal sides are equal]

Since BD and CE are bisectors of $\angle B$ and $\angle C$

$$\Rightarrow \angle ABD = \angle DBC = \angle BCE = ECA = \frac{\angle B}{2} = \frac{\angle C}{2} \quad …….(2)$$

Now,

Consider ΔEBC and ΔDCB

$\angle EBC = \angle DCB$ [$\because \angle B = \angle C$] from (1)

$BC = BC$ [Common side]

$\angle BCE = \angle CBD$ [$\because$ From (2)]

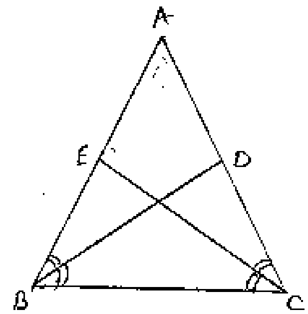
So, by ASA congruence criterion, we have $\boxed{\Delta EBC \cong \Delta DCB}$

Now,

$CE = BD$ [$\because$ Corresponding parts of congruent triangles are equal]

or $\boxed{BD = CE}$

$\therefore$ Hence proved



Since $AD \parallel BC$ and transversal AB cuts at A and B respectively

$\therefore \boxed{\angle DAO = \angle OBC}$ ……….(2) [alternate angle]

And similarly $AD \parallel BC$ and transversal DC cuts at D ad C respectively

$\therefore \boxed{\angle ADO = \angle OCB}$ ………(3) [alternate angle]

Since AB and CD intersect at O.

$\therefore \angle AOD = \angle BOC$ [Vertically opposite angles]

Now consider ΔAOD and ΔBOD

$\angle DAO = \angle OBC$ [$\because$ From (2)]

$AD = BC$ [$\because$ From (1)]

And $\angle ADO = \angle OCB$ [From (3)]

So, by ASA congruence criterion, we have

$\boxed{\Delta AOD \cong \Delta BOC}$

Now,

$AO = OB$ and $DO = OC$ [$\because$ Corresponding parts of congruent triangles are equal]

$\Rightarrow$ Lines AB and CD bisect at O.

$\therefore$ Hence proved

Exercise -10.3

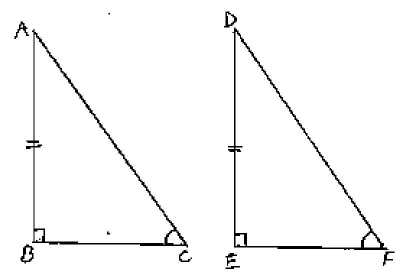
1. In two right triangles one side an acute angle of one are equal to the corresponding side and angle of the other. Prove that the triangles are congruent.

Sol:

Given that, in two right triangles one side and acute angle of one are equal to the corresponding side and angles of the other.

We have to prove that the triangles are congruent.

Let us consider two right triangles such that



$\angle B = \angle C = 90°$ ....(1)

$AB = DE$ ....(2)

$\angle C = \angle F$ ....(3)

Now observe the two triangles ABC and DEF

$\angle C = \angle F$ [From (3)]

$\angle B = \angle E$ [From (1)]

and $AB = DE$ [From (2)]

So, by AAS congruence criterion, we have $\boxed{\Delta ABC \cong \Delta DEF}$

$\therefore$ The two triangles are congruent

Hence proved

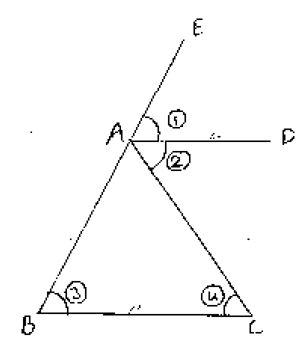
2. If the bisector of the exterior vertical angle of a triangle be parallel to the base. Show that the triangle is isosceles.

Sol:

Given that the bisector of the exterior vertical angle of a triangle is parallel to the base and we have to prove that the triangle is isosceles

Let ABC be a triangle such that AD is the angular bisector of exterior vertical angle EAC and $AD \parallel BC$

Let $\angle EAD = (1), \angle DAC = (2), \angle ABC = (3)$ and $\angle ACB = (4)$



We have,

(1) = (2) [$\because AD$ is bisector of $\angle EAC$]

(1) = (3) [Corresponding angles]

and (2) = (4) [alternative angle]

$\Rightarrow (3) = (4) \Rightarrow \boxed{AB = AC}$

Since, in $\Delta ABC,$ two sides AB and AC are equal we can say that ΔABC is isosceles

3. In an isosceles triangle, if the vertex angle is twice the sum of the base angles, calculate the angles of the triangle.

Sol:

Let ΔABC be isosceles such that $AB = AC$.

$\Rightarrow \angle B = \angle C$

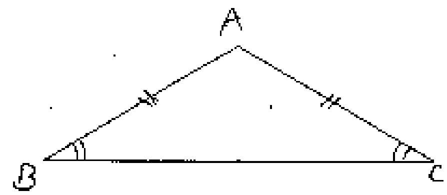
Given that vertex angle A is twice the sum of the base angles B and C.

i.e., $\angle A = 2(\angle B + \angle C)$

$\Rightarrow \angle A = 2(\angle B + \angle B)$ $[\because \angle B = \angle C]$

$\Rightarrow \angle A = 2(2\angle B)$

$\Rightarrow \boxed{\angle A = 4\angle B}$



Now,

We know that sum of angles in a triangle $= 180°$

$\Rightarrow \angle A + \angle B + \angle C = 180°$

$\Rightarrow 4\angle B + \angle B + \angle B = 180°$ $[\because \angle A = 4\angle B \text{ and } \angle B = \angle C]$

$\Rightarrow 6\angle B = 180°$

$\Rightarrow \angle B = \frac{180°}{6} = 30°$ $\therefore \boxed{\angle B = 30°}$

Since, $\angle B = \angle C \Rightarrow \angle B = \angle C = 30°$

And $\angle A = 4\angle B \Rightarrow \angle A = 4 \times 30° = 120°$

$\therefore$ Angles of the given triangle are $120°, 30°, 30°$.

4. PQR is a triangle in which PQ = PR and S is any point on the side PQ. Through S, a line is drawn parallel to QR and intersecting PR at T. Prove that PS = PT.

Sol:

Given that PQR is a triangle such that $PQ = PR$ and S is any point on the side PQ and $ST \parallel QR$.

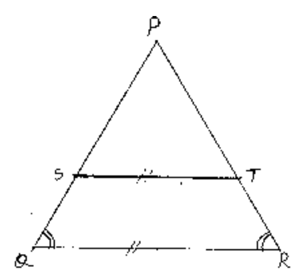
We have to prove $PS = PT$

Since, $PQ = PR \Rightarrow \Delta PQR$ is isosceles

$\Rightarrow \angle Q = \angle R$ (or) $\angle PQR = \angle PRQ$

Now,

$\angle PST = \angle PQR$ and $\angle PTS = \angle PRQ$ [Corresponding angles as $ST \parallel QR$]



Since, $\angle PQR = \angle PRQ \Rightarrow \boxed{\angle PST = \angle PTS}$

Now, In $\Delta PST, \angle PST = \angle PTS$

$\Rightarrow \Delta PST$ is an isosceles triangle

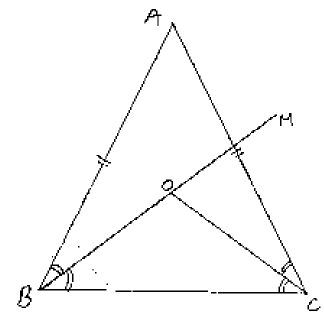
$\Rightarrow \boxed{PS = PT}$

5. In a ΔABC, it is given that AB = AC and the bisectors of ∠B and ∠C intersect at O. If M is a point on BO produced, prove that ∠MOC = ∠ABC.

Sol:

Given that in ΔABC,

$AB = AC$ and the bisector of $\angle B$ and $\angle C$ intersect at O and M is a point on BO produced



We have to prove $\angle MOC = \angle ABC$

Since,

$AB = AC \Rightarrow \Delta ABC$ is isosceles $\Rightarrow \angle B = \angle C$ (or) $\angle ABC = \angle ACB$

Now,

BO and CO are bisectors of $\angle ABC$ and $\angle ACB$ respectively

$\Rightarrow ABO = \angle OBC = \angle ACO = \angle OCB = \frac{1}{2}\angle ABC = \frac{1}{2}\angle ACB$ ……….(1)

We have, in ΔOBC

$\angle OBC + \angle OCB + \angle BOC = 180°$ ……….(2)

And also

$\angle BOC + \angle COM = 180°$ ……….(3) [Straight angle]

Equating (2) and (3)

$\Rightarrow \angle OBC + \angle OCB + \angle \cancel{BOC} = \angle \cancel{BOC} + \angle MOC$

$\Rightarrow \angle OBC + \angle OBC = \angle MOC$ $[\because \text{From (1)}]$

$\Rightarrow 2\angle OBC = \angle MOC$

$\Rightarrow 2\left(\frac{1}{2}\angle ABC\right) = \angle MOC$ $[\because \textit{From } (1)]$

$\Rightarrow \angle ABC = \angle MOC$

$\therefore \boxed{\angle MOC = \angle ABC}$

6. P is a point on the bisector of an angle ∠ABC. If the line through P parallel to AB meets BC at Q, prove that triangle BPQ is isosceles.

Sol:

Given that P is a point on the bisector of an angle $\angle ABC$, and $PQ \parallel AB$.

We have to prove that ΔBPQ is isosceles

Since,

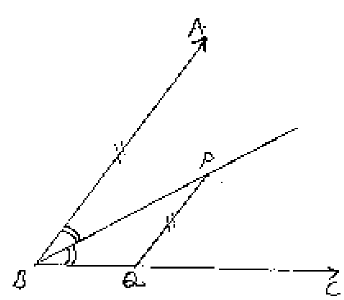
BP is bisector of $\angle ABC \Rightarrow \angle ABP = \angle PBC$ ………(1)

Now,

$PQ \parallel AB$

$\Rightarrow \angle BPQ = \angle ABP$ ………(2)

[alternative angles]



From (1) and (2), we get

$\boxed{\angle BPQ = \angle PBC \ (or) \ \angle BPQ = \angle PBQ}$

Now,

In ΔBPQ,

$\angle BPQ = \angle PBQ$

$\Rightarrow \Delta BPQ$ is an isosceles triangle.

$\therefore$ Hence proved

7. Prove that each angle of an equilateral triangle is 60°.

Sol:

Given to prove that each angle of an equilateral triangle is $60°$

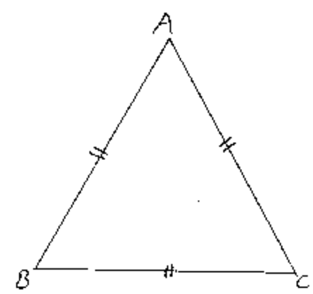
Let us consider an equilateral triangle ABC

Such that $AB = BC = CA$

Now,

$AB = BC \Rightarrow \angle A = \angle C$ ……..(1) [Opposite angles to equal sides are equal]

and $BC = AC \Rightarrow \angle B = \angle A$ ……..(2)



From (1) and (2), we get

$\boxed{\angle A = \angle B = \angle C}$ ……..(3)

We know that

Sum of angles in a triangle $= 180°$

$\Rightarrow \angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle A + \angle A + \angle A = 180°$ $\left[\because From(3)\right]$

$\Rightarrow 3\angle A = 180°$

$\Rightarrow \angle A = \dfrac{180°}{3} = 60°$

$\therefore \boxed{\angle A = \angle B = \angle C = 60°}$

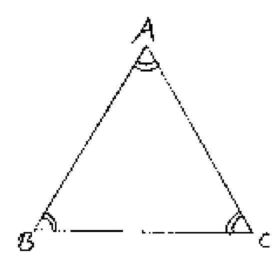
Hence, each angle of an equilateral triangle is $60°$.

8. Angles A, B, C of a triangle ABC are equal to each other. Prove that ΔABC is equilateral.

Sol:

Given that angles A, B, C of a triangle ABC equal to each other.

We have to prove that ΔABC is equilateral



We have, $\angle A = \angle B = \angle C$

Now,

$\angle A = \angle B \Rightarrow BC = AC$

[Opposite sides to equal angles are equal]

and $\angle B = \angle C \Rightarrow AC = AB$

From the above we get

$\boxed{AB = BC = AC}$

$\Rightarrow \Delta ABC$ is equilateral

9. ABC is a triangle in which ∠B = 2 ∠C . D is a point on BC such that AD bisects ∠BAC and AB = CD. Prove that ∠BAC = 72°.

Sol:

Given that in $\Delta ABC, \angle B = 2\angle C$ and D is a point on BC such that AD bisectors $\angle BAC$ and $AB = CD$.

We have to prove that $\angle BAC = 72°$

Now, draw the angular bisector of $\angle ABC$, which meets AC in P. join PD

Let $C = \angle ACB = y \Rightarrow \angle B = \angle ABC = 2\angle C = 2y$ and also

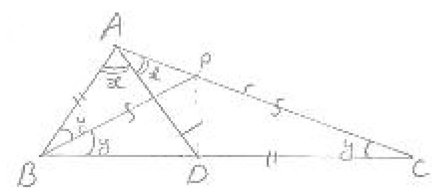
Let $\angle BAD = \angle DAC \Rightarrow \angle BAC = 2x$ [$\because AD$ is the bisector of $\angle BAC$]

Now, in ΔBPC,

$\angle CBP = y$ [$\because BP$ is the bisector of $\angle ABC$]

$\angle PCB = y$

$\Rightarrow \angle CBP = \angle PCB = y$ $\therefore \boxed{PC = BP}$



Consider, ΔABP and ΔDCP, we have

$\angle ABP = \angle DCP = y$

$AB = DC$ [Given]

And $PC = BP$ [From above]

So, by SAS congruence criterion, we have $\boxed{\Delta ABP \cong \Delta DCP}$

Now,

$\angle BAP = \angle CDF$ and $AP = DP$ [Corresponding parts of congruent triangles are equal]

$\Rightarrow \angle BAP = \angle CDP = 2x$

Consider, ΔAPD,

We have $AP = DP \Rightarrow \angle ADP = \angle DAP$

But $\angle DAP = x \Rightarrow \angle ADP = \angle DAP = x$

Now

In ΔABD,

$\angle ABD + \angle BAD + \angle ADB = 180°$

And also $\angle ADB + \angle ADC = 180°$ [Straight angle]

From the above two equations, we get

$\angle ABD + \angle BAD + \angle ADB = \angle ADB + \angle ADC$

$\Rightarrow 2y + x = \angle ADP + \angle PDC$

$\Rightarrow 2y + x = x + 2x$

$\Rightarrow 2y = 2x$

$\Rightarrow y = x\ (or)\ \boxed{x = y}$

We know,

Sum of angles in a triangle $= 180°$

So, in ΔABC,

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow 2x + 2y + y = 180°$ $[\because \angle A = 2x, \angle B = 2y, \angle C = y]$

$\Rightarrow 2(y) + 3y = 180°$ $[\because x = y]$

$\Rightarrow 5y = 180°$

$\Rightarrow y = \frac{180°}{5} = 36°$ $\therefore \boxed{x = y = 36°}$

Now,

$\angle A = \angle BAC = 2x = 2 \times 36° = 72°$

$\therefore \boxed{\angle BAC = 72°}$

10. ABC is a right angled triangle in which ∠A = 90° and AB = AC. Find ∠B and ∠C.

Sol:

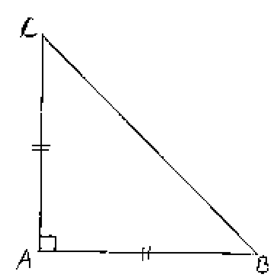
Given that ABC is a right angled triangle such that $\angle A = 90°$ and $AB = AC$

Since,

$AB = AC \Rightarrow \Delta ABC$ is also isosceles

$\therefore$ We can say that ΔABC is right angled isosceles triangle

$\Rightarrow \angle C = \angle B$ and $\angle A = 90°$ ……….(1)



Now, we have

Sum of angled in a triangle $= 180°$

$\Rightarrow \angle A + \angle B + \angle C = 180°$

$\Rightarrow 90° + \angle B + \angle B = 180°$ $[\because From (1)]$

$\Rightarrow 2\angle B = 180° - 90°$

$\Rightarrow \angle B = \frac{90°}{2} = 45°$

$\therefore \boxed{\angle B = \angle C = 45°}$

Exercise -10.4

1. In Fig. 10.92, it is given that AB = CD and AD = BC. Prove that ΔADC ≅ ΔCBA.

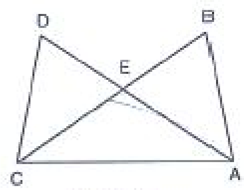


Fig. 10.92

Sol:

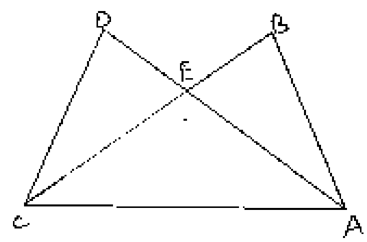
Given that in the figure $AB = CD$ and $AD = BC$.

We have to prove

$\Delta ADC \cong \Delta CBA$

Now,

Consider ΔADC and ΔCBA,



We have

$AB = CD$ [Given]

$BC = AD$ [Given]

And $AC = AC$ [Common side]

So, by SSS congruence criterion, we have

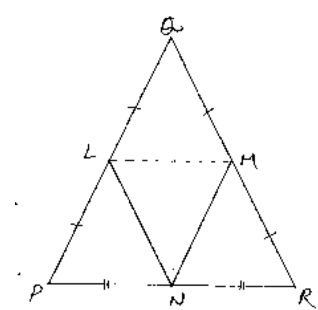
$\boxed{\Delta ADC \cong \Delta CBA}$

$\therefore$ Hence proved

2. In a ΔPQR, if PQ = QR and L, M and N are the mid-points of the sides PQ, QR and RP respectively. Prove that LN = MN.

Sol:

Given that in $\Delta PQR, PQ = QR$ and L, M and N are mid-points of PQ, QR and RP respectively



We have to prove $LN = MN$.

Join L and M, M and N, N and L

We have

$PL = LQ, QM = MR$ and $RN = NP$

[$\because L, M$ *and* N are mid-points of PQ, QR and RP respectively]

And also

$PQ = QR \Rightarrow PL = LQ = QM = MR = \frac{PQ}{2} = \frac{QR}{2}$ ....(1) Using mid-point theorem, we have

$MN \parallel PQ$ and $MN = \frac{1}{2}PQ \Rightarrow \boxed{MN = PL = LQ}$ ....(2)

Similarly, we have

$LN \parallel QR$ and $LN = \frac{1}{2} QR \Rightarrow \boxed{LN = QM = MR}$ ....(3)

From equation (1), (2) and (3), we have

$PL = LQ = QM = MR = MN = LN$

$\therefore \boxed{LN = MN}$

Exercise -10.5

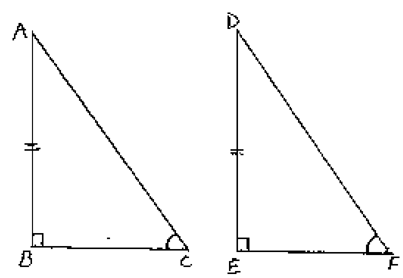
1. ABC is a triangle and D is the mid-point of BC. The perpendiculars from D to AB and AC are equal. Prove that the triangle is isosceles.

Sol:

Given that, in two right triangles one side and acute angle of one are equal to the corresponding side and angle of the other

We have to prove that the triangles are congruent

Let us consider two right triangles such that



$\angle B = \angle E = 90°$ ....(1)

$AB = DE$ ....(2)

$\angle C = \angle F$ ....(3)

Now observe the two triangles ABC and DEF

$\angle C = \angle F$ $[From\ (3)]$

$\angle B = \angle E$ $[From\ (1)]$

$and\ AB = DE$ $[From\ (2)]$

So, by AAS congruence criterion, we have

$\boxed{\Delta ABC \cong \Delta DEF}$

$\therefore$ The two triangles are congruent

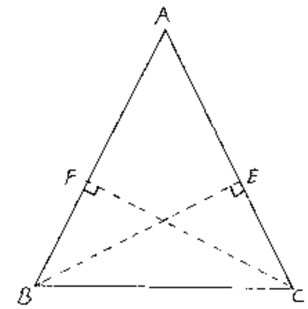
Hence proved

2. ABC is a triangle in which BE and CF are, respectively, the perpendiculars to the sides AC and AB. If BE = CF, prove that ΔABC is isosceles.

Sol:

Given that ABC is a triangle in which BE and CF are perpendicular to the sides AC and AB respectively such that $BE = CF$.

We have to prove that ΔABC is isosceles
Now, consider ΔBCF and ΔCBE,



We have

$\angle BFC = CEB = 90°$ [Given]
$BC = CB$ [Common side]
And $CF = BE$ [Given]

So, by RHS congruence criterion, we have $\boxed{\Delta BFC \cong CEB}$

Now,

$\angle FBC = \angle EBC$ [$\because$ Incongruent triangles corresponding parts are equal]
$\Rightarrow \angle ABC = \angle ACB$
$\Rightarrow AC = AB$ [$\because$ Opposite sides to equal angles are equal in a triangle]
$\therefore \Delta ABC$ is isosceles

3. If perpendiculars from any point within an angle on its arms are congruent, prove that it lies on the bisector of that angle.

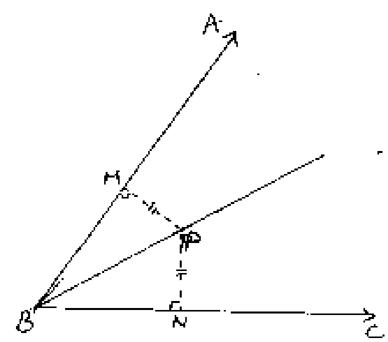
Sol:

Given that, if perpendicular from any point within, an angle on its arms is congruent, prove that it lies on the bisector of that angle

Now,

Let us consider an angle ABC and let BP be one of the arm within the angle

Draw perpendicular PN and PM on the arms BC and BA such that they meet BC and BA in N and M respectively.



Now, in ΔBPM and ΔBPN

We have $\angle BMP = BNP = 90°$ [given]
$BP = BP$ [Common side]
And $MP = NP$ [given]

So, by RHS congruence criterion, we have

$\boxed{\Delta BPM \cong \Delta BPN}$

Now,

$\angle MBP = \angle NBP$ [$\because$ Corresponding parts of congruent triangles are equal]
$\Rightarrow BP$ is the angular bisector of $\angle ABC$.
$\therefore$ Hence proved

4. In Fig. 10.99, AD ⊥ CD and CB ⊥. CD. If AQ = BP and DP = CQ, prove that ∠DAQ = ∠CBP.

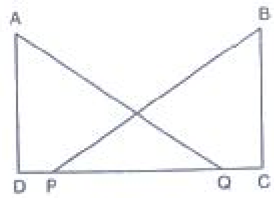


Fig. 10.99

Sol:

Given that, in the figure $AD \perp CD$ and $CB \perp CD$ and $AQ = BP, DP = CQ$

We have to prove that $\angle DAQ = \angle CBP$

Given that $DP = QC$

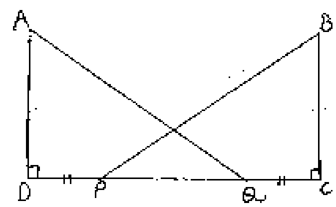
Add PQ on both sides

Given that $DP = QC$

Add PQ on both sides

$\Rightarrow DP + PQ = PQ + QC$

$\Rightarrow \boxed{DQ = PC}$ ....(1)



Now, consider triangle DAQ and CBP,

We have

$\angle ADQ = \angle BCP = 90°$ [given]

$AQ = BP$ [given]

And $DQ = PC$ [From (1)]

So, by RHS congruence criterion, we have $\boxed{\Delta DAQ \cong \Delta CBP}$

Now,

$\boxed{\angle DAQ = \angle CBP}$ [∵ Corresponding parts of congruent triangles are equal]

$\therefore$ Hence proved

5. ABCD is a square, X and Yare points on sides AD and BC respectively such that AY = BX. Prove that BY = AX and ∠BAY = ∠4BX.

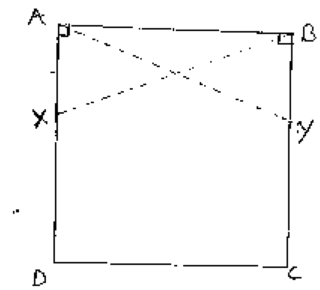
Sol:

Given that ABCD is a square, X and Y are points on sides AD and BC respectively such that $AY = BX$.

We have to prove $BY = AX$ and $\angle BAY = \angle ABX$

Join B and X, A and Y.

Since, ABCD is a square $\Rightarrow \angle DAB = \angle CBA = 90°$



$\Rightarrow \boxed{\angle XAB = \angle YBA = 90°}$ ....(1)

Now, consider triangle XAB and YBA

We have

$\angle XAB = \angle YBA = 90°$ [From (1)]

$BX = AY$ [given]

And $AB = BA$ [Common side]

So, by RHS congruence criterion, we have $\boxed{\Delta XAB \cong \Delta YBA}$

Now, we know that corresponding parts of congruent triangles are equal.

$\therefore \boxed{BY = AX \;\; and \;\; \angle BAY = \angle ABX}$

$\therefore$ Hence proved

6. Which of the following statements are true (T) and which are false (F):

(i) Sides opposite to equal angles of a triangle may be unequal.

(ii) Angles opposite to equal sides of a triangle are equal

(iii) The measure of each angle of an equilateral triangle is 60°

(iv) If the altitude from one vertex of a triangle bisects the opposite side, then the triangle may be isosceles.

(v) The bisectors of two equal angles of a triangle are equal.

(vi) If the bisector of the vertical angle of a triangle bisects the base, then the triangle may be isosceles.

(vii) The two altitudes corresponding to two equal sides of a triangle need not be equal.

(viii) If any two sides of a right triangle are respectively equal to two sides of other right triangle, then the two triangles are congruent.

(ix) Two right triangles are congruent if hypotenuse and a side of one triangle are respectively equal to the hypotenuse and a side of the other triangle.

Sol:

(i) False (F)

Reason: Sides opposite to equal angles of a triangle are equal

(ii) True (F)

Reason: Since the sides are equal, the corresponding opposite angles must be equal

(iii) True (T)

Reason: Since all the three angles of an equilateral triangles are equal and sum of the three angles is 180°, each angle will be equal to $\frac{180°}{3} \Rightarrow 60°$

(iv) False (F)

Reason: Here the altitude from thee vertex is also the perpendicular bisector of the opposite side.

$\Rightarrow$ The triangle must be isosceles and may be an equilateral triangle.

(v) True (T)

Reason: Since it an isosceles triangle, the lengths of bisectors of the two equal angles are equal

(vi) False (F)
Reason: The angular bisector of the vertex angle is also a median
$\Rightarrow$ The triangle must be an isosceles and also may be an equilateral triangle.

(vii) False (F)
Reason: Since two sides are equal, the triangle is an isosceles triangle.
$\Rightarrow$ The two altitudes corresponding to two equal sides must be equal.

(viii) False (F)
Reason: The two right triangles may or may not be congruent

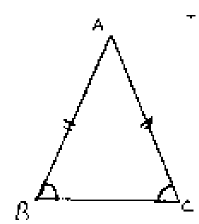
(ix) True (T)
Reason: According to RHS congruence criterion the given statement is true.

7. Fill the blanks in the following so that each of the following statements is true.

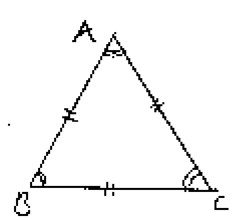
(i) Sides opposite to equal angles of a triangle are ......
(ii) Angle opposite to equal sides of a triangle are .....
(iii) In an equilateral triangle all angles are .....
(iv) In a ΔABC if $\angle A = \angle C$, then AB = ...
(v) If altitudes CE and BF of a triangle ABC are equal, then AB = ...
(vi) In an isosceles triangle ABC with AB = AC, if BD and CE are its altitudes, then BD is …… CE.
(vii) In right triangles ABC and DEF, if hypotenuse AB = EF and side AC = DE, then $\Delta ABC \cong \Delta$ ……

Sol:

(i) Sides opposite to equal angles of a triangle are equal
(ii) Angles opposite to equal sides of a triangle are equal

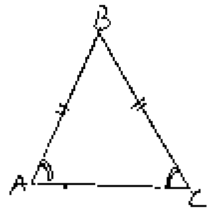


(iii) In an equilateral triangle all angles are equal
Reason: Since all sides are equal in a equilateral triangle, the angles opposite to equal sides will be equal



(iv) In a ΔABC if $\angle A = \angle C$, then $AB = BC$

Reason: Since, the sides opposite to equal angles are equal, the side opposite to $\angle A$ i.e., BC and $\angle C$ i.e., AB are equal



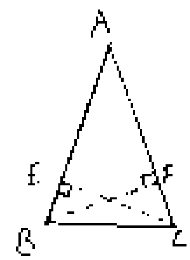
(v) If altitudes CE and BF of a triangle ABC are equal, then

$AB = AC$

Reason: From RHS congruence criterion $\Delta BEC \cong \Delta CFB$

$\Rightarrow \angle EBC = \angle FCB \Rightarrow \angle ABC = \angle ACB \Rightarrow AC = AB$

[$\because$ Sides opposite to equal angels are equal]



(vi) In an isosceles triangle ABC with $AB = AC$, if BD and CE are its altitudes, then BD is equal to CE

Reason: Since angles opposite to equal sides are equal, so

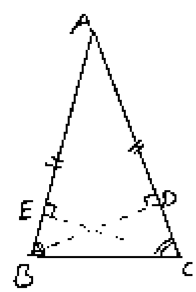
$\angle ABC = \angle ACB$

$\Rightarrow \angle EBC = \angle DCB$

So, by ASA congruence criterion

$\Delta EBC \cong \Delta DCB$

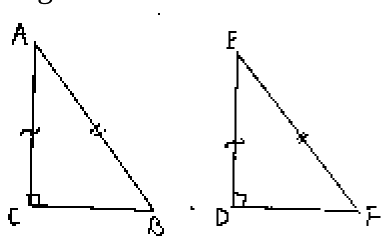
$\Rightarrow CE = BD$ [Corresponding parts of congruent triangles are equal]



(vii) In right triangles ABC and DEF, if hypotenuse $AB = EF$ and side $AC = DE$, then.

$\Delta ABC \cong \Delta EFD$

Reason: From RHS congruence criterion we have $\Delta ABC \cong \Delta EFD$



Exercise -10.6

1. In ΔABC, if ∠A = 40° and ∠B = 60°. Determine the longest and shortest sides of the triangle.

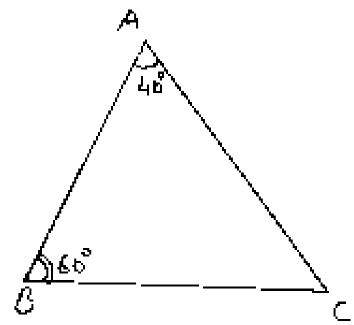
Sol:

Given that in ΔABC, $\angle A = 40°$ and $\angle B = 60°$

We have to find longest and shortest side

We know that,

Sum of angles in a triangle $180°$



$\Rightarrow \angle A + \angle B + \angle C = 180°$

$\Rightarrow 40° + 60° + \angle C = 180°$

$\Rightarrow \angle C = 180° - (40° + 60°)$

$= 180° - 100° = 80°$

$\therefore \boxed{\angle C = 80°}$

Now,

$\Rightarrow 40° < 60° < 80° \Rightarrow \angle A < \angle B < \angle C$

$\Rightarrow \angle C$ is greater angle and $\angle A$ is smaller angle

Now, $\angle A < \angle B < \angle C$

$\Rightarrow BC < AAC < AB$

[$\because$ Side opposite to greater angle is larger and side opposite to smaller angle is smaller]

$\therefore AB$ is longest and BC is smallest or shortest side.

2. In a ΔABC, if ∠B = ∠C = 45°, which is the longest side?

Sol:

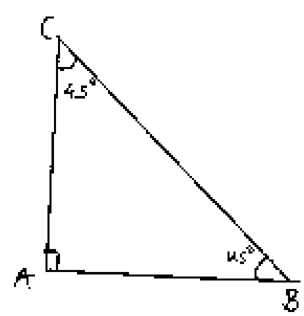
Given that in ΔABC,

$\angle B = \angle C = 45°$

We have to find longest side

We know that,

Sum of angles in a triangle $= 180°$



$\Rightarrow \angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle A + 45° + 45° = 180°$

$\Rightarrow \angle A = 180° - (45° + 45°) = 180° - 90° = 90°$

$\therefore \boxed{\angle A = 90°}$

3. In ΔABC, side AB is produced to D so that BD = BC. If ∠B = 60° and ∠A = 70°, prove that: (i) AD > CD (ii) AD > AC

Sol:

Given that in ΔABC, side AB is produced to D So that $BD = BC$ and $\angle B = 60°, \angle A = 70°$

We have to prove that

(i) $AD > CD$ (ii) $AD > AC$

First join C and D

Now, in ΔABC

$\angle A + \angle B + \angle C = 180°$ [$\because$ Sum of angles in a triangle $= 180°$]

$\Rightarrow \angle C = 180° - 70° - 60°$

$= 180° - 130° = 50°$

$\therefore \boxed{\angle C = 50°} \Rightarrow \angle ACB = 50°$ ....(1)

And also in ΔBDC,

$\angle DBC = 180° - \angle ABC$ [$\because ABD$ is a straight angle]

$= 180° - 60° = 120°$

and also $BD = BC$ [given]

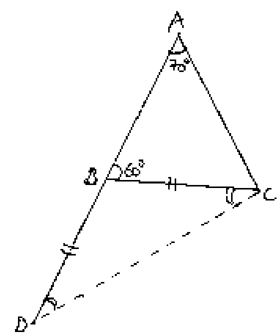
$\Rightarrow \angle BCD = \angle BDC$ [$\because$ Angles opposite to equal sides are equal]

Now,

$\angle DBC + \angle BCD + \angle BDC = 180°$ [$\because$ Sum of angles in a triangle $= 180°$]

$\Rightarrow 120° + \angle BCD + \angle BCD = 180°$

$\Rightarrow 2\angle BCD = 180° - 120° \Rightarrow \angle BCD = \dfrac{60°}{2} = 30°$



$\therefore \boxed{\angle BCD = \angle BDC = 30°}$ ....(2)

Now, consider ΔADC,

$\angle BAC \Rightarrow \angle DAC = 70°$ [given]

$\angle BDC \Rightarrow \angle ADC = 30°$ [$\because$ From (2)]

$\angle ACD = \angle ACB + \angle BCD$

$= 50° + 30°$ [$\because$ From (1) and (2)]

$= 80°$

Now, $\angle ADC < \angle DAC < \angle ACD$

$\Rightarrow \boxed{AC < DC < AD}$

[$\because$ Side opposite to greater angle is longer and smaller angle is smaller]

$\Rightarrow AD > CD$ and $AD > AC$

$\therefore$ Hence proved

Or

We have, $\angle ACD > \angle DAC$ and $\angle ACD > \angle ADC$

$\Rightarrow \boxed{AD > DC}$ and $\boxed{AD > AC}$

[$\because$ Side opposite to greater angle is longer and smaller angle is smaller]

4. Is it possible to draw a triangle with sides of length 2 cm, 3 cm and 7 cm?

Sol:

Given lengths of sides are 2cm, 3cm and 7cm we have to check whether it is possible to draw a triangle with ten the given lengths of sides

We know that,

A triangle can be drawn only when the sum of any two sides is greater than the third side.

So, let's check the rule.

$2+3 \ngtr 7$ *or* $2+3<7$

$2+7>3$

and $3+7>2$

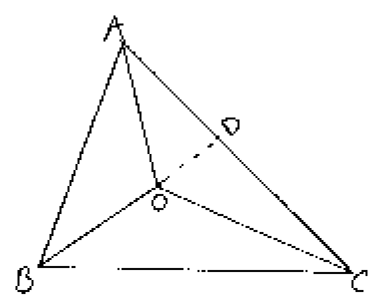
Here, $2+3 \ngtr 7$ So, the triangle does not exit.

5. O is any point in the interior of ΔABC. Prove that

(i) AB + AC > OB + OC

(ii) AB + BC + CA > OA + QB + OC

(iii) OA + OB + OC > $\frac{1}{2}$ (AB + BC + CA)



Sol:

Given that O is any point in the interior of ΔABC

We have to prove

(i) AB + AC > OB + OC

(ii) AB + BC + CA > OA + QB + OC

(iii) OA + OB + OC > $\frac{1}{2}$ (AB + BC + CA)

We know that, in a triangle the sum of any two sides is greater than the third side

So, we have

In ΔABC

$AB+BC>AC$

$BC+AC>AB$

$AC+AB>BC$

In ΔOBC

$OB+OC>BC$ ....(1)

In ΔOAC

$OA+OC>AC$ ....(2)

In ΔOAB

$OA+OB>AB$ ....(3)

Now, extend (or) produce BO to meet AC in D.

Now, in ΔABD, we have

$AB+AD>BD$

$\Rightarrow AB+AD>BO+OD$ ....(4) $[\because BD=BO+OD]$

Similarly in ΔODC, we have

$OD + DC > OC$ ....(5)

(i) Adding (4) and (5), we get

$AB + AD + OD + DC > BO + OD + OC$

$\Rightarrow AB + (AD + DC) > OB + OC$

$\Rightarrow \boxed{AB + AC > OB + OC}$ ....(6)

Similarly, we have

$BC + BA > OA + OC$ ....(7)

$and\ CA + CB > OA + OB$ ....(8)

(ii) Adding equation (6), (7) and (8), we get

$AB + AC + BC + BA + CA + CB > OB + OC + OA + OC + OA + OB$

$\Rightarrow 2AB + 2BC + 2CA > 2OA + 2OB + 2OC$

$\Rightarrow 2(AB + BC + CA) > 2(OA + OB + OC)$

$\Rightarrow \boxed{AB + BC + CA > OA + OB + OC}$

(iii) Adding equations (1), (2) and (3)

$OB + OC + OA + OC + OA + OB > BC + AC + AB$

$\Rightarrow 2OA + 2OB + 2OC > AB + BC + CA$

We get $\Rightarrow 2(OA + OB + OC) > AB + BC + CA$

$\therefore \boxed{(OA + OB + OC) > \frac{1}{2}(AB + BC + CA)}$

6. Prove that the perimeter of a triangle is greater than the sum of its altitudes.

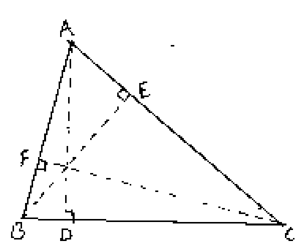
Sol:

Given: A ΔABC in which $AD \perp BC, BE \perp AC$ and $CF \perp AB.$

To prove:

$AD + BE + CF < AB + BC + AC$

Figure:



Proof:

We know that of all the segments that can be drawn to a given line, from a point not lying on it, the perpendicular distance i.e., the perpendicular line segment is the shortest.

Therefore,

$AD \perp BC$

$\Rightarrow AB > AD \;\; and \;\; AC > AD$

$\Rightarrow \boxed{AB + AC > 2AD}$ ....(1)

Similarly $BE \perp AC$

$\Rightarrow BA > BE$ and $BC > BE$

$\Rightarrow \boxed{BA + BC > 2BE}$ ......(2)

And also $CF \perp AB$

$\Rightarrow CA > CF$ and $CB > CF$

$\Rightarrow \boxed{CA + CB > 2CF}$ ......(3)

Adding (1), (2) and (3), we get

$AB + AC + BA + BC + CA + CB > 2AD + 2BE + 2CF$

$\Rightarrow 2AB + 2BC + 2CA > 2\left(AD + BE + CF\right)$

$\Rightarrow 2\left(AB + BC + CA\right) > 2\left(AD + BE + CF\right)$

$\Rightarrow \boxed{AB + BC + CA > AD + BE + CF}$

$\Rightarrow$ The perimeter of the triangle is greater than the sum of its altitudes

$\therefore$ Hence proved

7. In Fig. 10.131, prove that: (i) CD + DA + AB + BC > 2AC (ii) CD + DA + AB > BC

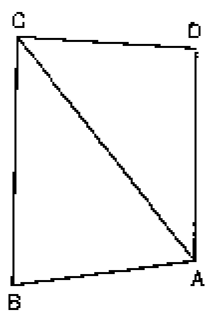


Fig. 10.131

Sol:

Given to prove

(i) $CD + DA + AB + BC > 2AC$

(ii) $CD + DA + AB > BC$

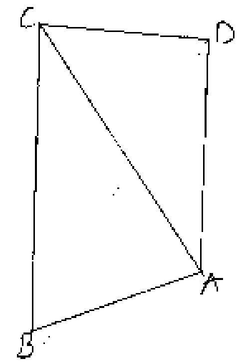
From the given figure,

We know that, in a triangle sum of any two sides is greater than the third side

(i) So,

In ΔABC, we have

$AB + BC > AC$ ....(1)



In ΔADC, we have

$CD + DA > AC$ ....(2)

Adding (1) and (2) we get

$AB + BC + CD + DA > AC + AC$

$\Rightarrow \boxed{CD + DA + AB + BC > 2AC}$

(ii) Now, in ΔABC, we have

$\boxed{AB + AC > BC}$ ....(3)

and in ΔADC, we have

$CD + DA > AC$

Add AB on both sides

$\Rightarrow \boxed{CD + DA + AB > AC + AB}$ ....(4)

From equation (3) and (4), we get

$CD + DA + AB > AC + AB > BC$

$\Rightarrow \boxed{CD + DA + AB > BC}$

$\therefore$ Hence proved

8. Which of the following statements are true (T) and which are false (F)?

(i) Sum of the three sides of a triangle is less than the sum of its three altitudes.

(ii) Sum of any two sides of a triangle is greater than twice the median drawn to the third side.

(iii) Sum of any two sides of a triangle is greater than the third side.

(iv) Difference of any two sides of a triangle is equal to the third side.

(v) If two angles of a triangle are unequal, then the greater angle has the larger side opposite to it.

(vi) Of all the line segments that can be drawn from a point to a line not containing it, the perpendicular line segment is the shortest one.

Sol:

(i) False (F)

Reason: Sum of these sides of a triangle is greater than sum of its three altitudes

(ii) True (F)

(iii) True (T)

(iv) False (F)

Reason: The difference of any two sides of a triangle is less than third side.

(v) True (T)

Reason: The side opposite to greater angle is longer and smaller angle is shorter in a triangle

(vi) True (T)

Reason: The perpendicular distance is the shortest distance from a point to a line not containing it.

9. Fill in the blanks to make the following statements true.

(i) In a right triangle the hypotenuse is the .... side.

(ii) The sum of three altitudes of a triangle is ..... than its perimeter.

(iii) The sum of any two sides of a triangle is .... than the third side.

(iv) If two angles of a triangle are unequal, then the smaller angle has the side opposite to it.

(v) Difference of any two sides of a triangle is than the third side.

(vi) If two sides of a triangle are unequal, then the larger side has .... angle opposite to it.

Sol:

(i) In a right triangle the hypotenuse is the largest side

Reason: Since a triangle can have only one right angle, other two angles must be less than $90°$

$\Rightarrow$ The right angle is the largest angle

$\Rightarrow$ Hypotenuse is the largest side.

(ii) The sum of three altitudes of a triangle is less than its perimeter

(iii) The sum of any two sides of a triangle is greater than the third side.

(iv) If two angles of a triangle are unequal, then the smaller angle has the smaller side opposite to it.

(v) Difference of any two sides of a triangle is less than the third side.

(vi) If two sides of a triangle are unequal, then the larger side has greater angle opposite to it.

Exercise -11.1

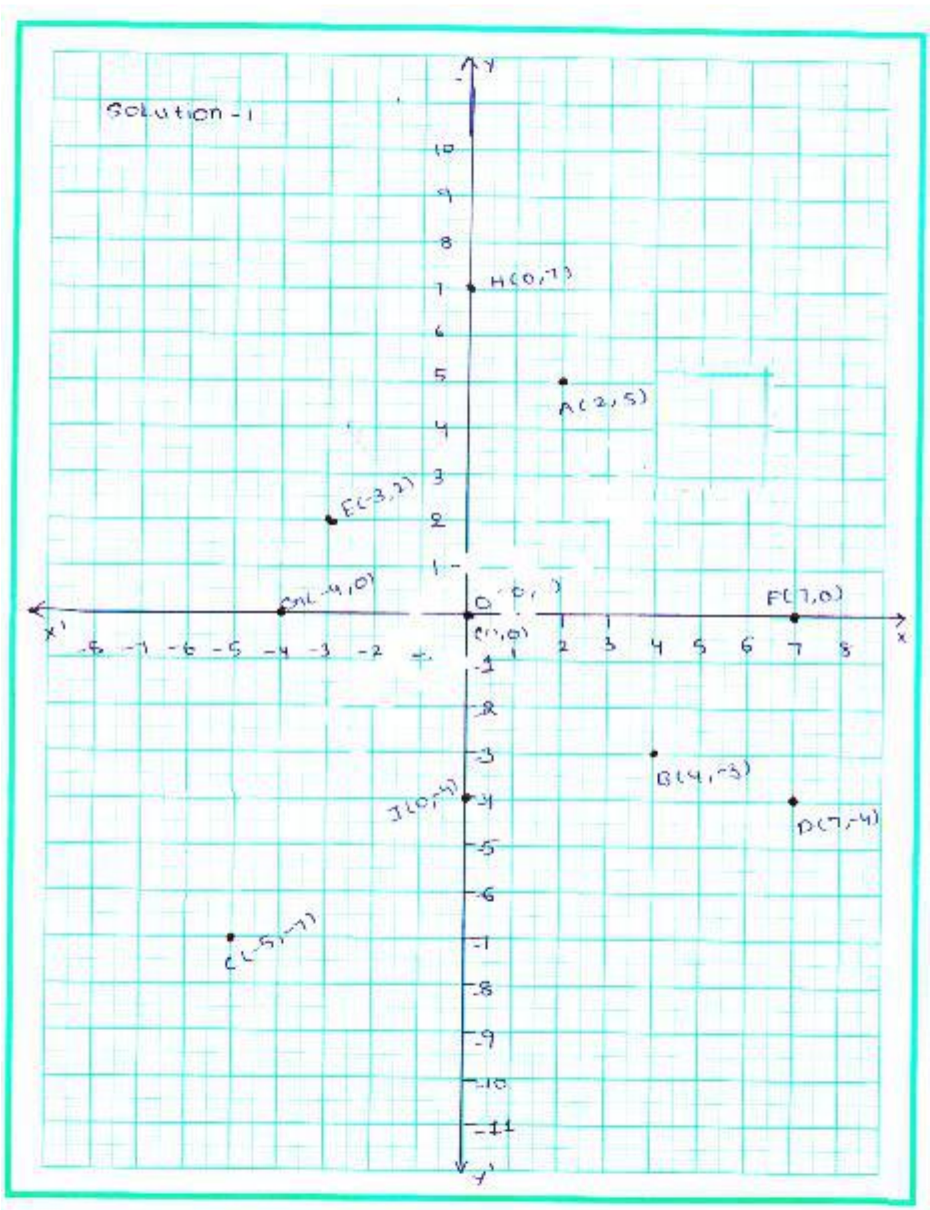
1. Plot the following points on the graph paper:

(i) (2, 5) (ii) (4, -3) (iii) (-5, -7) (iv) (7, -4) (v) (-3, 2)
(vi) (7, 0) (vii)(-4, 0) (viii)(0, 7) (ix) (0, -4) (x) (0, 0)

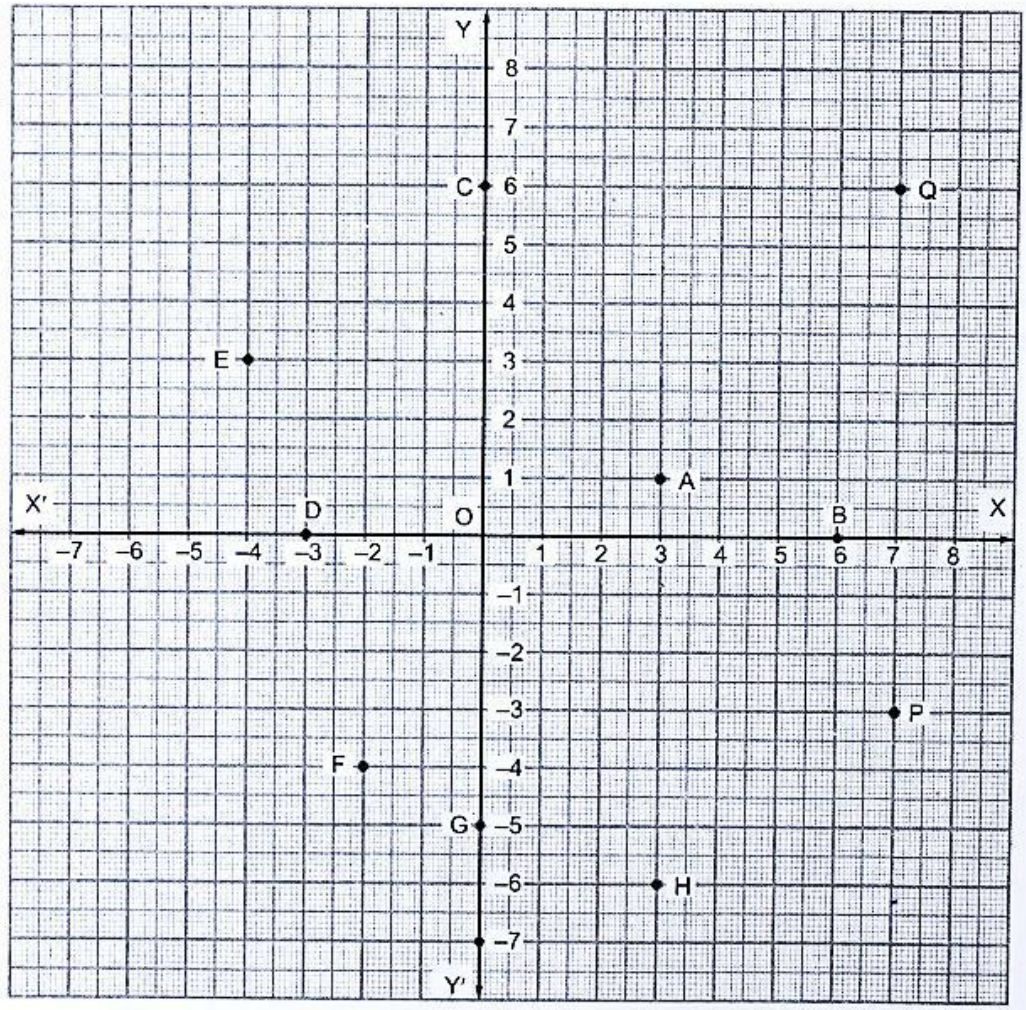
Sol:

The given points are A (2, 5) B (4, -3), c(-5, 7) d(7, -4) e(-3, 2) f(7, 0) g(-4, 0) h(0, 7) & i(0, -4)

The given points represented on the graph as follows:



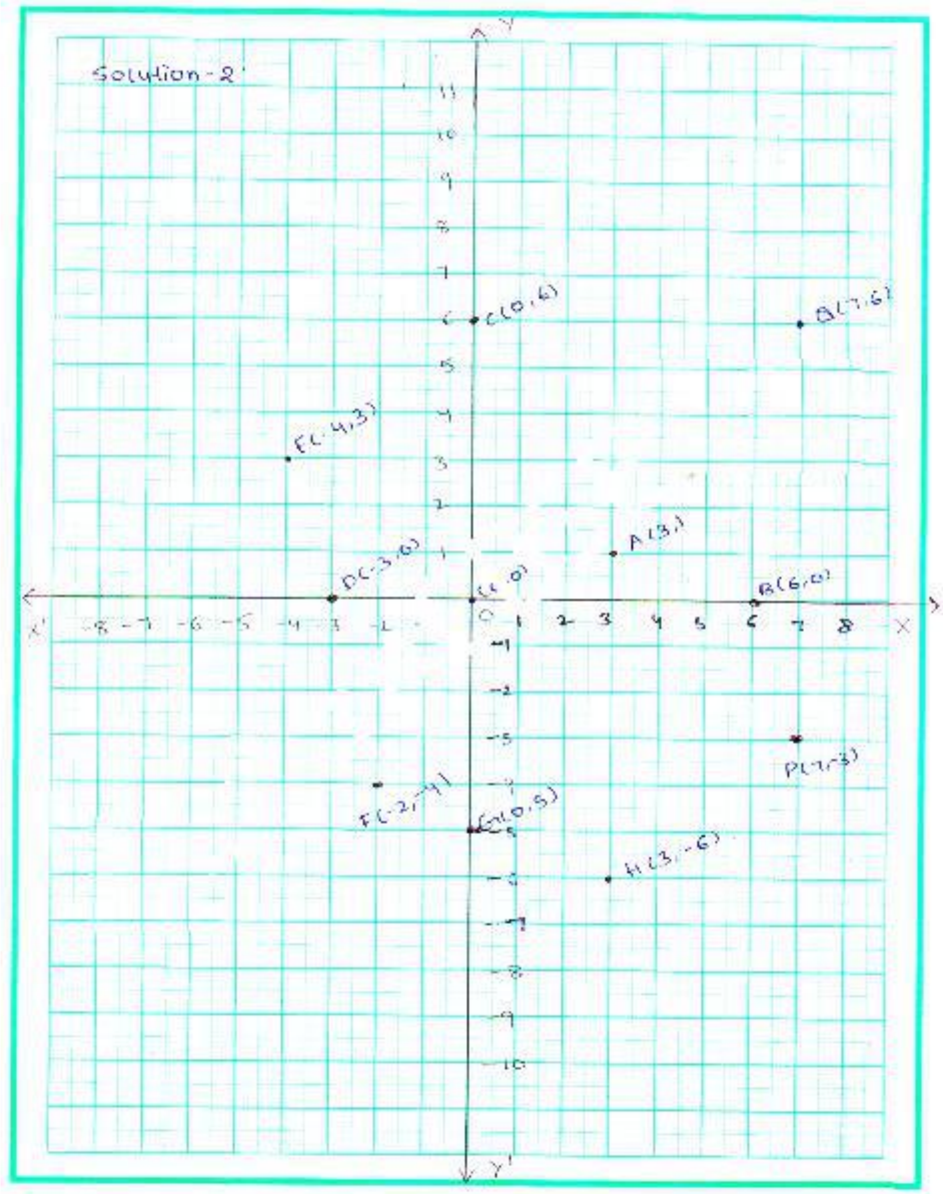
2. Write the coordinates of each of the following points marked in the graph paper:



Sol:

The given points are A (3, 1) B (6, 0), c(0, 6) d(-3, 0) e(-4, 3) f(-2, -4) g(0, -5) h(3, -6) p(7,−3) & q(7, 6)

The Graph of coordinates of given points as follows:


Solution-2
C(0,6)
B(7,6)
F(-4,3)
A(3,1)
D(-3,0)
B(6,0)
P(7,-3)
H(3,-6)
X'
X
Y'

Exercise – 12.1

1. Find the area of a triangle whose sides are respectively 150 cm, 120 cm and 200 cm.

Sol:

The triangle whose sides are

a = 150 cm

b = 120 cm

c = 200 cm

The area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Here 1s = semi perimeter of triangle

2s = a + b + c

S = $\frac{a+b+c}{2} = \frac{150+200+120}{2} = 235cm$

∴ area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{235(235-150)(235-200)(235-120)}$

= $\sqrt{235(85)(35)(115)}cm^2$

= $8966.56\ cm^2$

2. Find the area of a triangle whose sides are 9 cm, 12 cm and 15 cm.

Sol:

The triangle whose sides are a = 9cm, b = 12 cm and c = 15 cm

The area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Here 1s = semi-perimeter of a triangle

2s = a + b + c

S = $\frac{a+b+c}{2} = \frac{9+12+15}{2} = \frac{36}{2} = 18\ cm$

∴ area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{18(18-9)(18-12)(18-15)} = \sqrt{18(9)(6)(3)}$

= $\sqrt{18\ cm\ \times 3\ cm\ \times 54cm^2}$ = 54 cm^2.

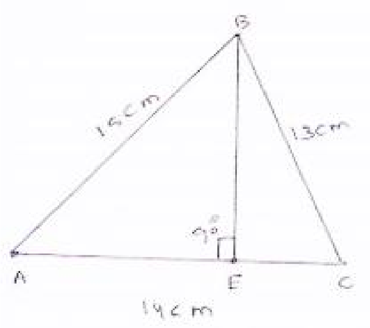
3. Find the area of a triangle two sides of which are 18 cm and 10 cm and the perimeter is 42cm.

Sol:

$21\sqrt{11}cm^2$

4. In a ΔABC, AB = 15 cm, BC = 13 cm and AC = 14 cm. Find the area of ΔABC and hence its altitude on AC.

Sol:
The triangle sides are



Let a = AB = 15 cm, BC = 13cm = b.
c = AC = 14 cm say.
Now,
2s = a + b + c
⇒ S = $\frac{1}{2}(a+b+c)$
⇒ s = $\left(\frac{15+13+14}{2}\right) cm$
⇒ s = 21 cm
∴ area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$
= $\sqrt{21(21-15)(21-14)(21-13)}\ cm^2$
= $\sqrt{21 \times 6 \times 8 \times 7}$ cm^2
= 84 cm^2
Let BE be perpendicular ($\perp^{er}$) to AC
Now, area of triangle = 84 cm^2
⇒ $\frac{1}{2} \times BE \times AC = 84$
⇒ BE = $\frac{84 \times 2}{AC}$
⇒ BE = $\frac{168}{14} = 12cm$
∴ Length of altitude on AC is 12 cm.

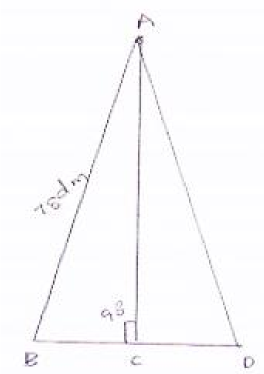
5. The perimeter of a triangular field is 540 m and its sides are in the ratio 25 : 17 : 12. Find the area of the triangle.
Sol:
The sides of a triangle are in the ratio 25 : 17 : 12

Let the sides of a triangle are a = 25x, b = 17 x and c = 12x say.
Perimeter = 25 = a + b + c = 540 cm
⇒ 25x + 17x + 12x = 540 cm
⇒ 54x = 540cm
⇒ $x = \frac{540}{54}$
⇒ $x = 10\ cm$
∴ The sides of a triangle are a = 250 cm, b = 170 cm and c = 120 cm
Now, Semi perimeter s = $\frac{a+b+c}{2}$
= $\frac{540}{2} = 270\ cm$
∴ The area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$
= $\sqrt{270(270-250)(270-170)(270-120)}$
= $\sqrt{270(20)(100)(150)}$
= $\sqrt{(9000)(9000)}$
= $9000\ cm^2$
∴ The area of triangle = 900 cm^2.

6. The perimeter of a triangle is 300 m. If its sides are in the ratio 3 : 5 : 7. Find the area of the triangle.
Sol:
Given that
The perimeter of a triangle = 300 m
The sides of a triangle in the ratio 3 : 5 : 7
Let 3x, 5x, 7x be the sides of the triangle
Perimeter ⇒ 2s = a + b + c
⇒ $3x + 5x + 17x = 300$
⇒ $15x = 300$
⇒ $x = 20m$
The triangle sides are $a = 3x$
$= 3\ (20)m = 60\ m$
$b = 5x = 5(20)\ m = 100m$
$c = 7x = 140\ m$
Semi perimeter $s = \frac{a+b+c}{2}$
$= \frac{300}{2} m$
$= 150m$
∴ The area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{150(150-60)(150-100)(150-140)}$
$= \sqrt{150 \times 10 \times 90 \times 50}$
$= \sqrt{1500 \times 1500 \times 3}\ cm^2$
$\therefore$ Δle Area $= 1500\sqrt{3}\ cm^2$

7. The perimeter of a triangular field is 240 dm. If two of its sides are 78 dm and 50 dm, find the length of the perpendicular on the side of length 50 dm from the opposite vertex.
Sol:
ABC be the triangle, Here a = 78 dm = AB,
BC = b = 50 dm



Now, perimeter = 240 dm
⇒ AB + BC + CA = 240 dm
⇒ AC = 240 – BC – AB
⇒ AC = 112 dm
Now, 2s = AB + BC + CA
⇒ 2s = 240
⇒ s = 120 dm
∴ Area of ΔABC = $\sqrt{s(s-a)(s-b)(s-c)}$ by heron's formula
$= \sqrt{120(120-78)(120-50)(120-112)}$
$= \sqrt{120 \times 42 \times 70 \times 8}$
$= 1680\ dm^2$
$Let\ AD\ be\ perpendicuälar\ on\ BC$
Area of ΔABC = $\frac{1}{2} \times AD \times BC$ (area of triangle = $\frac{1}{2} \times b \times h$)
= $\frac{1}{2} \times AD \times BC = 1680$
⇒ AD = $\frac{2 \times 1680}{50} = 67.2\ dm$

8. A triangle has sides 35 cm, 54 cm and 61 cm long. Find its area. Also, find the smallest of its altitudes.

Sol:

The sides of a triangle are a = 35 cm, b = 54 cm and c = 61 cm

Now, perimeter a + b + c = 25

⇒ S = $\frac{1}{2}(35 + 54 + 61)$

⇒ s = 75 cm

By using heron's formula

∴Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{75(75-35)(75-54)(75-61)}$

= $\sqrt{75(40)(21)(14)}$ = 939.14cm^2

∴ The altitude will be a smallest when the side corresponding to it is longest Here, longest side is 61 cm

[∴ Area of Δle = $\frac{1}{2} \times b \times h$] = $\frac{1}{2} \times base \times height$

∴ $\frac{1}{2} \times h \times 61 = 939.14$

⇒ $h = \frac{939.14 \times 2}{61} = 30.79\ cm$

$Hence\ the\ length\ of\ the\ smallest\ altitude\ is\ 30.79\ cm$

9. The lengths of the sides of a triangle are in the ratio 3 : 4 : 5 and its perimeter is 144 cm. Find the area of the triangle and the height corresponding to the longest side.

Sol:

Let the sides of a triangle are 3x, 4x and 5x.

Now, a = 3x, b = 4x and c = 5x

The perimeter 2s = 144

⇒ 3x + 4x + 5x = 144 [∵ a + b + c = 2s]

⇒ 12x = 144

⇒ x = 12

∴ sides of triangle are a = 3(x) = 36cm

b = 4(x) = 48 cm

c = 5(x) = 60 cm

Now semi perimeter s = $\frac{1}{2}(a+b+c) = \frac{1}{2}(144) = 72cm$

By heron's formulas ∴ Area of Δle = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{72(72-36)(72-48)(72-60)}$

= $864cm^2$

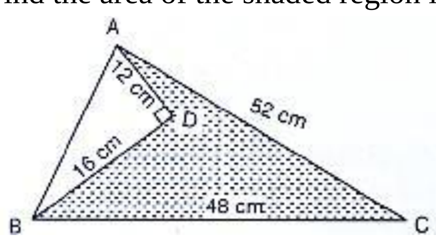
Let *l* be the altitude corresponding to longest side, ∴ $\frac{1}{2} \times 60 \times l = 864$

$\Rightarrow l = \frac{864\times 2}{60}$
$\Rightarrow l = 28.8cm$
Hence the altitude one corresponding long side = 28.8 cm

10. The perimeter of an isosceles triangle is 42 cm and its bašе is $\left(\frac{3}{2}\right)$ times each of the equal sides. Find the length of each side of the triangle, area of the triangle and the height of the triangle.

Sol:
Let 'x' be the measure of each equal sides
$\therefore$ Base = $\frac{3}{2}x$
$\therefore$ x + x + $\frac{3}{2}x = 42$ [$\because$ Perimeter = a + b + c = 42 cm]
$\Rightarrow \frac{7}{2}x = 42$
$\Rightarrow$ x = 12 cm
$\therefore$ Sides are a = x = 12 cm
b = x = 12 cm
c = x = $\frac{3}{2}$(12) cm = 18 cm
By heron's formulae
$\therefore$ Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}cm^2$
$= \sqrt{21(9)(9)(21-18)}\ cm^2$
$= \sqrt{(21)(9)(9)(3)}cm^2$
$= 71.42cm^2$
$\therefore$ Area of triangle = 71.42 cm^2

11. Find the area of the shaded region in Fig. Below.

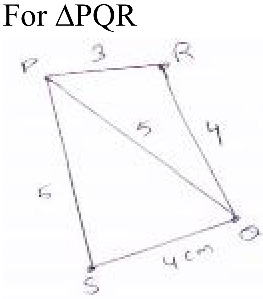


Sol:
Area of shaded region = Area of ΔABC – Area of ΔADB
Now in ΔADB
$\Rightarrow AB^2 = AD^2 + BD^2$ --(i)
$\Rightarrow$ Given that AD = 12 cm BD = 16 cm
Substituting the values of AD and BD in the equation (i), we get

$AB^2 = 12^2 + 16^2$
$AB^2 = 144 + 256$
AB = $\sqrt{400}$
AB = 20 cm
∴ Area of triangle = $\frac{1}{2} \times AD \times BD$
= $\frac{1}{2} \times 12 \times 16$
= 96 cm^2
Now
In ΔABC, S $= \frac{1}{2}(AB + BC + CA)$
$= \frac{1}{2} \times (52 + 48 + 20)$
$= \frac{1}{2}(120)$
= 60 cm
By using heron's formula
We know that, Area of Δle ABC = $\sqrt{s(s-a)(s-b)(s-c)}$
$= \sqrt{60(60-20)(60-48)(60-52)}$
$= \sqrt{60(40)(12)(8)}$
= 480 cm^2
= Area of shaded region = Area of ΔABC – Area of ΔADB
$= (480 - 96) cm^2$
= 384 cm^2
∴ Area of shaded region = 384 cm^2

Exercise – 12.2

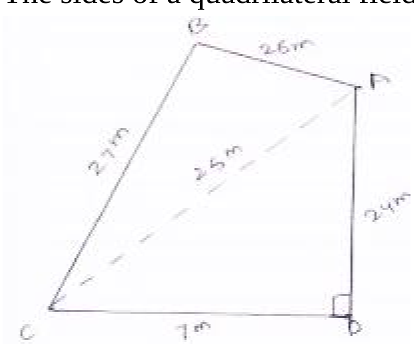
1. Find the area of a quadrilateral ABCD is which AB = 3 cm, BC = 4 cm, CD = 4 cm, DA = 5 cm and AC = 5 cm.
Sol:
For ΔPQR



$PQ^2 = QR^2 + RP^2$

$(5^2) = (3)^2 + (4)^2$ [$\because PR = 3\ QR = 4\ and\ PQ = 5$]
So, ΔPQR is a right angled triangle. Right angle at point R.
Area of ΔABC = $\frac{1}{2} \times QR \times RP$
= $\frac{1}{2} \times 3 \times 4$
= $6cm^2$
For ΔQPS
Perimeter = 2s = AC + CD + DA = (5 + 4 + 5)cm = 14 cm
S = 7 cm
By Heron's formulae
Area of Δle $\sqrt{s(s-a)(s-b)(s-c)}\ cm^2$
Area of Δle PQS = $\sqrt{7(7-5)(7-4)(7-3)} cm^2$
= $\sqrt{7 \times 2 \times 2 \times 3}\ cm^2$
= $2\sqrt{21} cm^2$
= $(2 \times 4.583) cm^2$
= $9.166\ cm^2$
Area of PQRS = Area of PQR + Area of ΔPQS = $(6 + 9.166)cm^2 = 15.166cm^2$

2. The sides of a quadrangular field, taken in order are 26 m, 27 m, 7m are 24 m respectively. The angle contained by the last two sides is a right angle. Find its area.
Sol:
The sides of a quadrilateral field taken order as AB = 26m



BC = 27 m
CD = 7m and DA = 24 m
Diagonal AC is joined
Now ΔADC
By applying Pythagoras theorem
$\Rightarrow AC^2 = AD^2 + CD^2$
$\Rightarrow$ AC = $\sqrt{AD^2 + CD^2}$
$\Rightarrow$ AC = $\sqrt{24^2 + 7^2}$

$\Rightarrow$ AC = $\sqrt{625} = 25\ m$

Now area of ΔABC

S = $\frac{1}{2}(AB + BC + CA) = \frac{1}{2}(26 + 27 + 25)$ =

$$= \frac{78}{2} = 39m.$$

By using heron's formula

Area (ΔABC) = $\sqrt{S(S - AD)(S\_BC)(S - CA)}$

$= \sqrt{39(39 - 26)(39 - 21)(39 - 25)}$

= $\sqrt{39 \times 14 \times 13 \times 12 \times 1}$

= 291.849 cm^2

Now for area of ΔADC

S = $\frac{1}{2}(AD + CD + AC)$

= $\frac{1}{2}(25 + 24 + 7) = 28m$

By using heron's formula

∴ Area of ΔADC = $\sqrt{S(S - AD)(S - DC)(S - CA)}$

= $\sqrt{28(28 - 24)(28 - 7)(28 - 25)}$

= $84m^2$

∴ Area of rectangular field ABCD = area of ΔABC + area of ΔADC

= $291.849 + 84$

= $375.8m^2$

3. The sides of a quadrilateral, taken in order are 5, 12, 14 and 15 meters respectively, and the angle contained by the first two sides is a right angle. Find its area.

Sol:

Given that sides of quadrilateral are AB = 5 m, BC = 12 m, CD = 14 m and DA = 15 m

AB = 5m, BC = 12m, CD = 14 m and DA = 15 m

Join AC

Area of ΔABC = $\frac{1}{2} \times AB \times BC$ $\left[\because Area\ of\ \Delta le = \frac{1}{2}(3x + 1)\right]$

= $\frac{1}{2} \times 5 \times 12$

= 30 cm^2

In ΔABC By applying Pythagoras theorem.

$AC^2 = AB^2 + BC^2$

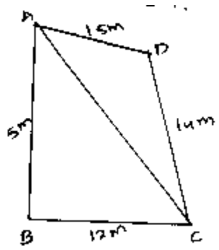
$\Rightarrow AC = \sqrt{5^2 + 12^2}$

$= \sqrt{25 + 144}$

$= \sqrt{169}$ = 13 m

$Now\ in\ \Delta ADC$

Let 2s be the perimeter



∴ 2s = (AD + DC + AC)

⇒ S = $\frac{1}{2}(15 + 14 + 13) = \frac{1}{2} \times 42 = 21m$

By using Heron's formula

∴ Area of ΔADC = $\sqrt{S(S - AD)(S - DC)(S - AC)}$

$= \sqrt{21(21 - 15)(21 - 14)(21 - 13)}$

$= \sqrt{21 \times 6 \times 7 \times 8}$

$= 84m^2$

∴ $Area\ of\ quadrilateral\ ABCD = area\ of\ (\Delta ABC) + Area\ of\ (\Delta ADC)$ = 30 + 84 = 114 m^2

4. A park, in the shape of a quadrilateral ABCD, has ∠C = 900, AB = 9 m, BC = 12 m, CD = 5 m and AD = 8 m How much area does it occupy?

Sol:

Given sides of a quadrilaterals are AB = 9, BC = 12, CD = 05, DA = 08

Let us joint BD

In ΔBCD applying Pythagoras theorem.

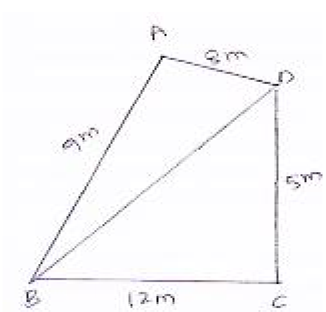
$BD^2 = BC^2 + CD^2$

$= (12)^2 + (5)^2$

$= 144 + 25$

$= 169$

$BD = 13m$



Area of ΔBCD = $\frac{1}{2} \times BC \times CD = \left[\frac{1}{2} \times 12 \times 5\right] m^2 = 30\ m^2$

For ΔABD

S = $\frac{perimeter}{2} = \frac{(9+8+13)}{2} = 15cm$

By heron's formula = $\sqrt{s(s-a)(s-b)(s-c)}$

Area of the triangle = $\sqrt{15(15-9)(15-8)(15-13)}m^2$

= $\sqrt{15(6)(7)(2)}m^2 = 6\sqrt{35}\, m^2 = 35.496m^2$

Area of park = Area of ΔABD + ΔABD + Area of BCD

= 35.496 + 30 m^2

= 65.5 m^2 (approximately)

5. Two parallel side of a trapezium are 60 cm and 77 cm and other sides are 25 cm and 26 cm. Find the area of the trapezium.

Sol:

Given that two parallel sides of trapezium are AB = 77 and CD = 60 cm

Other sides are BC = 26 m and AD = 25 cm.

Join AE and CF

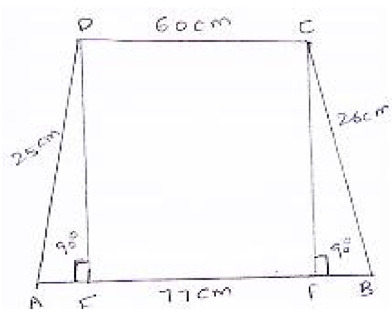
Now, DE ⊥ AB and CF ⊥ AB

∴ DC = EF = 60 cm

Let AE = x

⇒ BF = 77 – 60 – x = 17 – x

In ΔADE, $DE^2 = AD^2 - AE^2 = 25^2 - x^2$ [∵ Pythagoras theorem]



And in ΔBCF, $CF^2 = BC^2 - BF^2$ [∵ By Pythagoras theorem]

⇒ CF = $\sqrt{26^2 - (17-x)^2}$

But DE = CF ⇒ $DE^2 = CF^2$

⇒ $25^2 - x^2 = 26^2 - (17-x)^2$

⇒ $25^2x^2 = 25^2 - (289 + x^2 - 34x)$ [∵ $(a-b)^2 = a^2 - 2ab + b^2$]

⇒ 625 – $x^2 = 676 - 289 - x^2 + 34x$

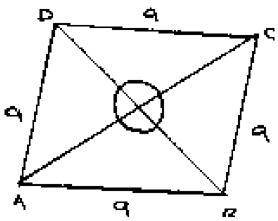
⇒ 34x = 238

⇒ x = 7

∴ DE = $\sqrt{25^2 - x^2} = \sqrt{625 - 7^2} = \sqrt{516} = 24cm$

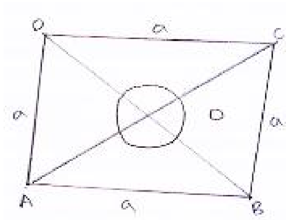
∴ Area of trapezium = $\frac{1}{2}(sum\ of\ parallel\ sides) \times height = \frac{1}{2}(60 \times 77) \times 24 =$ $1644cm^2$

6. Find the area of a rhombus whose perimeter is 80 m and one of whose diagonal is 24 m.
Sol:
Given that,
Perimeter of rhombus = 80m
Perimeter of rhombus = 4 × side



⇒ 4a = 80
⇒ a = 20m
Let AC = 24 m
∴ OA = $\frac{1}{2}AC = \frac{1}{2} \times 24 = 12m$
In ΔAOB
$OB^2 = AB^2 - OA^2$ [By using Pythagoras theorem]
⇒ $OB = \sqrt{20^2 - 12^2}$
$= \sqrt{400 - 144}$
$= \sqrt{256} = 16\, m$
Also BO = OD [Diagonal of rhombus bisect each other at 90°]
∴ BD = 20B = 2 ×16 = 32 m
∴Area of rhombus = $\frac{1}{2} \times 32 \times 24 = 384m^2$ [∵Area of rhombus = $\frac{1}{2} \times BD \times AC$]

7. A rhombus sheet, whose perimeter is 32 m and whose one diagonal is 10 m long, is painted on both sides at the rate of Rs 5 per m^2. Find the cost of painting.
Sol:
Given that,
Perimeter of a rhombus = 32 m
We know that,
Perimeter of rhombus = 4 × side
⇒ 49 = 32m
⇒ a = 8 m
Let AC = 10 = OA = $\frac{1}{2}AC$
= $\frac{1}{2} \times 10$
= $5m$



By using Pythagoras theorem:

$\therefore OB^2 = AB^2 - OA^2$

$\Rightarrow$ OB = $\sqrt{AB^2 - OA^2}$

$\Rightarrow$ OB = $\sqrt{8^2 - 5^2}$

$\Rightarrow$ OB = $\sqrt{64 - 25}$

$\Rightarrow$ OB = $\sqrt{39}\, m$

Now, BD = 2OB = $2\sqrt{39}m$

$\therefore$ Area of sheet = $\frac{1}{2} \times BD \times AC = \frac{1}{2} \times 2\sqrt{39} \times 10 = 10\sqrt{39}m^2$

$\therefore$ Cost of printing on both sides at the rate of Rs 5 per m^2 = Rs $2 \times 10\sqrt{39} \times 5$

= Rs. 625.00

8. Find the area of a quadrilateral ABCD in which AD = 24 cm, ∠BAD = 90° and BCD forms an equilateral triangle whose each side is equal to 26 cm. (Take $\sqrt{3}$ = 1.73)

Sol:

Given that, a quadrilateral ABCD in which AD = 24 cm, ∠BAD = 90°

BCD is equilateral triangle and sides BC = CD = BD = 26 cm

In ΔBAD By using Pythagoras theorem

$BA^2 = BD^2 - AD^2$

$\Rightarrow BA = \sqrt{BD^2 - AD^2}$

$= \sqrt{676 - 576}$

$= \sqrt{100} = 10\ cm$

$Area\ of\ \Delta BAD = \frac{1}{2} \times BA \times AD$

$= \frac{1}{2} \times 10 \times 24$

$= 120cm^2$

$Area\ of\ \Delta BCD = \frac{\sqrt{3}}{4} \times (26)^2 = 292.37cm^2$

$\therefore Area\ of\ quadrilateral$

ABCD = Area of ΔBAD + area of ΔBCD

= 120 + 292.37

= 412.37 cm^2

9. Find the area of a quadrilateral ABCD in which AB = 42 cm, BC = 21 cm, CD = 29 cm, DA =34 cm and diagonal BD =20 cm.

Sol:

Given that

Sides of a quadrilateral are AB = 42 cm, BC = 21 cm, CD = 29 cm

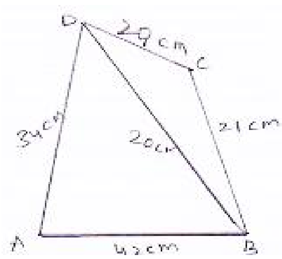
DA = 34 cm and diagonal BD = 20 cm

Area of quadrilateral = area of ΔADB + area of ΔBCD.

Now, area of ΔABD

Perimeter of ΔABD

We know that



2s = AB + BD + DA

⇒ S = $\frac{1}{2}(AB + BD + DA)$

= $\frac{1}{2}(34 + 42 + 20) = 96$

=$\frac{96}{2}$

= $48\ cm$

Area of ΔABD = $\sqrt{S(S - AB)(S - BD)(S - DA)}$

= $\sqrt{48(48 - 42)(48 - 20)(48 - 34)}$

= $\sqrt{48(14)(6)(28)}$

= $336\ cm^2$

Also for area of ΔBCD,

Perimeter of ΔBCD

2s = BC + CD + BD

⇒ S = $\frac{1}{2}(29 + 21 + 20) = 35\ cm$

By using heron's formulae

Area of ΔBCD = $\sqrt{s(s - bc)(s - cd)(s - db)}$

= $\sqrt{35(35 - 21)(35 - 29)(35 - 20)}$

= $\sqrt{210 \times 210} cm^2$

= $210 cm^2$

∴ Area of quadrilateral ABCD = 336 + 210 = 546 cm^2

10. Find the perimeter and area of the quadrilateral ABCD in which AB = 17 cm, AD =9 cm, CD = l2cm, ∠ACB = 90° and AC=l5cm.

Sol:

The sides of a quadrilateral ABCD in which AB = 17 cm, AD = 9 cm, CD = 12 cm, ∠ACB = 90° and AC = 15 cm

Here, By using Pythagoras theorem

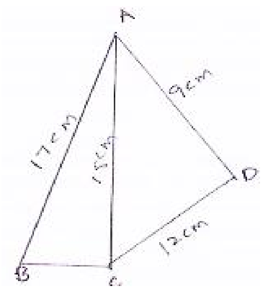
BC = $\sqrt{17^2 - 15^2} = \sqrt{289 - 225} = \sqrt{64} = 8cm$

Now, area of ΔABC = $\frac{1}{2} \times 8 \times 15 = 60\ cm^2$

For area of Δle ACD,

Let a = 15 cm, b = 12 cm and c = 9 cm

Therefore, S = $\frac{15+12+9}{2} = \frac{36}{2} = 18\ cm$



Area of ACD = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{18(18-15)(18-12)(18-9)}$

= $\sqrt{18 \times 18 \times 3 \times 3}$

$= \sqrt{(18 \times 3)^2}$

= 54 cm^2

∴ Thus, the area of quadrilateral ABCD = 60 + 54 = 114 cm^2

11. The adjacent sides of a parallelogram ABCD measure 34 cm and 20 cm, and the diagonal AC measures 42 cm. Find the area of the parallelogram.

Sol:

Given that adjacent sides of a parallelogram ABCD measure 34 cm and 20 cm, and the diagonal AC measures 42 cm.

Area of parallelogram = Area of ΔADC + area of ΔABC

[∵ Diagonal of a parallelogram divides into two congruent triangles]

= 2 × $[Area\ of\ \Delta ABC]$

Now for Area of ΔABC

Let 2s = AB + BC + CA [∵ Perimeter of ΔABC]

⇒ S = $\frac{1}{2}(AB + BC + CA)$

⇒ S = $\frac{1}{2}(34 + 20 + 42)$

$= \frac{1}{2}(96) = 48\ cm$

∴ Area of ΔABC = $\sqrt{s(s-ab)(s-bc)(s-ca)}$ $[heron'sformula]$

$= \sqrt{48(48-34)(48-20)(48-42)}$

$= \sqrt{48(14)(28)(6)} = 336cm^2$

$\therefore Area\ of\ parallelogram\ ABCD = 2[Area\ of\ \Delta ABC] = 2 \times 336 = 672\ cm^2$

12. Find the area of the blades of thc magnetic compass shown in Fig.. 12.27. (Take $\sqrt{11}$ = 3.32).

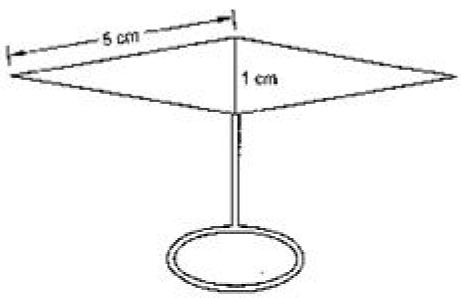


Fig.12.27

Sol:

Area of the blades of magnetic compass = Area of ΔADB + Area of ΔCDB

Now, for area of ΔADB

Let, 2s = AD + DB + BA (Perimeter of ΔADB)

Semi perimeter (S) = $\frac{1}{2}(5+1+5) = \frac{11}{2}cm$

By using heron's formulae

Now, area of ΔADB = $\sqrt{s(s-ad)(s-bd)(s-ba)}$

$= \sqrt{\frac{11}{2}\left(\frac{11}{2}-5\right)\left(\frac{11}{2}-1\right)\left(\frac{11}{2}-5\right)}$

= 2.49 cm^2

= Also, area of triangle ADB = $Area\ of\ \Delta le\ CDB$

∴ Area of the blades of magnetic compass

$= 2 \times (area\ of\ \Delta ADB)$

= 2 × 2.49

= 4.98 m^2

13. A hand fan is made by stitching lo equal size triangular strips of two different types of paper as shown in Fig. 12.28. The dimensions of equal strips are 25 cm, 25 cm and 14 cm. Find the area of each type of paper needed to make the hand fan.

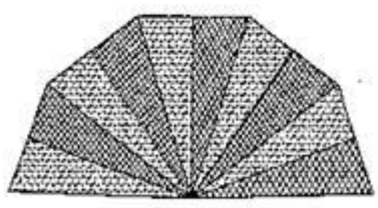


Fig. 12.28

Sol:
Given that the sides of ΔAOB are
AO = 24 cm
OB = 25 cm
BA = 14 cm
Area of each equal strips = Area of Δle AOB
Now, for area of ΔAOB
Perimeter of ΔAOB
Let 2s = AO + OB + BA
$\Rightarrow s = \frac{1}{2}(AO + OB + BA)$
$= \frac{1}{2}(25 + 25 + 14) = 32\ cm$
$\therefore By\ using\ Heron's formulae$
$Area\ of\ (\Delta AOB) = \sqrt{s(s - ao)(s - ob)(s - ba)}$
= $\sqrt{32(32 - 25)(32 - 25)(32 - 14)}$
= $\sqrt{32(7)(4)(18)}$
= 168 cm^2
∴ Area of each type of paper needed to make the hand fan = 5 × $(area\ of\ \Delta AOB)$
= 5 × 168
= 840 cm^2

14. A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 13 cm, 14 cm and 15 cm and the parallelogram stands on the base 14 cm, find the height of the parallelogram.
Sol:
The sides of a triangle DCE are
DC = 15 cm, CE = 13 cm, ED = 14 cm
Let h be the height of parallelogram ABCD
Given,
Perimeter of ΔDCE

2s = DC + CE + ED

$\Rightarrow$ S = $\frac{1}{2}(15 + 13 + 4)$

$\Rightarrow S = \frac{1}{2}(42)$

$\Rightarrow S = 21\ cm$

$Area\ of\ \Delta DCE = \sqrt{s(s - dc)(s - ce)(s - ed)}$ [By heron's formula]

$= \sqrt{21(21 - 15)(21 - 13)(21 - 14)}$

$= \sqrt{21 \times 7 \times 8 \times 6}$

$= \sqrt{84 \times 84}$

$= 84 cm^2$

Given that

Area of $\Delta^{le}\ DCE = \ area\ of\ ABCD$

= Area of parallelogram ABCD = = $84 cm^2$

$\Rightarrow 24 \times h = 84$ [∴ Area of parallelogram = base × height]

$\Rightarrow$ h = 6 cm

Exercise – 13.1

1. Express the following linear equations in the form ax + by + c = 0 and indicate the values of a, b and c in each case:

(i) $-2x+3y=12$

(ii) $x-\frac{y}{2}-5=0$

(iii) $2x+3y=9\cdot3\overline{5}$

(iv) $3x=-7y$

(v) $2x+3=0$

(vi) $y-5=0$

(vii) $4=3x$

(viii) $y=\frac{x}{2}$

Sol:

(i) We have

$$-2x+3y=12$$

$$\Rightarrow \quad -2x+3y-12=0$$

On comparing this equation with $ax+by+c=0$ we obtain $a=-2, b=3$ and $c=-12$.

(ii) Given that

$$x-\frac{y}{2}-5=0$$

$$1x-\frac{y}{2}-5=0$$

On comparing this equation with $ax+by+c=0$ we obtain $a=1, b=\frac{-1}{2}$ and $c=-5$

(iii) Given that

$$2x+3y=9\cdot3\overline{5}$$

$$\Rightarrow \quad 2x+3y-9\cdot3\overline{5}=0$$

On comparing this equation with $ax+by+c=0$ we get $a=2, b=3$ and $c=-9\cdot3\overline{5}$

(iv) $3x=-7y \Rightarrow 3x+7y+0=0$

On comparing this equation with $ax+by+c=0$ we get $a=3, b=7$ and $c=0$.

(v) We have

$$2x+3=0$$

$$2x+0(y)+3=0$$

On comparing this equation with $ax+by+c=0$ we get $a=2, b=0$ and $c=3$

(vi) Given that

$$y-5=0$$

$$\Rightarrow \quad 0x+1y-5=0$$

On comparing this equation with $ax+by+c=0$ we get $a=0, b=1$ and $c=-5$

(vii) We have

$4 = x$

$-3x + 0 \cdot y + 4 = 0$

On comparing the equation with $ax + by + c = 0$ we get $a = -3, b = 0$ and $c = 4$

(viii) Given that,

$y = \frac{x}{2}$

$\Rightarrow 2y = x$

$\Rightarrow x - 2y + 0 = 0$

On comparing this equation with $ax + by + c = 0$ we get $a = 1, b = -2$ and $c = 0$

2. Write each of the following as an equation in two variables:

(i) $2x = -3$

(ii) $y = 3$

(iii) $5x = \frac{7}{2}$

(iv) $y = \frac{3}{2}x$

Sol:

(i) We have

$2x = -3$

$\Rightarrow 2x + 3 = 0$

$\Rightarrow 2x + 0 \cdot y + 3 = 0$

(ii) We have,

$y = 3$

$y - 3 = 0$

$\Rightarrow 0 \cdot x + 1 \cdot y - 3 = 0$

(iii) Given

$5x = \frac{7}{2}$

$10x - 7 = 0$

$10x + 0 \cdot y - 7 = 0$

(iv) We have

$y = \frac{3}{2}x$

$3x - 2y = 0$

$3x - 2y + 0 = 0$

3. The cost of ball pen is Rs. 5 less than half of the cost of fountain pen. Write this statement as a linear equation in two variables.

Sol:

Let us assume the cost of the ball pen be Rs. x and that of a fountain pen to be y. then according to given statements

We have

$$x = \frac{y}{2} - 5$$

$$\Rightarrow 2x = y - 10$$

$$\Rightarrow 2x - y + 10 = 0$$

Exercise – 13.2

1. Write two solutions for each of the following equations:

(i) 3x + 4y = 7

(ii) x = 6y

(iii) x + πy = 4

(iv) $\frac{2}{3}x - y = 4$

Sol:

(i) Given that $3x + 4y = 7$

Substituting $x = 0$ in this equation, we get

$$3 \times 0 + 4y = 7$$

$$\Rightarrow y = \frac{7}{4}$$

So, $\left(0, \frac{7}{4}\right)$ is a solution of the given equation substituting $x = 1$, in given equation, we get

$$\Rightarrow 3 \times 1 + 4y = 7$$

$$\Rightarrow 4y = 7 - 3$$

$$\Rightarrow = 4$$

$$\Rightarrow y = 1$$

So, $(1,1)$ is a solution of the given equation

$\therefore \left(0, \frac{7}{4}\right)$ and $(1,1)$ are the solutions for the given equation.

(ii) We have

$$x = 6y$$

Substituting $y = 0$ in this equation, we get $x = 6 \times 0 = 0$

So, $(0,0)$ is a function of the given equation substituting $y=1,$ in the given equation, we set $x=6\times1=6$

So, $(6,1)$ is a solution of the given equation.

$\therefore$ we obtain $(0,0)$ and $(6,1)$ as solutions of the given equation.

(iii) We have

$x+\pi y=4$

Substituting $y=0$ in this equation, we get

$x+\pi(0)=4$

$\Rightarrow x=4$

So, $(y,0)$ is a solution of the give equation.

$\therefore$ we obtain $(4,0)$ and $(4-x)$ as solutions of the given equation.

(iv) Given that

$\frac{2}{3}x-y=4$

Substituting $y=0$ in this equation we get

$\frac{2}{3}x-0=4$

$\Rightarrow x=4\times\frac{3}{2}$

$\Rightarrow x=6$

So, $(6,0)$ is a solution of the given equation

Substituting $y=1$ in the given equation, we get

$\frac{2}{3}\times-1=4$

$\frac{2}{3}x=5\Rightarrow x=\frac{15}{2}$

So, $\left(\frac{15}{2},1\right)$ is a solution of the given equation.

$\therefore$ We obtain $(6,0)$ *and* $\left(\frac{15}{2},1\right)$ as solutions of the given equation.

2. Write two solutions of the form x = 0, y = a and x = b, y = 0 for each of the following equations:

(i) 5x – 2y = 10 (ii) −4x + 3y = 12 (iii) $2x+3y=24$

Sol:

(i) Given that

$5x-2y=10$

Substituting $x=0$ in the equation $5x-2y=10$

We get $5\times 0-2y=10$

$\Rightarrow y=\frac{-10}{2}=-5$

Thus $x=0$ and $y=-5$ is a solution of $5x-2y=10$

Substituting $y=0,$ we get

$\Rightarrow 5x-2\times 0=10$

$\Rightarrow 5x=10$

$\Rightarrow x=2$

Thus, $x=2$ and $y=0$ is a solution of $5x-2y=10$

Thus $x=0, y=-5$ and $x=2, y=0$ are two solutions of $5x-2y=10$

(ii) Given that,

$-4x+3y=12$

Substituting $x=0$ in the equation

$-4x+3y=12,$ we get

$\Rightarrow -4\times 0+3y=12$

$\Rightarrow 3y=12$

$\Rightarrow y=4$

Thus $x=0$ and $y=4$ is a solution of $-4x+3y=12$

Substituting $y=0$ in the equation

$-4x+3y=12,$ we get

$\Rightarrow -4x+3\times 0=12$

$\Rightarrow -4x=12$

$\Rightarrow x=\frac{12}{-4}=-3$

Thus, $x=-3$ and $y=0$ is a solution of $-4x+3y=12.$

Thus $x=0, y=4$ and $x=-3, y=0$ are two solutions of $-4x+3y=12$

(iii) Given that

$2x+3y=24$

Substituting $x=0$ in the given equation

$2x+3y=24,$ We get

$\Rightarrow 2\times 0+3y=24$

$\Rightarrow 3y=24$

$\Rightarrow y=\frac{24}{3}=8$

Thus, $x=0$ and $y=8$ is a solution of $2x+3y=24$

Substituting $y=0$ in $2x+3y=24$, we get $2x+3\times 0=24$

$\Rightarrow 2x=24$

$\Rightarrow x=\frac{24}{2}=12$

Thus $x=12$ and $y=0$ is a solution of $2x+3y=24$

Thus $x=0, y=-8$ and $x=12, y=0$ are two solutions of $2x+3y=24$

3. Check which of the following are solutions of the equation 2x – y = 6 and which are not:

(i) (3, 0) (ii) (0, 6) (iii) (2, −2) (iv) $\left(\sqrt{3}, 0\right)$ (v) $\left(\frac{1}{2}, -5\right)$

Sol:

In the equation $2x-y=6$ we get

$\text{LHS}=2x-y$ and $\text{RHS}=6$

(i) Substituting $x=3$ and $y=0$ in $2x-y=6$, we get

$LHS=2\times 3-0=6-0=6=\text{RHS}$

So, $x=3, y=0$ or $(3,0)$ is a solution of $2x-y=6$

(ii) Substituting $x=0$ and $y=6$ in $2x-y=6$, we get

$LHS=2\times 0-6=-6\neq RHS$

So, $(0,6)$ is not a solution of the equation $2x-y=6$

(iii) Substituting $x=2, y=-2$ in $2x-y=6$, we get

$LHS=2\times 2(-2)=4+2=6=RHS$

So, $(2,-2)$ is a solution of $2x-y=6$

(iv) Substituting $x=\sqrt{3}$ and $y=0$ in $2x-y=6$, we get

$LHS=2\times\sqrt{3}-0=2\sqrt{3}\neq RHS$

So, $\left(\sqrt{3},0\right)$ is not a solution of the equation $2x-y=6$

(v) Substituting $x=\frac{1}{2}$ and $y=-5$ in $2x-y=6$, we get

$LHS=2\times\frac{1}{2}-(-5)=1+5=6=RHS$

So, $\left(\frac{1}{2},-5\right)$ is a solution of the $2x-y=6$

4. If x = −1, y = 2 is a solution of the equation 3x + 4y = k, find the value of k.
Sol:
Given that
$3x+4y=k$
It is given that $x=-1$ and $y=2$ is a solution of the equation $3x+4y=k$
$\therefore 3\times(-1)\times 4\times 2=k$
$\Rightarrow -3+8=k$
$\Rightarrow k=5$
$\Rightarrow k=5$

5. Find the value of λ, if x = $-\lambda$ and y = $\frac{5}{2}$ is a solution of the equation x + 4y – 7 = 0.
Sol:
Given that
$x+4y-7=0$
It is given that $x=-\lambda$ and $y=\frac{5}{2}$ is a solution of the equation $x+4y-7=0$
$\therefore -1+4\times\frac{5}{2}-7=0$
$\Rightarrow -\lambda+10-7=0$
$\Rightarrow -\lambda=-3$
$\Rightarrow \lambda=3$

6. If x = 2α + 1 and y = α – 1 is a solution of the equation 2x−3y + 5 = 0, find the value of α.
Sol:
We have
$2x-3y+5=0$
It is given that $x=2\alpha+1$ and $=\alpha-1$ is a solution of the equation $2x-3y+5=0$
$\therefore 2(2\alpha+1)-3(\alpha+1)5=0$
$\Rightarrow 4\alpha+2-3\alpha+3+5=0$
$\Rightarrow \alpha+10=0$
$\Rightarrow \alpha=-10$

7. If x = 1 and y = 6 is a solution of the equation 8x – ay + a^2 = 0, find the value of a.
Sol:
Given that
$8x-ay+a^2=0$
It is given that $x=1$ and $y=6$ is a solution on the equation $8x-ay+a^2=0$

$\therefore 8\times1-a\times6+a^2=0$

$\Rightarrow 8-6a+a^2=0$

$\Rightarrow a^2-6a+8=0$

$\Rightarrow a^2-4a-2a+8=0$

$\Rightarrow a(a-4)(a-2)=0$

$\Rightarrow a-4=0$ or $a-2=0$

$a-4=0$ or $a=2$

Hence $a=4$ *or* $a=2$

Exercise – 13.3

1. Draw the graph of each of the following linear equations in two variables:

(i) x + y = 4

(ii) x – y = 2

(iii) –x + y = 6

(iv) y = 2x

(v) 3x + 5y = 15

(vi) $\frac{x}{2}-\frac{y}{3}=3$

(vii) $\frac{x-2}{3}=y-3$

(viii) $2y=-x+1$

Sol:

(i) We have $x+y=4$

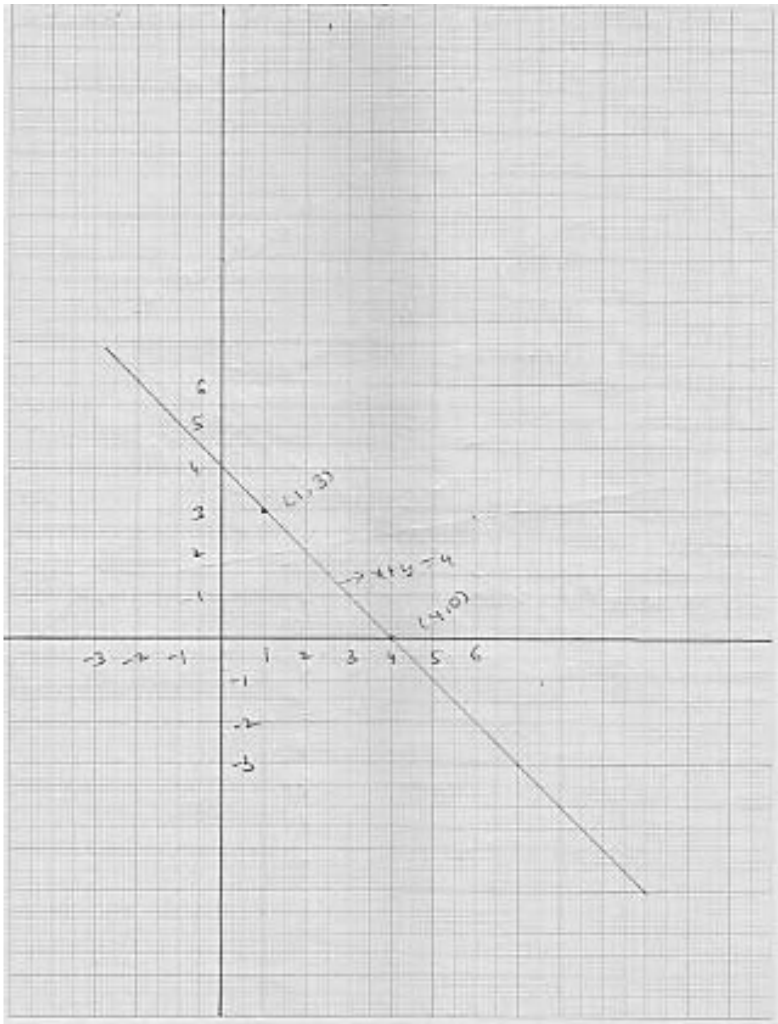
$x=4-y$

Putting $y=0$, we get $x=4-0=4$

Putting $y=3$, we get $x=4-3=1$

Thus, we get the following table giving the two points on the line represented by the equation $x+y=4$

Graph for the equation $x+y=4$



(ii) We have

$x - y = 2$

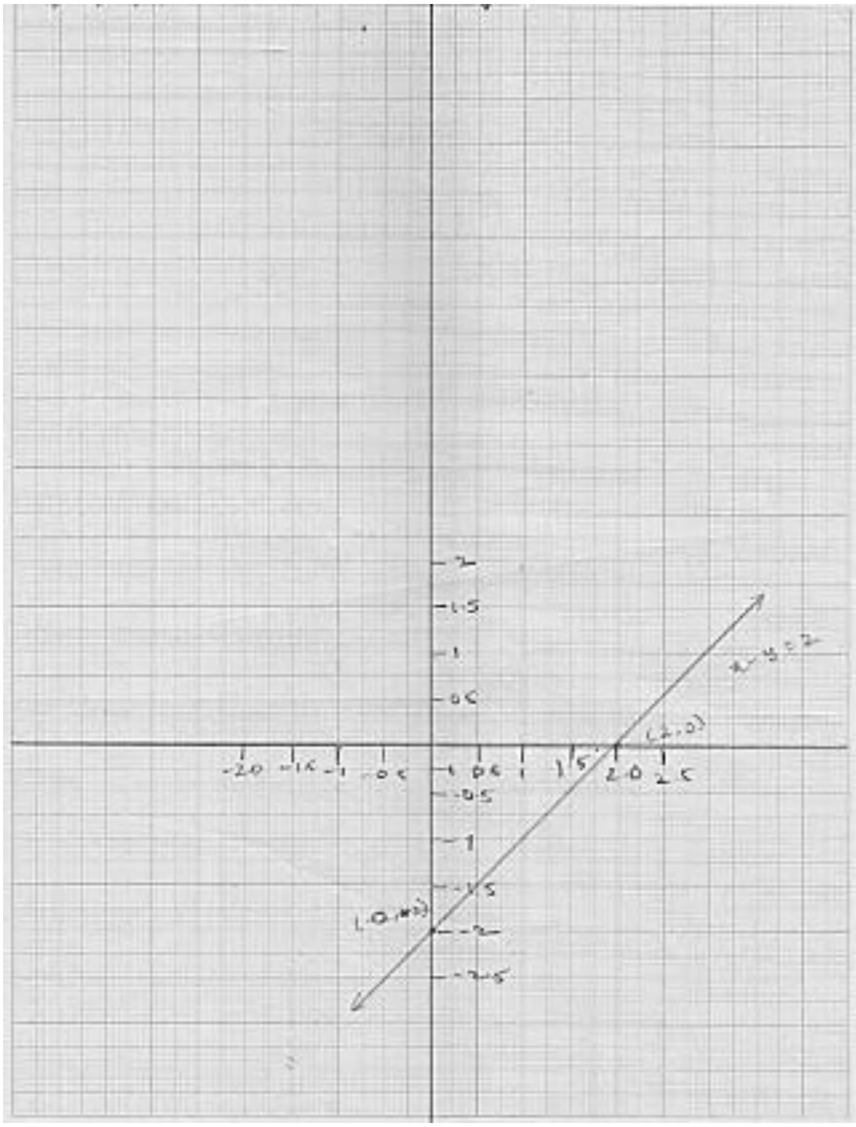
$x = 2 + y \quad ....(i)$

Putting $y = 0,$ we get $x = 2 + 0 = 2$

Putting $y = -2,$ we get $x = 2 - 2 = 0$

Thus, we get the following table giving the two points on the line represented by the equation $x - y - 2$

Graph for the equation $x - y = 2$



(iii) We have

$-x + y = 6$

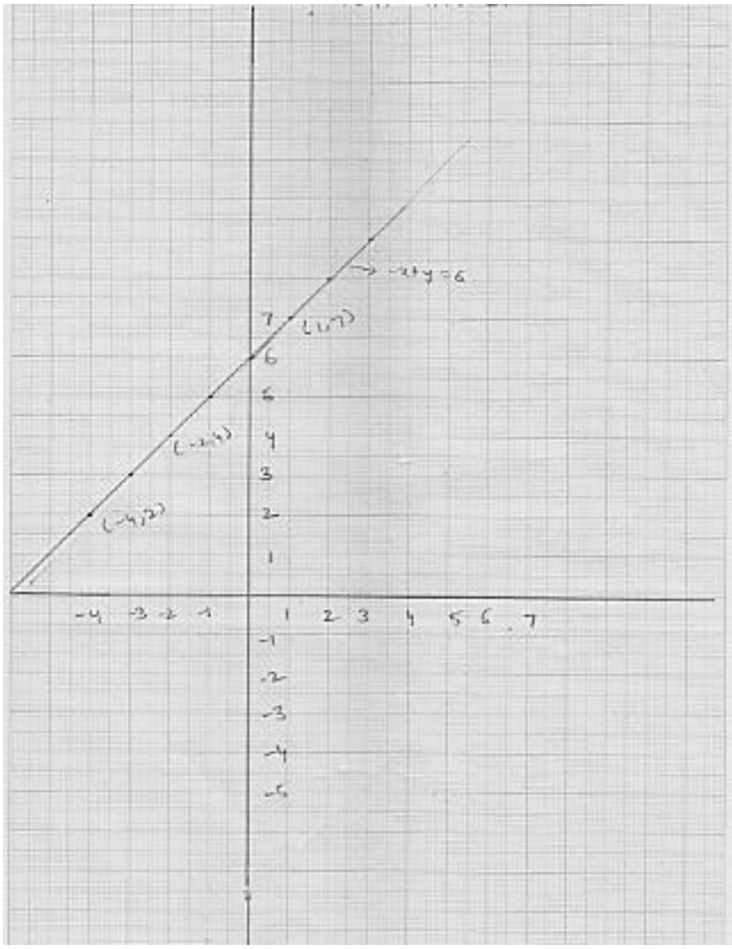
$\Rightarrow x = 6 + x$

Putting $y = -4$, we get $y = 6 - 4 = 2$

Putting $x = -3$ we get $y = 6 - 3 = 3$

Thus, we get the following table giving the two points on the line represented by the equation $-x + y = 6$

Graph for the equation $-x + y = 6$.



(iv) We have

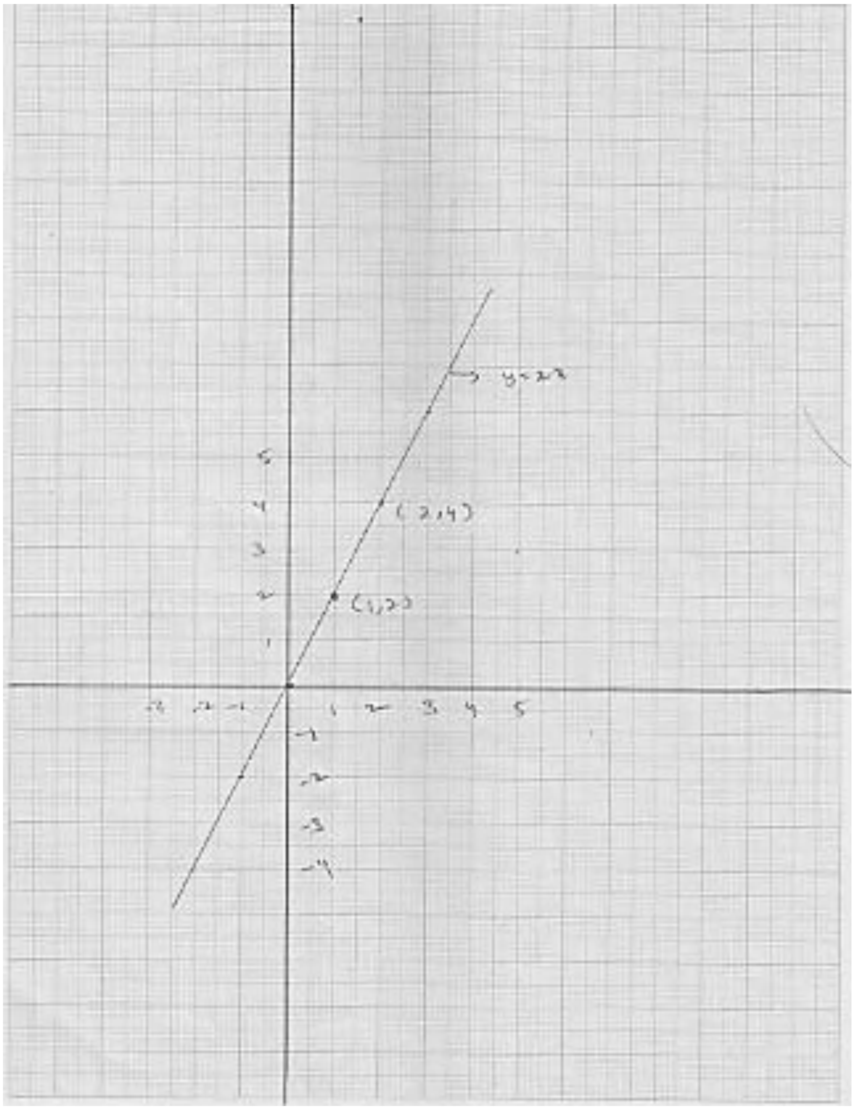
$y = 2x \qquad ....(i)$

Putting $x = 0,$ we get $y = 2 \times 0 = 0$

Putting $x = 1$ we get $y = 2 \times 1 = 2$

Thus, we get the following table giving the two points on the line represented by the equation $y = 2x$

Graph for the equation $y = 2x$



(v) We have

$3x+5y=15$

$3x=15-5y$

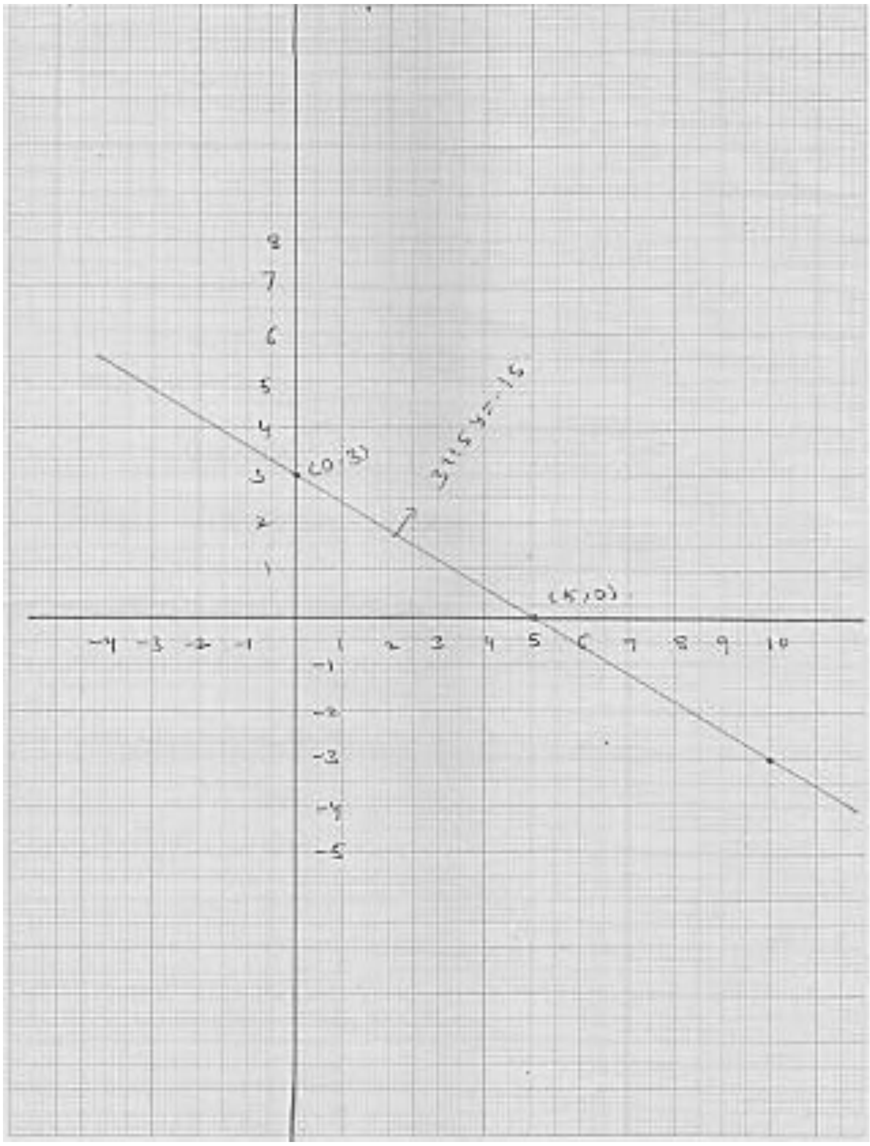
$x=\frac{15-5y}{3}$

Putting $y=0,$ we get $x=\frac{15-5\times 0}{3}=5$

Putting $y=3$ we get $x=\frac{15-5\times 3}{3}=0$

Thus, we get the following table giving the two points on the line represented by the equation $3x+5y-15$

Graph for the equation $3x+5y-15$



(vi) We have

$$\frac{x}{2} - \frac{y}{3} = 2$$

$$\Rightarrow \frac{3x - 2y}{6} = 2$$

$$\Rightarrow 3x - 2y = 12$$

$$\Rightarrow 3x = 12 + 2y$$

$$\Rightarrow x + \frac{12 + 2y}{3}$$

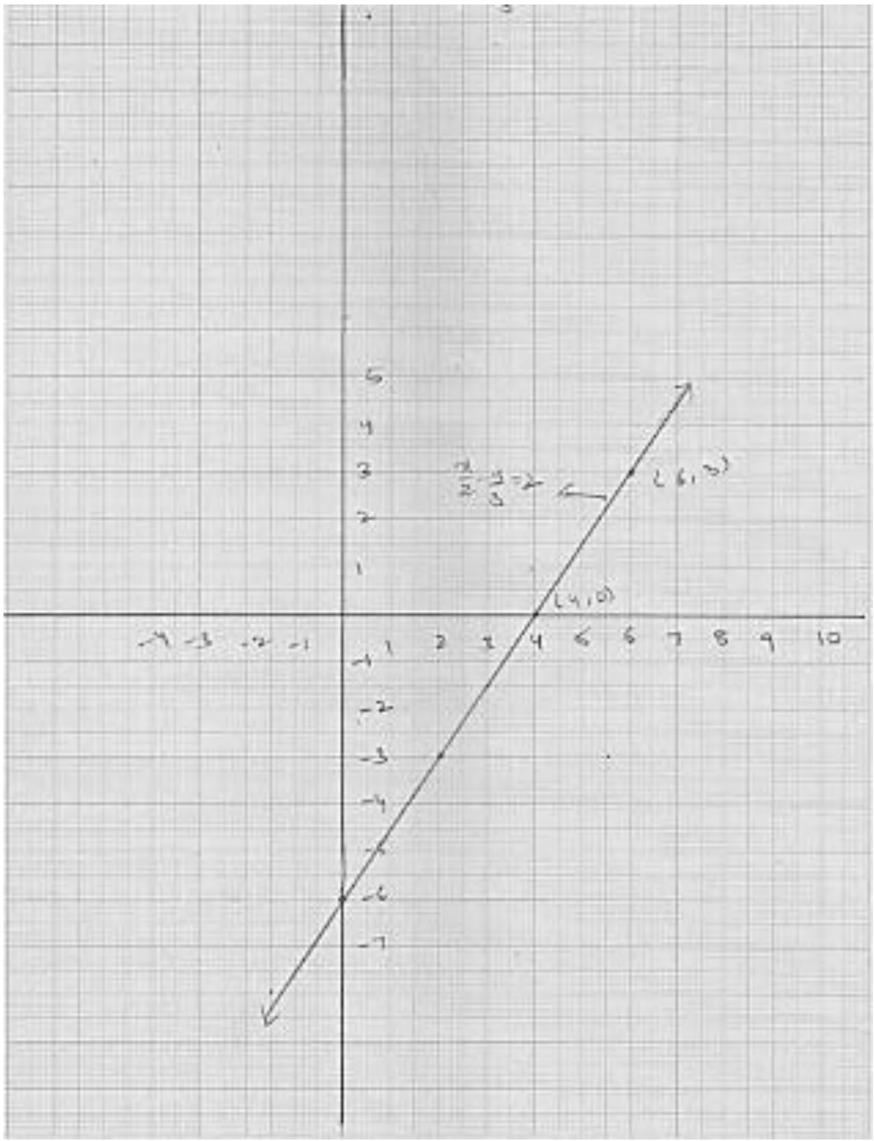
Putting $y = -6$, we get $x = \dfrac{12 + 2(-6)}{3} = 0$

Putting $y=-3$, we get $x=\frac{12+2(-3)}{3}=2$

Putting $y=0$ we get $x=\frac{12+0}{3}=4$

Thus, we get the following table giving the two points on the line represented by the equation $\frac{x}{2}-\frac{y}{3}=2$

Graph for the equation $\frac{x}{2}-\frac{y}{3}=2$



(vii) We have,

$\frac{x-2}{3} = y-3$

$\Rightarrow x-2 = 3(y-3)$

$\Rightarrow x-2 = 3y-9$

$\Rightarrow x = 3y-9+2$

$\Rightarrow x = 3y-7$

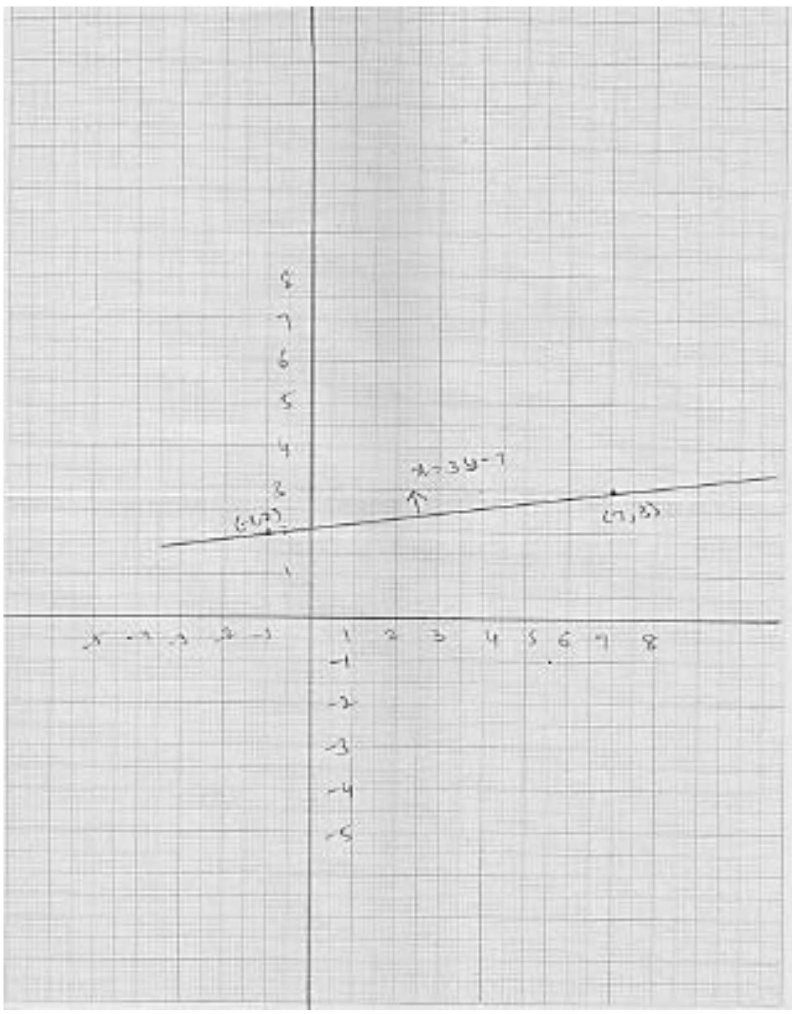
Putting $y=0$, we get $x-0=-7 \Rightarrow x=-7$

Putting $y=2$, we get $x-3(2)=-7 \Rightarrow x=-1$

Putting $y=3$, we get $x=3(3)-7 \Rightarrow x=2$

Thus, we get the following table giving the two points on the line represented by the equation $\frac{x-2}{y} = y-3$

Graph for the equation $\frac{x-2}{y} = y-3$



(viii) We have

$2y = -x + 1$

$\Rightarrow x - 1 = 2y$ ......(1)

Putting $y = 0$, we get $x = 1 - 2 \times 0 = 1$

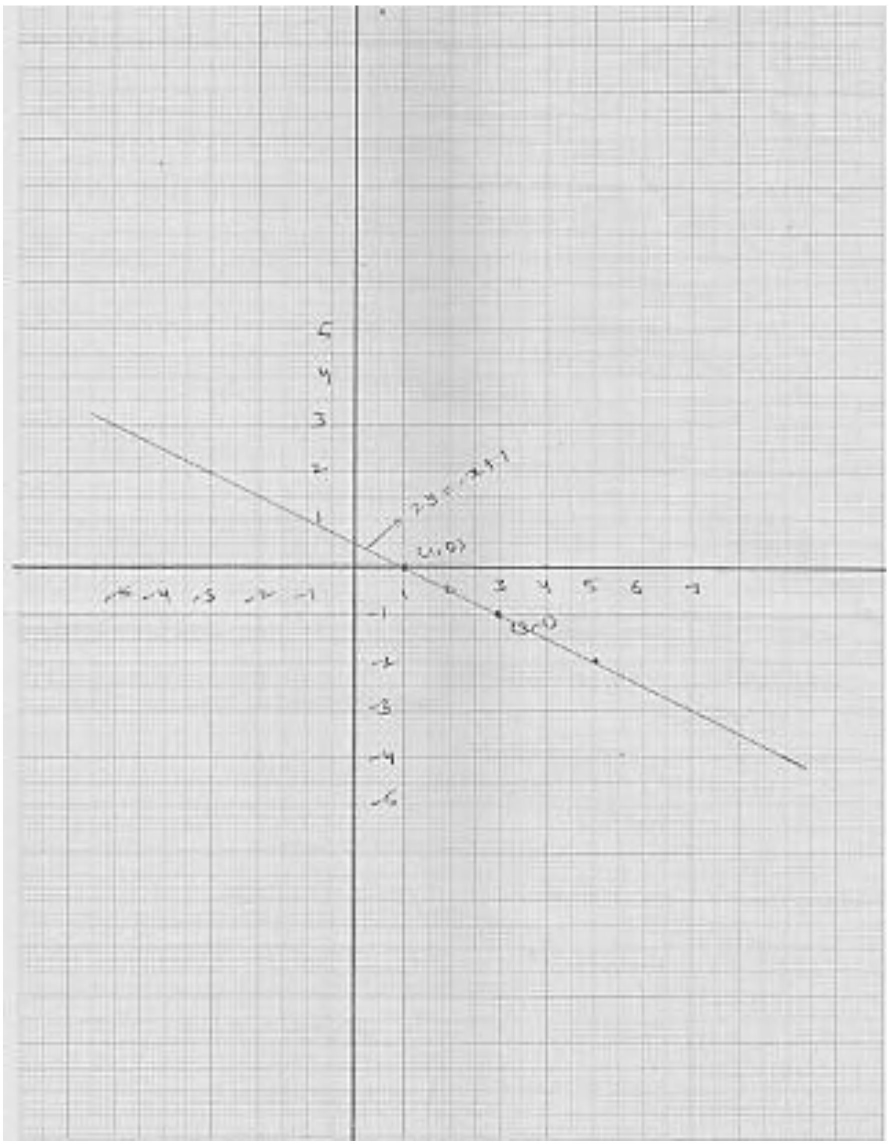
Putting $y = -1$, we get $x = 1 - 2(-1) = 3$

Thus, we have the following table giving the two points on the line represented by the equation

$2y = x + 3$

$2y = -x + 1$

Graph for the equation $2y = -x + 1$



2. Give the equations of two lines passing through (3, 12). How many more such lines are there, and why?

Sol:

The equation of two lines passing through

$(3,12)$ are

$4x - y = 0$

$3x - y + 3 = 0$ ....(i)

There are infinitely many lines passing through $(3,12)$

3. A three-wheeler scooter charges Rs 15 for first kilometer and Rs 8 each for every subsequent kilometer. For a distance of x km, an amount of Rs y is paid. Write the linear equation representing the above information.

Sol:

Total fare of Rs *y* for covering distance of *x* kilometers is given by

$y = 15 + 8(x - 1)$

$\Rightarrow \quad y = 15 + 8x - 8$

$\Rightarrow \quad y = 8x + 7$

This is the required linear equation for the given information

4. A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Aarushi paid Rs 27 for a book kept for seven days. If fixed charges are Rs x and per day charges are Rs y. Write the linear equation representing the above information.

Sol:

Total charges paid by Aarushi is given by

$27 = x + 4y$

$\Rightarrow \quad x + 4y = 27$

This is the required linear equation for the given information.

5. A number is 27 more than the number obtained by reversing its digits. If its unit's and ten's digit are x and y respectively, write the linear equation representing the above statement.

Sol:

Total original number is $10y + x$

The new number is obtained after reversing the order of digits is $10x + y$

According to question

$\Rightarrow \quad 10y + x = 10x + y + 27$

$\Rightarrow \quad 9y - 9x = 27$

$\Rightarrow \quad y - x = 3$

$\Rightarrow \quad x - y + 3 = 0$

This is the required linear equation for the given information.

6. The sum of a two digit number and the number obtained by reversing the order of its digits is 121. If units and ten's digit of the number are x and y respectively then write the linear equation representing the above statement.

Sol:

Total original number is $10y + x$

The new number is obtained after reversing the order of digits is $(10x + y)$

According to problem

$$(10y + x) + (10x + y) = 121$$

$\Rightarrow \quad 11x + 11y = 121$

$\Rightarrow \quad 11(x + y) = 121$

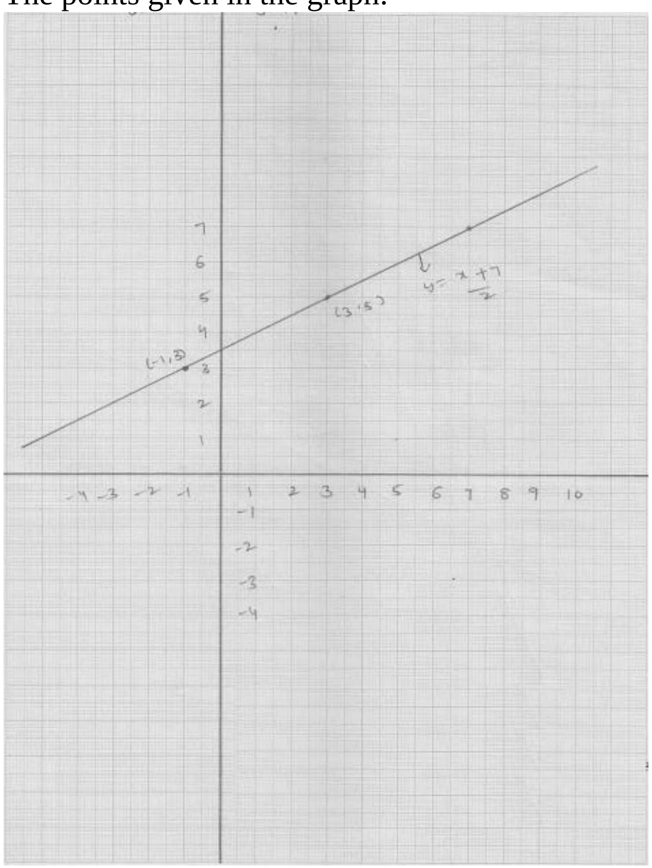
$\Rightarrow \quad x + y = 11$

Thus is the required linear equation for the given information

7. Plot the points (3, 5) and (− 1, 3) on a graph paper and verify that the straight line passing through these points also passes through the point (1, 4).

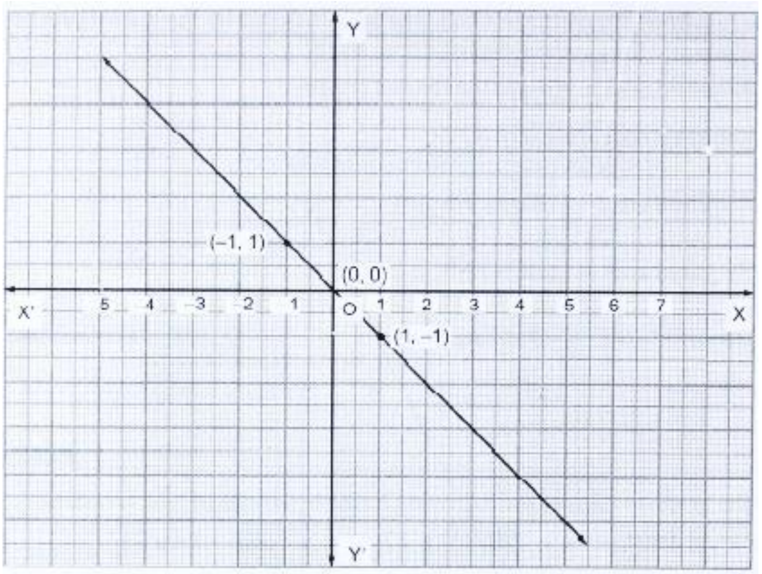
Sol:

The points given in the graph:



It is clear from the graph the straight lines passes through these points also pass a through $(1,4)$.

8. From the choices given below, choose the equation whose graph is given in Fig. below.
(i) y = x (ii) x + y = 0 (iii) y = 2x (iv) 2 + 3y = 7x



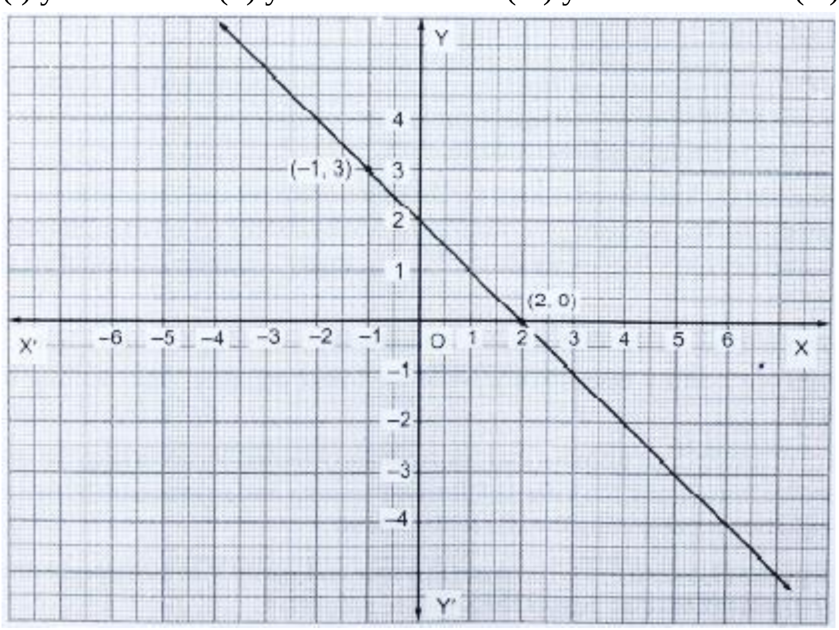
[Hint: Clearly, (-1, 1) and (1, -1) satisfy the equation x + y = 0**]**

Sol:

Clearly $(-1,1)$ and $(1,-1)$ satisfy the equation $x+y=0$

$\therefore$ The equation whose graph is given by $x+y=0$

9. From the choices given below, choose the equation whose graph is given in fig. below.
(i) y = x + 2 (ii) y = x – 2 (iii) y = −x + 2 (iv) x + 2y = 6



[Hint: Clearly, (2, 0) and (−1, 3) satisfy the equation y = −x + 2]

Sol:

Clearly $(2,0)$ and $(-1,3)$ satisfy the equation $y=-x+2$

$\therefore$ The equation whose graph is given by $y=-x+2$

10. If the point (2, -2) lies on the graph of the linear equation 5x + ky = 4, find the value of k.

Sol:

It is given that $(2,-2)$ is a solution of the equation $5x+ky=4$

$\therefore \quad 5\times 2+k\times(-2)=4$

$\Rightarrow \quad 10-2k=4$

$\Rightarrow \quad -2k=4-10$

$\Rightarrow \quad -2k=-6$

$\Rightarrow \quad k=3.$

11. Draw the graph of the equation 2x + 3y = 12. From the graph, find the coordinates of the point: (i) whose y-coordinates is 3. (ii) whose x-coordinate is −3.

Sol:

Graph of the equation $2x+3y=12$:

We have,

$2x+3y=12$

$\Rightarrow \quad 2x=12-3y$

$\Rightarrow \quad x=\dfrac{12-3y}{2}$

Putting $y=2,$ we get $x=\dfrac{12-3\times 2}{2}=3$

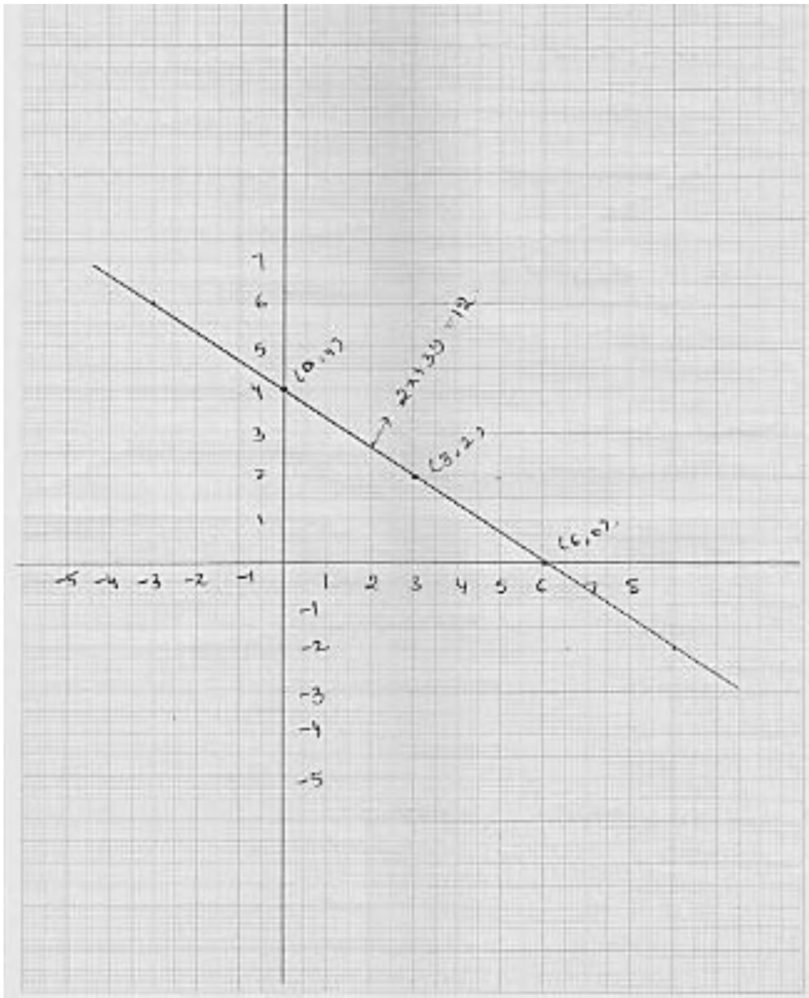
Putting $y=-4,$ we get $x=\dfrac{12-3\times 4}{2}=0$

Thus, $(3,0)$ and $(0,4)$ are two points on the line $2x+3y=12$

The graph of line represents by the equation $2x+3y=12$

x	0	3
y	4	2

Graph of the equation $2x+3y=12$



(i) To find coordinates of the points when $y = 3,$ we draw a line parallel to $x - axis$ and passing through $(0,3)$ this lines meets the graph of $2x + 3y = 12$ at a point p from which we draw a line parallel to $y - axis$ which process $x - axis$ at $x = \frac{3}{2}$, so the coordinates of the required points are $\left(\frac{3}{2}, 3\right)$.

(ii) To find the coordinates of the points when $x = -3$ we draw a line parallel to $y - axis$ and passing through $(-3, 0)$. This lines meets the graph of $2x + 3y = 12$ at a point p from which we draw a line parallel to $x - axis$ crosses $y - axis$ at $y = 6,$ so, the coordinates of the required point are $(-3, 6)$.

12. Draw the graph of each of the equations given below. Also, find the coordinates of the points where the graph cuts the coordinate axes:

(i) 6x − 3y = 12 (ii) −x + 4y = 8 (iii) 2x + y = 6 (iv) 3x + 2y + 6 = 0

Sol:

(i) We have

$$6x - 3y = 12$$

$$\Rightarrow \quad 3(2x - y) = 12$$

$$\Rightarrow \quad 2x - y = 4$$

$$\Rightarrow \quad 2x - 4 = y$$

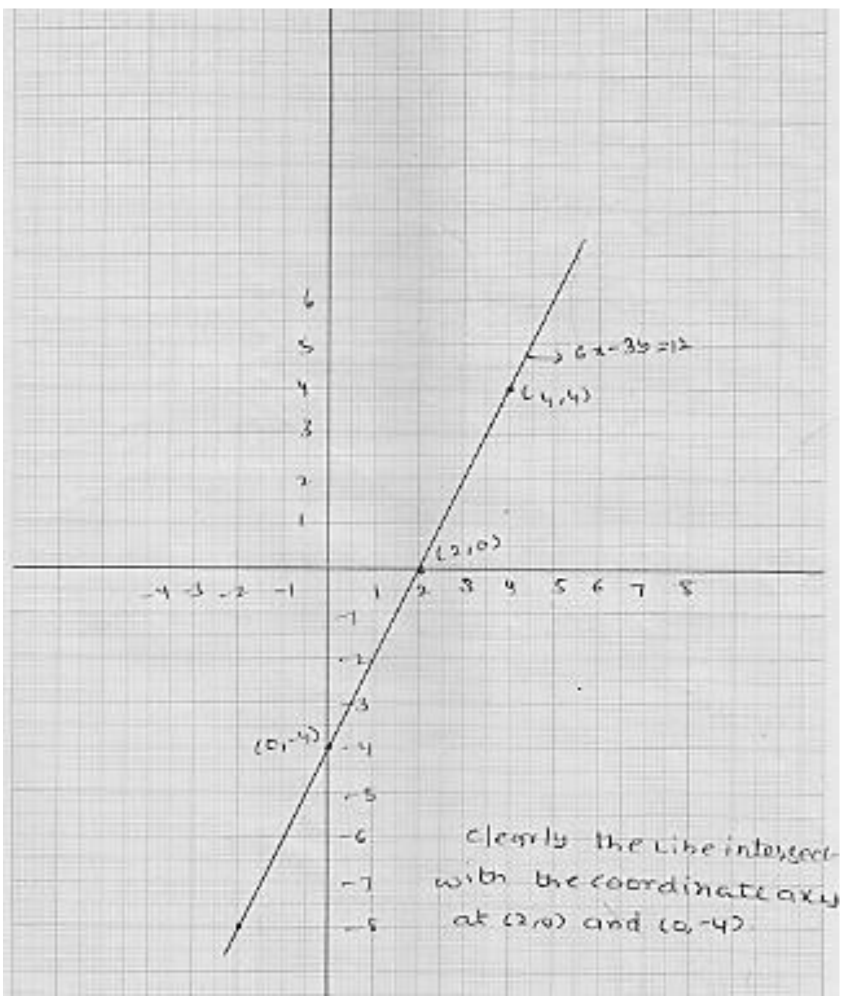
$$\Rightarrow \quad y = 2x - 4 \qquad \text{......}(i)$$

Putting $x = 0$ in (i), we get $y = -4$

Putting $x = 2$ in (i), we get $y = 0$

Thus, we obtain the following table giving coordinates of two points on the line represented by the equation $6x - 3y = 12$.

The graph of the line $6x - 3y = 12$



(ii) We have

$$-x+4y=8$$

$\Rightarrow$ $4y-8=x$

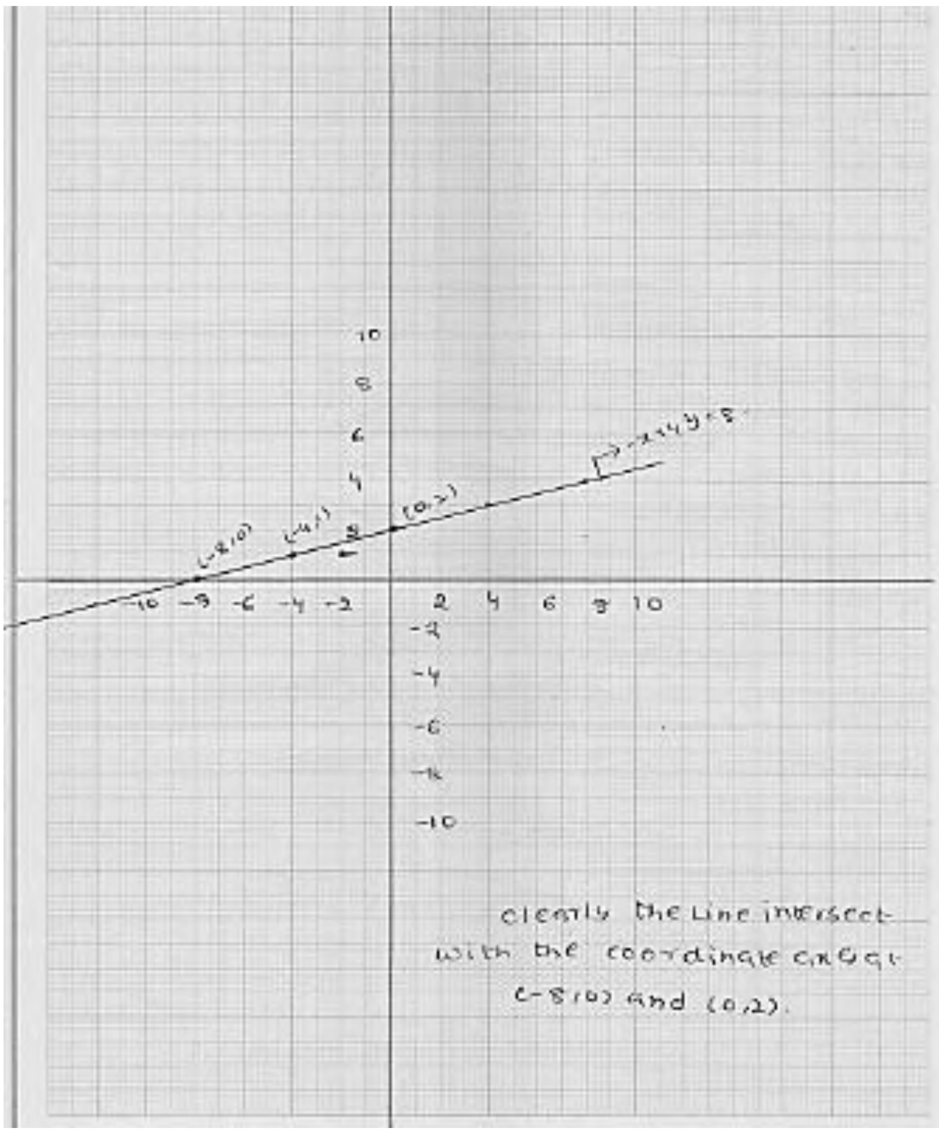
$\Rightarrow$ $x=4y=8$

Putting $y=1$ in (i), we get $x=4\times1-8=-4$

Putting $y=2$ in (i), we get $x=4\times2-8=0$

Thus, we obtain the following table giving coordinates of two points on the line represented by the equation $-x+4y=-8$

Graph of the equation $-x+4y=8$



(iii) We have

$$2x+y=6$$

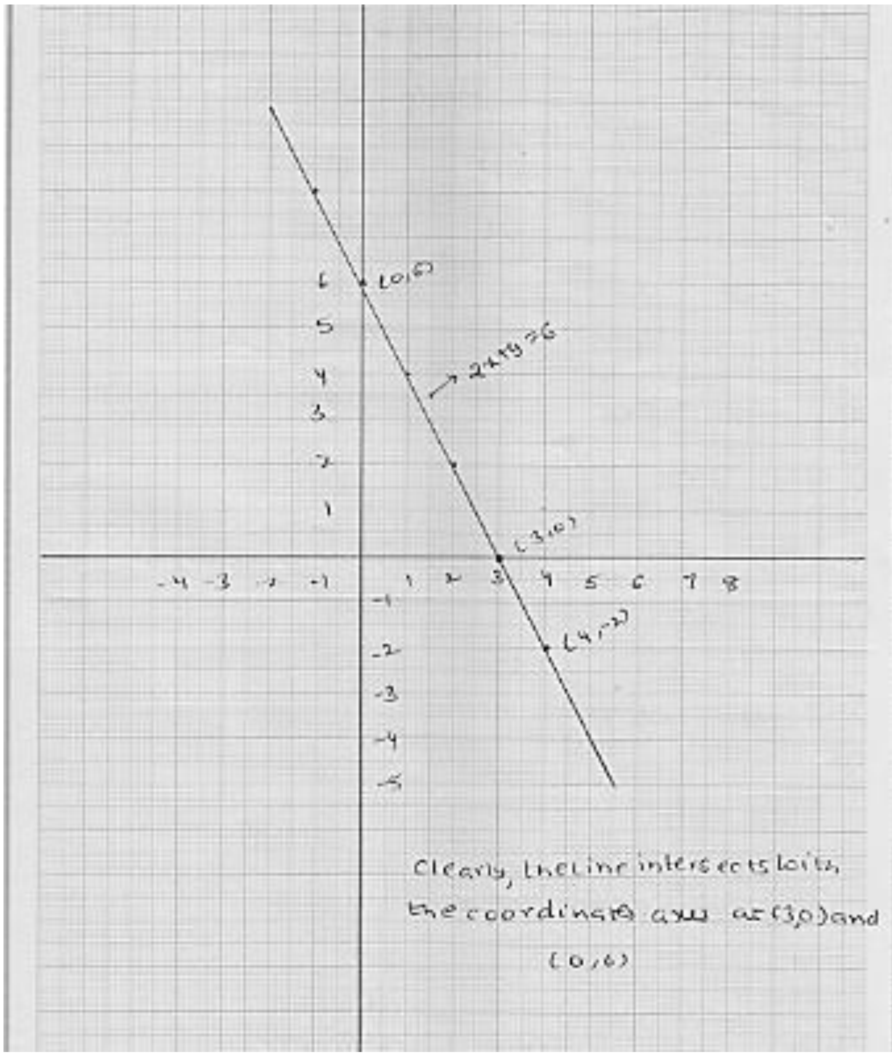
$\Rightarrow$ $y=6-2x$ ....(i)

Putting $x = 3$ in (i), we get $y = 6 = 2 \times 3 = 0$

Putting $x = 4$ in (i), we get $y = 6 - 2 \times 4 = -2$

Thus, we obtain the following table giving coordinates of two points on the line represented by the equation $2x + y = 6$

Graph of the equation $2x + y = 6$



(iv) We have

$$3x + 2y + 6 = 0$$

$$\Rightarrow \quad 2y = -6 - 3x$$

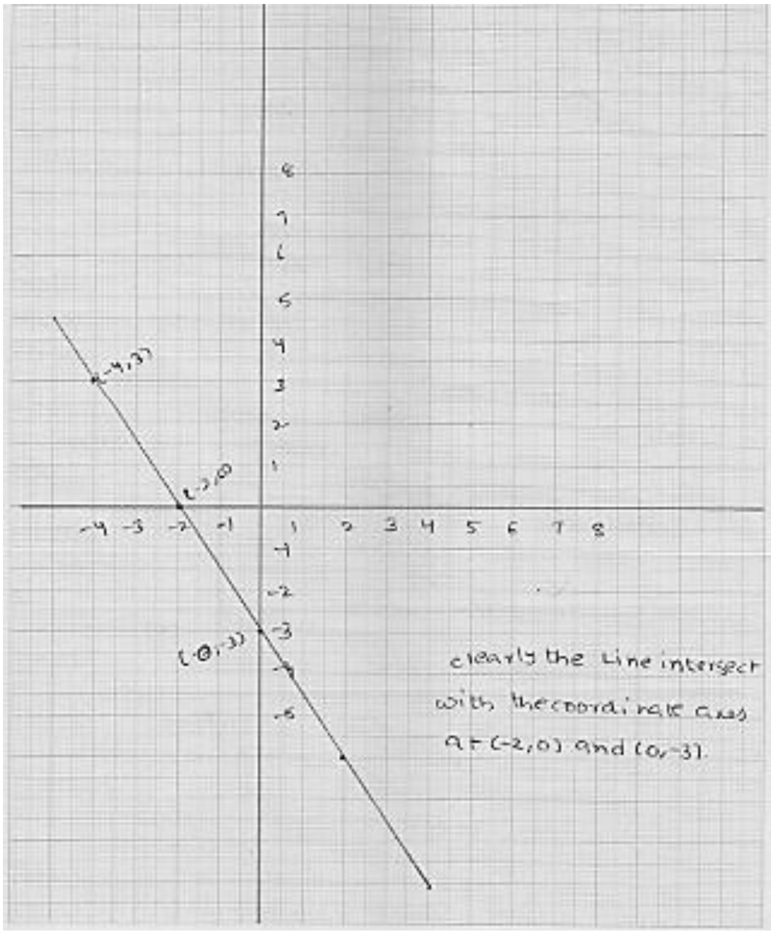
$$\Rightarrow \quad y = \frac{-6 - 3x}{2}$$

Putting $x = -2$ in (i), we get $x = \frac{6 - 3(-2)}{2} = 0$

Putting $x = -4$ in (i), we get $y = \frac{6 - 3(-4)}{2} = 3$

Thus, we obtain the following table giving coordinates of two points on the line represented by the equation $3x+2y+6=0$

Graph of the equation $3x-2y+6=0$



13. Draw the graph of the equation 2x + y = 6. Shade the region bounded by the graph and the coordinate axes. Also, find the area of the shaded region.

Sol:

We have

$2x+y=6$

$y=6-2x$ ....(i)

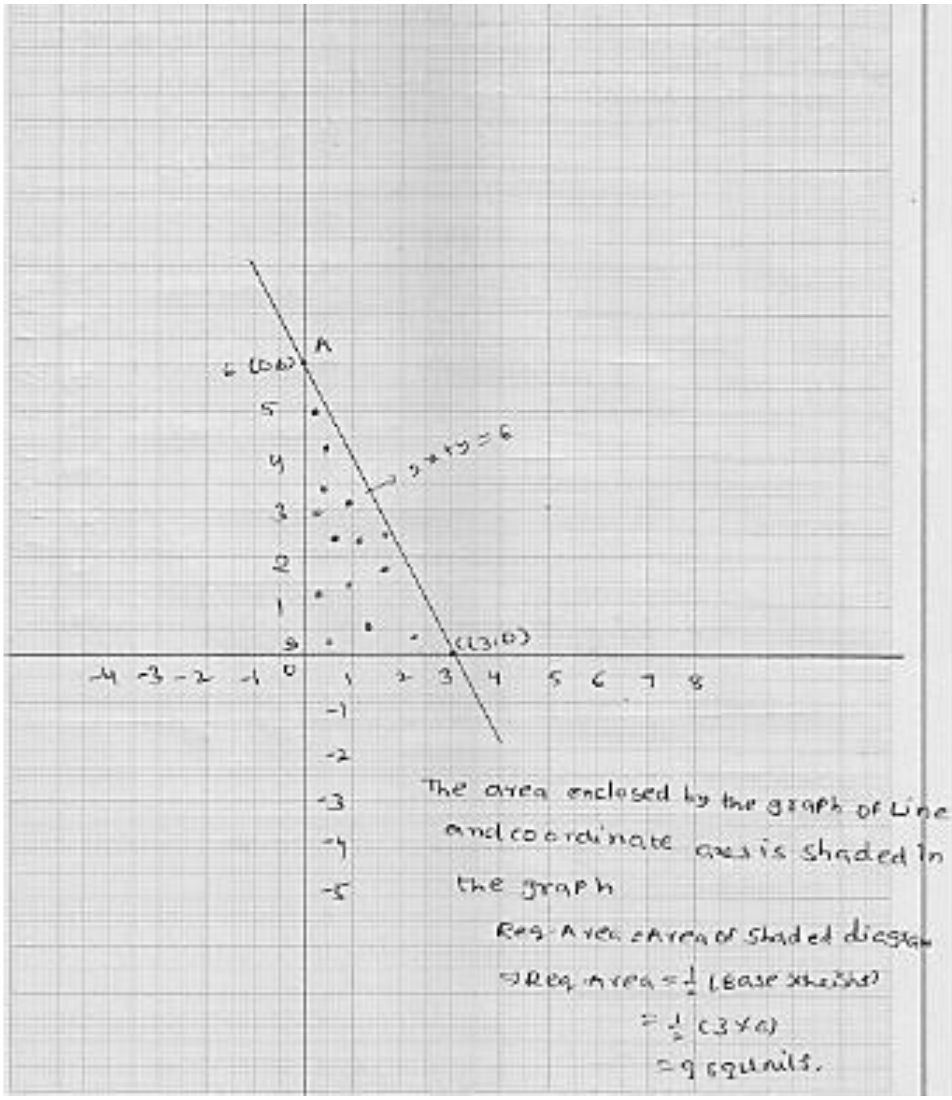
Putting $x=3$ in (i), we get $y=6-2\times 3=0$

Putting $x=0$ in (i), we get $y=6-2\times 0=6$

Thus, we obtained the following table giving coordinates of two points on the line represented by the equation $2x+y=6$

x	3	0
y	0	6

The graph of line $2x+y=6$



14. Draw the graph of the equation $\frac{x}{3}+\frac{y}{4}=1$. Also, find the area of the triangle formed by the line and the co-ordinates axes.

Sol:

We have

$$\frac{x}{3}+\frac{y}{4}=1$$

$$\Rightarrow \quad 4x+3y=12$$

$$\Rightarrow \quad 4x=12-3y$$

$$\Rightarrow \quad x=\frac{12-3y}{4}$$

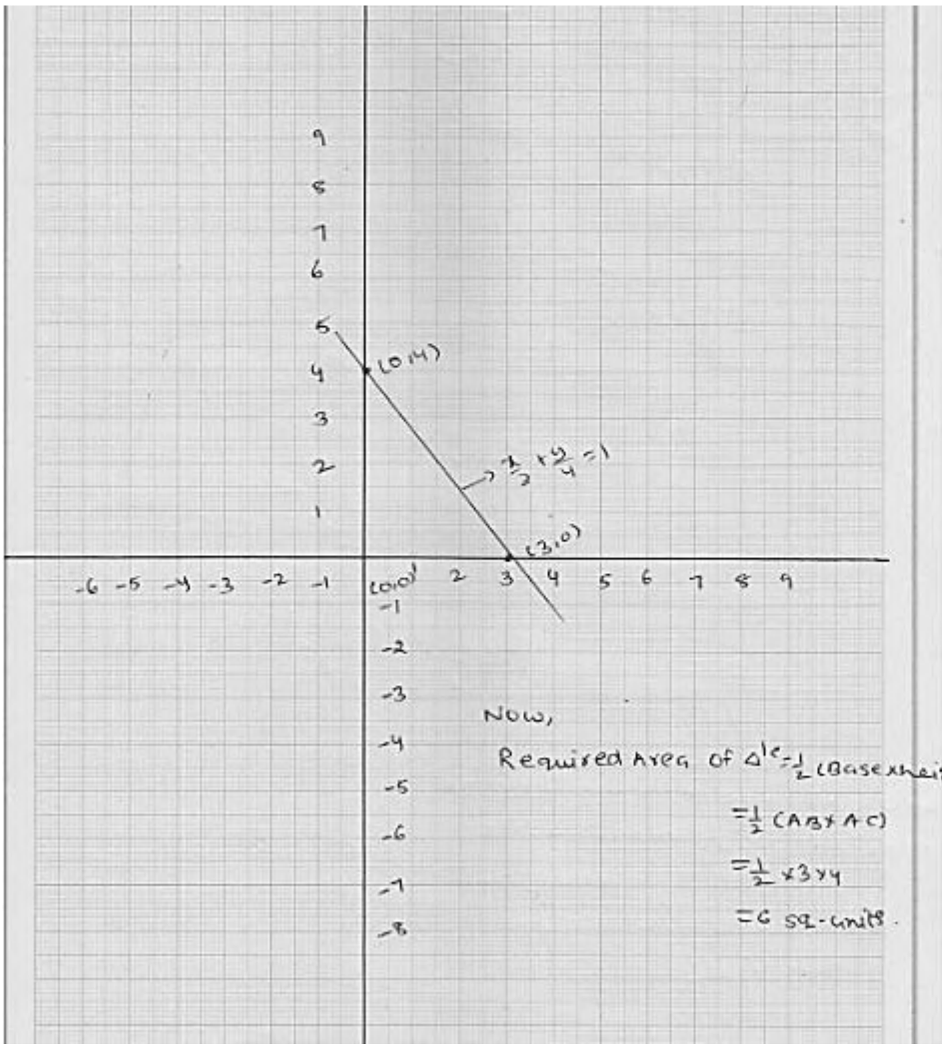
Putting $y=0$ in (i), we get $x=\frac{12-3\times 0}{4}=3$

Putting $y=-4$ in (ii), we get $x=\frac{12-3\times 4}{4}=0$

Thus, we obtained the following table giving coordinates of two points on the line represents by the equation $\frac{x}{3}+\frac{y}{4}=1$.

x	0	3
y	4	0

The graph of line $\frac{x}{3}+\frac{y}{4}=1$.



15. Draw the graph of y = | x |.

Sol:

We have

$y=|x|$ ....(i)

Putting $x=0$, we get $y=0$

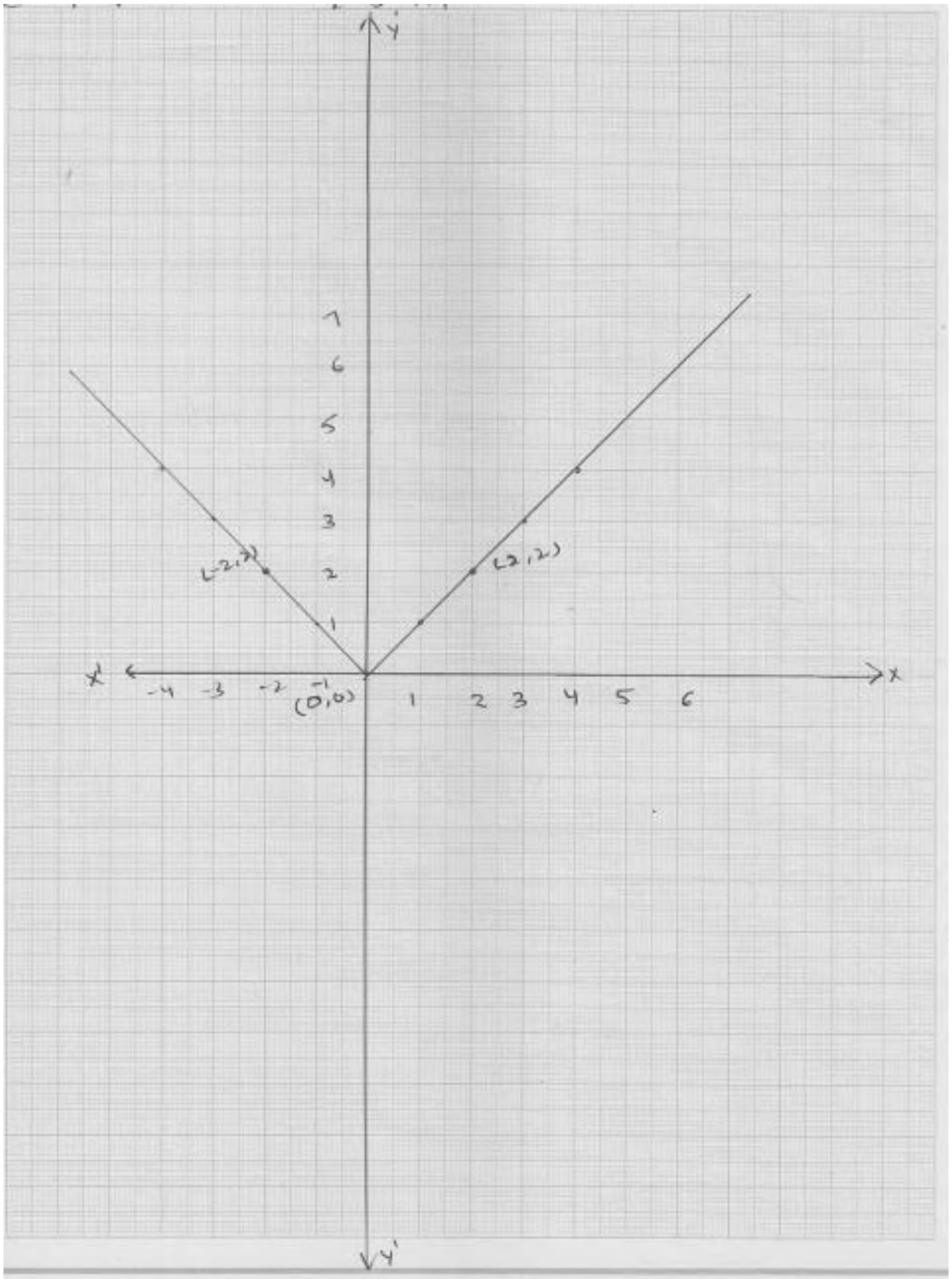
Putting $x = 2,$ we get $y = 2$

Putting $x = 2,$ we get $y = -12$

Thus, we have the following table for the two points on graph of $|x|$

x	0	2	-2
y	0	2	2

Graph of line equation $y = |x|$



16. Draw the graph of y = | x | + 2.

Sol:

We have

$y = |x| + 2$ ......(i)

Putting $x = 0$, we get $y = 2$..........

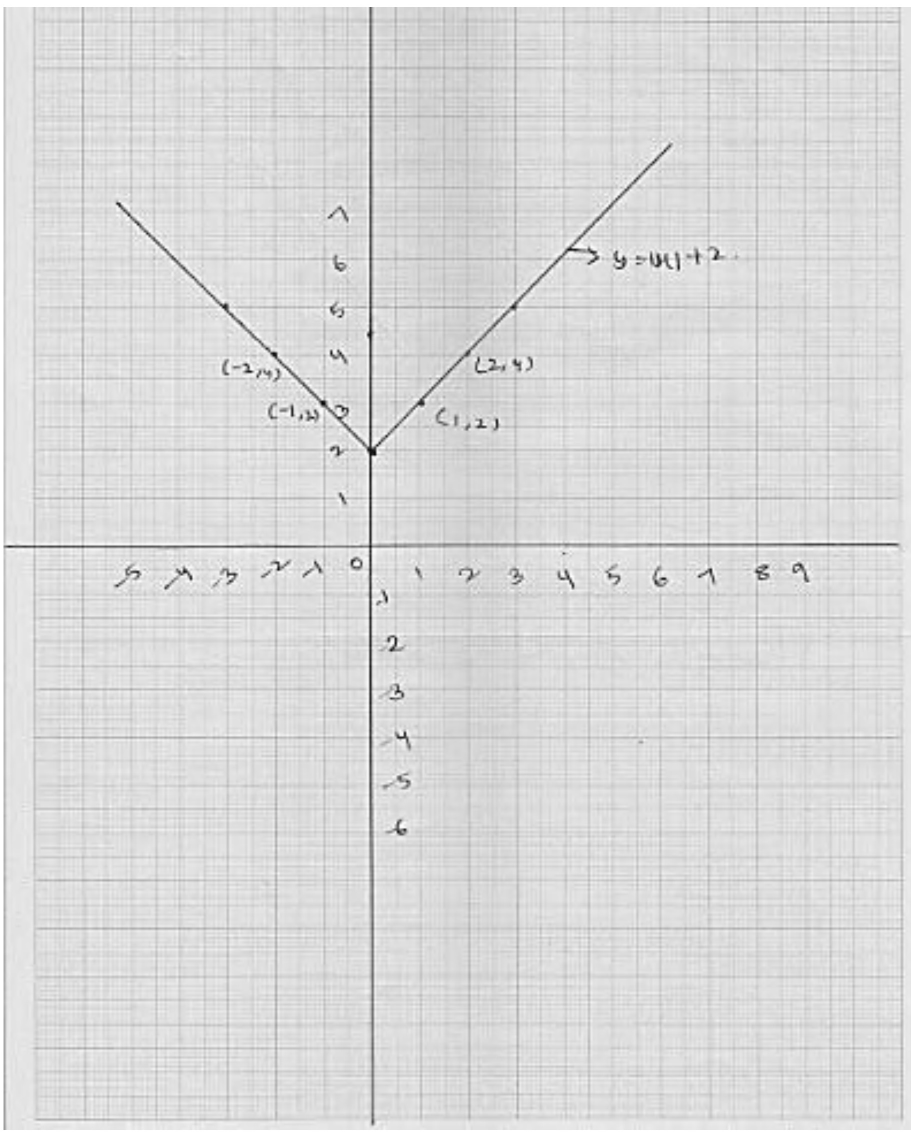
Putting $x = 1$, we get $y = 3$

Putting $x = -1$, we get $y = 3$

Thus, the we have the following table for the points on graph of $|x| + 2$

x	0	1	1
y	2	3	3

Graph of line equation $y = |x| + 2$



17. Draw the graphs of the following linear equations on the same graph paper: 2x + 3y = 12, x – y = 1.
Find the coordinates of the vertices of the triangle formed by the two straight lines and the y-axis. Also, find the area of the triangle.

Sol:

Graph of the equation $2x+3y-12=0$

We have

$$2x+3y=12$$

$$\Rightarrow \quad 2x=12-3y$$

$$\Rightarrow \quad x=\frac{12-3y}{2}$$

Putting $y=4,$ we get $x=\frac{12-3\times 4}{2}=0$

Putting $y=2,$ we get $x=\frac{12-3\times 2}{2}=3$

Thus, we have the following table for the p table for the points on the line $2x+3y=12$

x	0	3
y	4	2

Plotting points $A(0,4), B(3,2)$ on the graph paper and drawing a line passing through them we obtain graph of the equation.

Graph of the equation

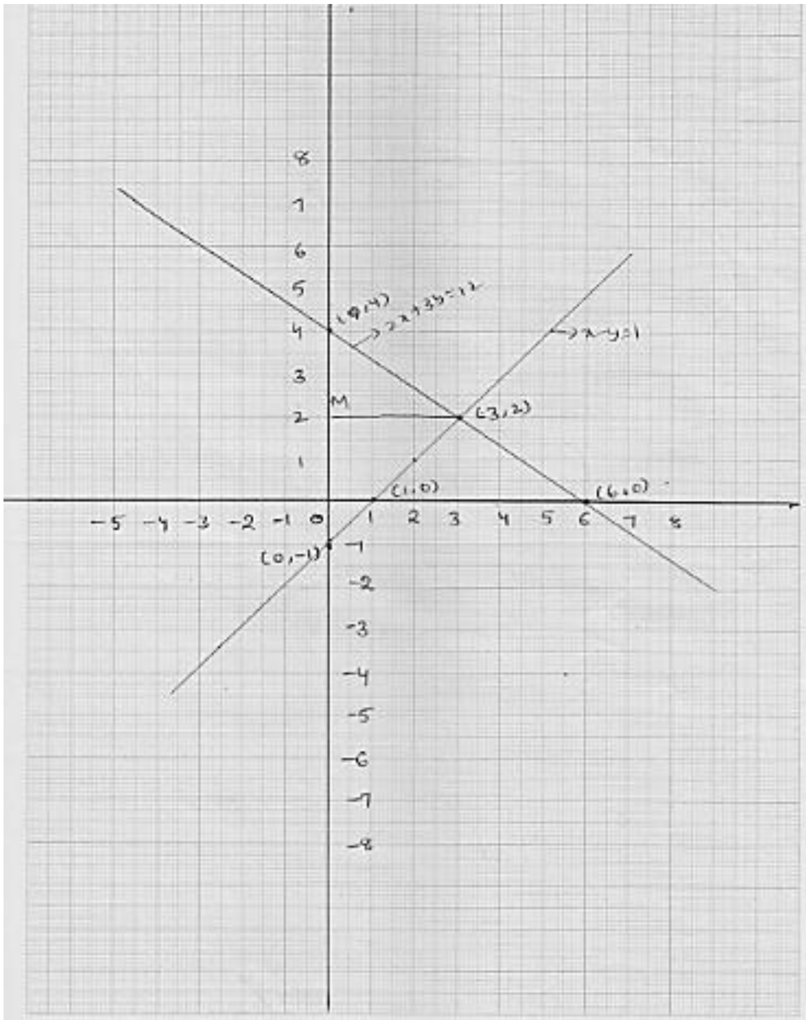
Graph of the equation $x-y-1$:

We have $x-y=1 \Rightarrow x=1+y$

Thus, we have the following table for the points the line $x-y=1$

x	1	0
y	0	-1

Plotting points $C(1,0)$ and $D(0,-1)$ on the same graph paper drawing a line passing through the m, we obtain the graph of the line represents by the equation $x-y=1$.



Clearly two lines intersect at $A(3,2)$.

The graph of time $2x+3y=12$ intersect with $y-axis$ at $B(0,4)$ and the graph of the line $x-y=1$ intersect with $y-axis$ at $C(0,-1)$.

So, the vertices of the triangle formed by thee two straight lines and $y-axis$ are $A(3,2)$ and $B(0,4)$ and $C(0,-1)$

Now,

Area of $\Delta ABC=\frac{1}{2}[\text{Base}\times\text{ Height}]$

$=\frac{1}{2}(BC\times AB)$

$=\frac{1}{2}(5+3)$

$=\frac{15}{2}$ sq.units

18. Draw the graphs of the linear equations 4x − 3y + 4 = 0 and 4x + 3y − 20 = 0. Find the area bounded by these lines and x-axis.

Sol:

We have

$$4x-3y+4=0$$

$\Rightarrow$ $4x-3y=4$

$\Rightarrow$ $x=\frac{3y-4}{4}$

Putting $y=0,$ we get $x=\frac{3\times 0-4}{4}=-1$

Putting $y=4,$ we get $x=\frac{3\times 4-4}{4}=2$

Thus, we have the following table for the p table for the points on the line $4x-3y+4=0$

x	-1	2
y	0	4

We have

$$4x+3y-20=0$$

$\Rightarrow$ $4x=20-3y$

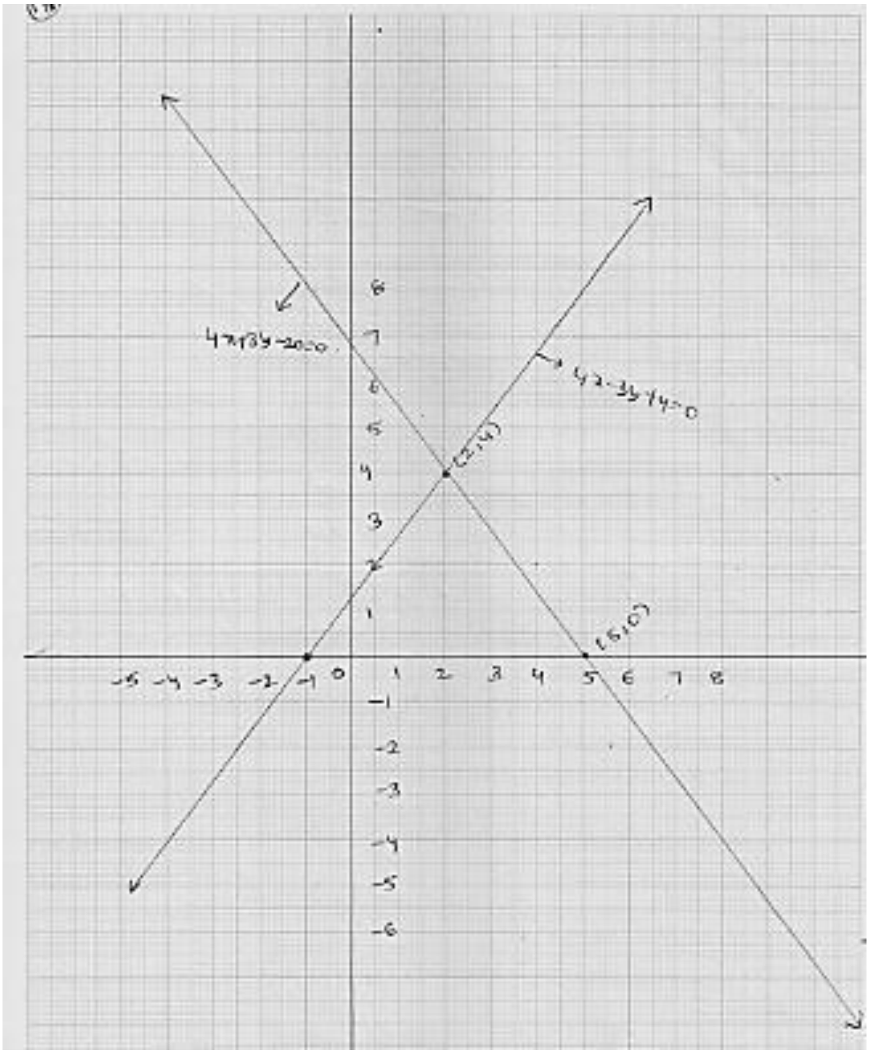
$\Rightarrow$ $x=\frac{20-3y}{4}$

Putting $y=0,$ we get $x=\frac{20-3\times 0}{4}=5$

Putting $y=4,$ we get $x=\frac{20-3\times 4}{4}=2.$

Thus, we have the following table for the p table for the points on the line $4x-3y-20=0$

x	0	2
y	0	4



Clearly, two lines intersect at $A(2,4)$.

The graph of the lines $4x-3y+4=0$ and $4x+3y-20=0$ intersect with $y-axis$ at $a+B(-1,0)$ and $c(5,0)$ respectively

$\therefore$ Area of $\Delta ABC=\frac{1}{2}[\text{Base}\times\text{height}]$

$=\frac{1}{2}(BC\times AB)$

$=\frac{1}{2}(6\times 4)$

$=3\times 4$

$=12$ sq.units

$\therefore$ Area of $\Delta ABC=12$ sq.units

19. The path of a train A is given by the equation 3x + 4y − 12 = 0 and the path of another train B is given by the equation 6x + 8y − 48 = 0. Represent this situation graphically.

Sol:

We have,

$$3x+4y-12=0$$

$$\Rightarrow \quad 3x=12-4y$$

$$\Rightarrow \quad 3x=\frac{12-4y}{3}$$

Putting $y=0,$ we get $x=\frac{12-4\times 0}{3}=4$

Putting $y=3,$ we get $x=\frac{12-4\times 3}{3}=0$

Thus, we have the following table for the points on the line $3x+4y-12=0$:

x	4	0
y	0	3

We have

$$6x+8y-48=0$$

$$6x+8y=48$$

$$6x=48-8y$$

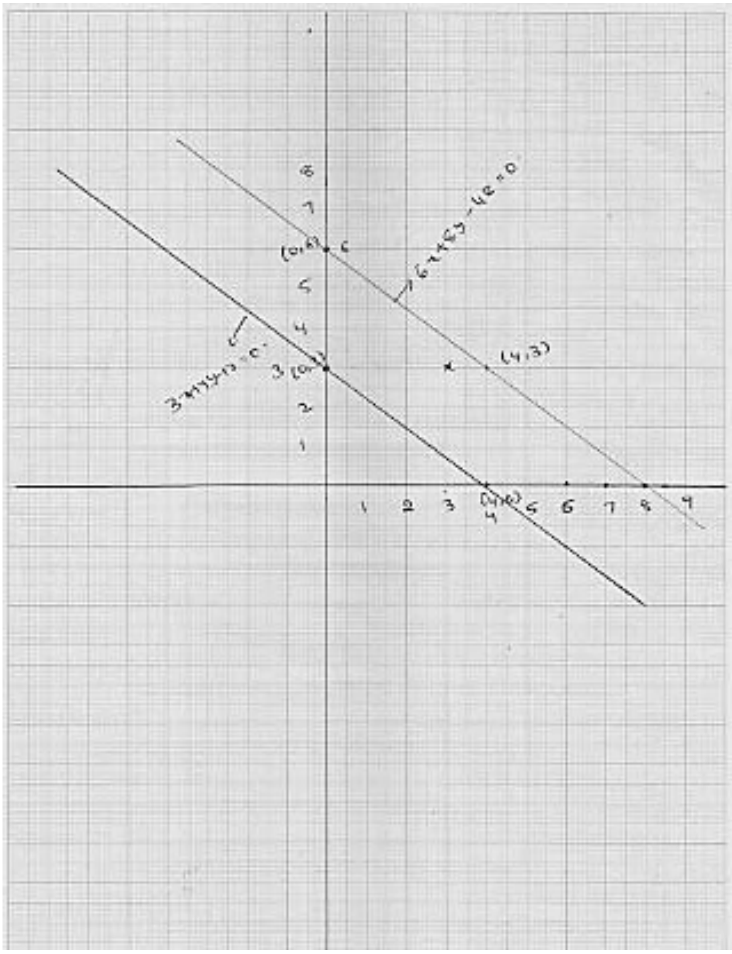
$$x=48-\frac{8y}{6}$$

Putting $y=6,$ we get $x=\frac{48-8\times 6}{6}=0$

Putting $y=4,$ we get $x=\frac{48-8\times 3}{6}=4$

Thus, we have the following table for the points on the line $6x+8y-48=0$

x	0	4
y	6	3



20. Ravish tells his daughter Aarushi, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be". If present ages of Aarushi and Ravish are x and y years respectively, represent this situation algebraically as well as graphically.

Sol:

It is given that seven year ago Harish was seven times a sold as his daughter

$\therefore 7(x-y)=y-7$

$\Rightarrow 7x-49=y-7$

$\Rightarrow 7x-42=y \qquad ....(i)$

It is also given that after three years from now Ravish shall be three times a sold as her daughter

$\therefore 3(x+3)=y+3 \Rightarrow 3x+9=y+3 \Rightarrow 3x+6=y \qquad ......(ii)$

Now, $y=7x-42$ [using (i)]

Putting $x=6$, we get $y=7\times 6-42=0$

Putting $x=5$, we get $y=7\times 5-42=-7$

Thus, we have following table for the points on the
Line $7x-42=y$:

x	6	5
y	0	-7

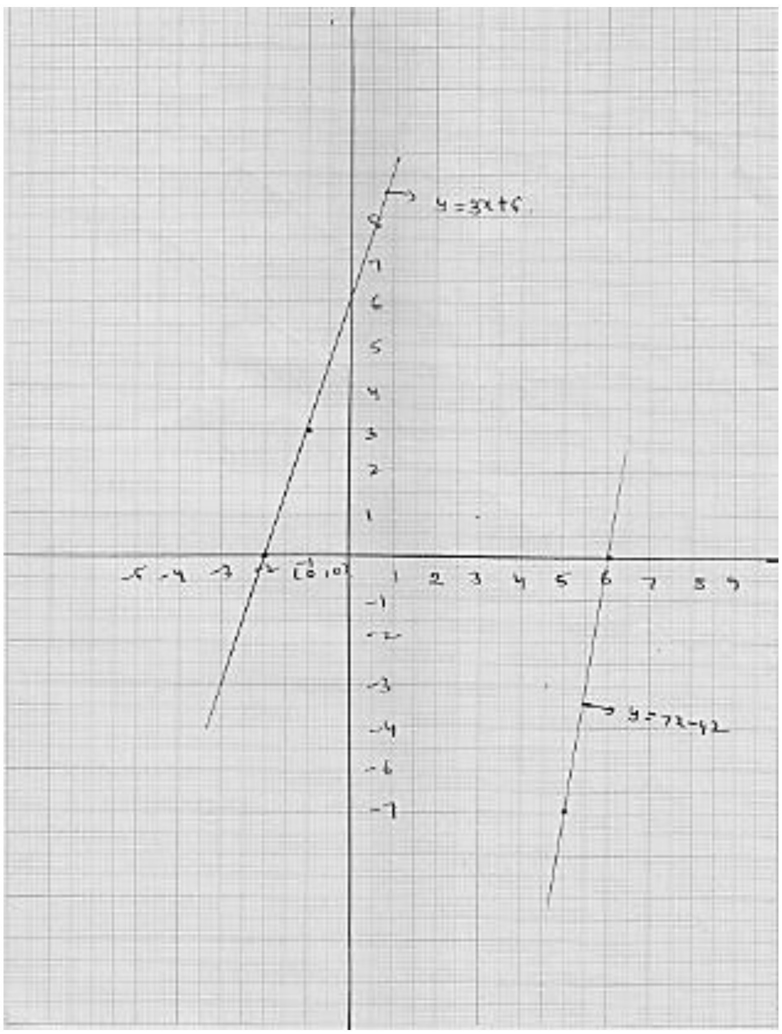
We have,
$y=3x+6$ [using (ii)]

Putting $x=-2,$ we get $y=3\times(-2)+6=0$

Putting $x=-1,$ we get $y=3\times(-1)+6=3$

Thus, we have following table for the points on the
Line $y=3x+6$:

x	-1	-2
y	3	0



21. Aarushi was driving a car with uniform speed of 60 km/h. Draw distance-time graph. From the graph, find the distance travelled by Aarushi in

(i) $2\frac{1}{2}$ Hours (ii) $\frac{1}{2}$ Hour

Sol:

Let x be the time and y be the distance travelled by Aarushi

It is given that speed of car is $60 km/h$

We know that $\text{speed} = \frac{\text{distance}}{\text{speed}}$

$\Rightarrow 60 = \frac{y}{x}$

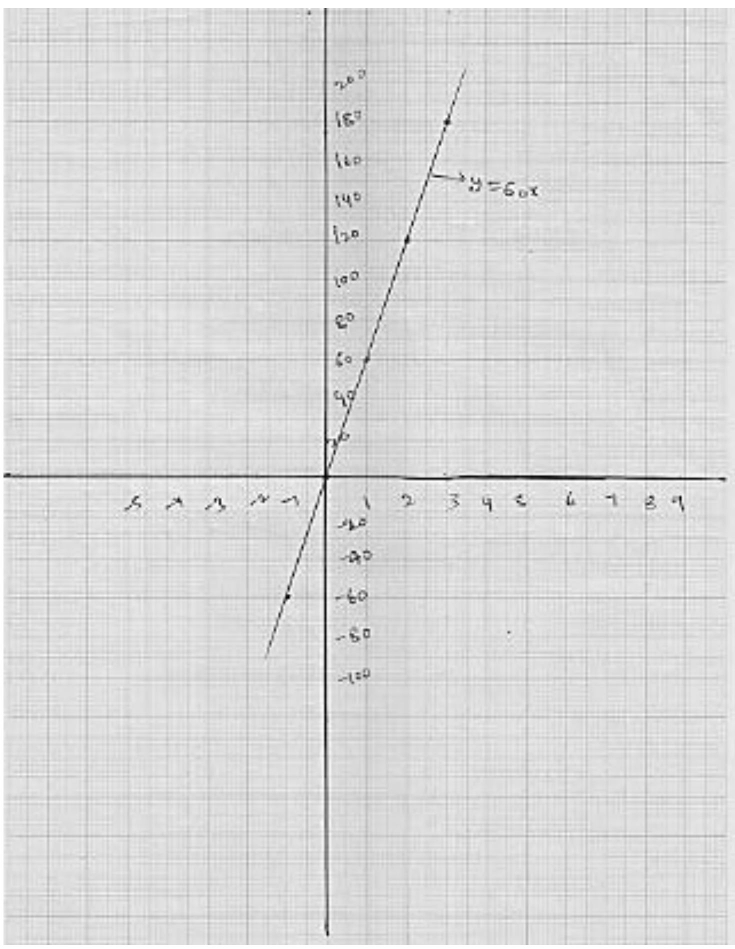
$\Rightarrow y = 60x$

Putting $x = 1$, we get $y = 60$

Putting $x = 2$, we get $y = 120$

Thus, we have the following table for the points on the line $y = 60x$

x	1	2
y	60	120



Exercise – 13.4

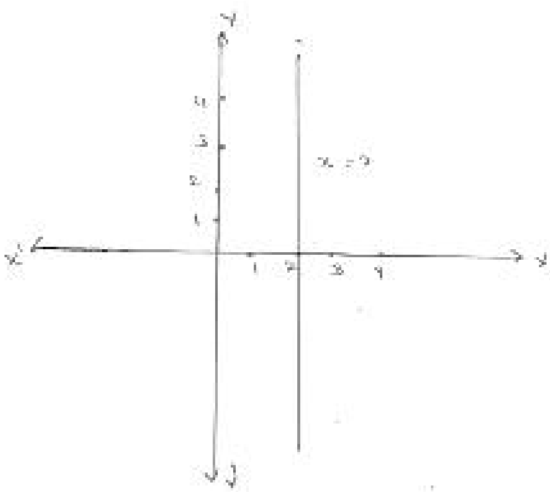
1. Give the geometric representations of the following equations

(a) on the number line (b) on the Cartesian plane:

(i) $x = 2$ (ii) $y + 3 = 0$ (iii) $y = 3$ (iv) $2x + 9 = 0$ (v) $3x - 5 = 0$

Sol:

(i)

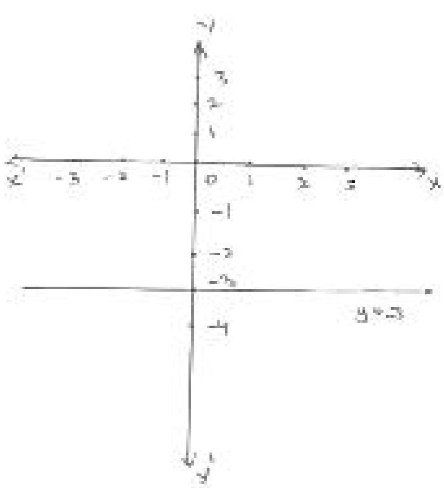


$x = 2$

Point A represents $x = 2$ number line

On Cartesian plane, equation represents all points on $y - axis$ for which $x = 2$

(ii)



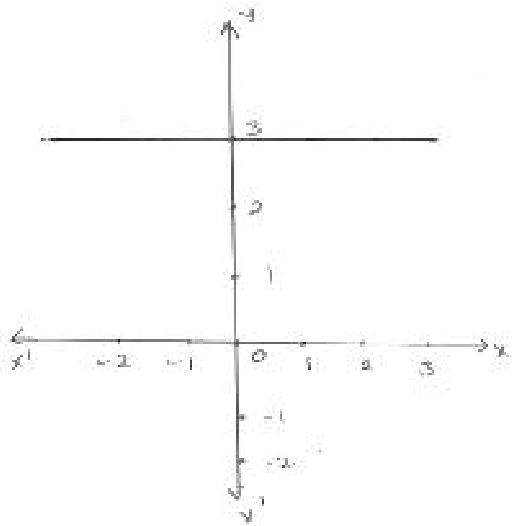
$y+3=0$

$y=-3$

Point A represents -3 on number line

On Cartesian plane equation represents all the points on $x-axis$ for which $y=-3$.

(iii)

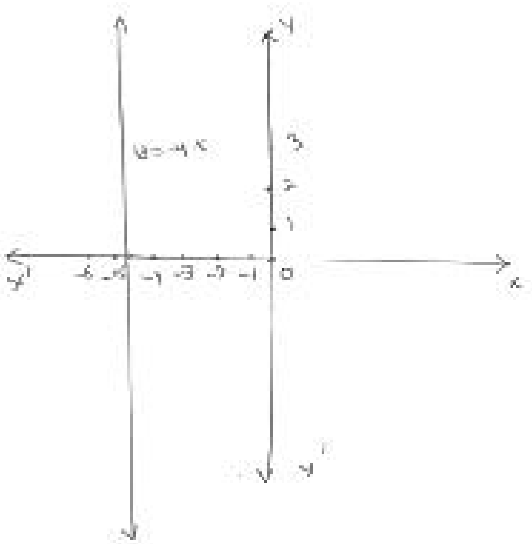


$y=3$.

Point A represents 3 on number line

On Cartesian plane, equation represents all points on $x-axis$ for which $y=3$

(iv)



$2x+9=0$

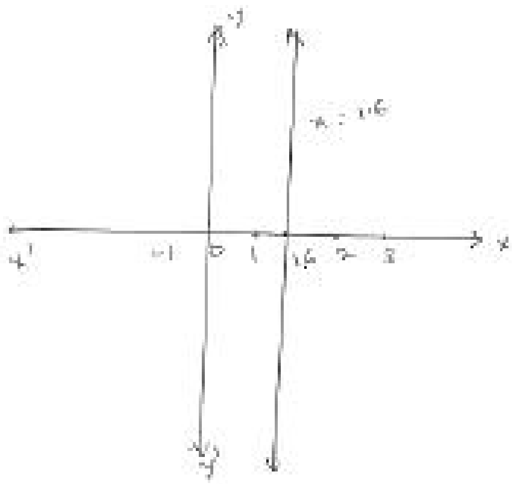
$2x=-9$

$x=\frac{-9}{2}=-4\cdot5$

Point A represents $-4\cdot5$ on number line

On Cartesian plane, equation represents all points on $y-axis$ for which $x=-4\cdot5$

(v)



$3x-5=0$

$3x=5$

$x=\frac{5}{3}=1\cdot6$ (Approx)

Point A represents $1\frac{1}{2}(\text{or})\frac{5}{3}$ on number line

On Cartesian plane, equation represents all points on $y-axis$ for which $x=16$

2. Give the geometrical representation of 2x + 13 = 0 as an equation in
(i) one variable (ii) two variables

Sol:

(i)

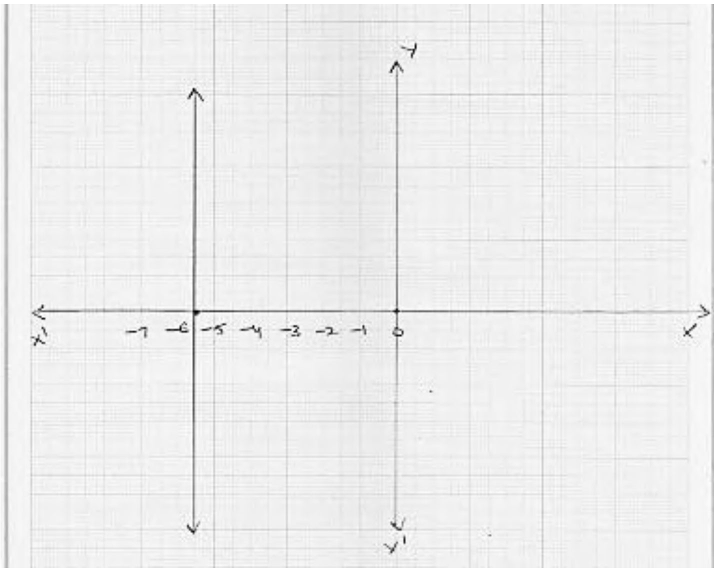
One variable representation of $2x+13=0$

$2x=-13$

$x=\frac{-13}{2}=-6\frac{1}{2}$

Points A represents $\frac{-13}{2}$

(ii)



Two variable representation of $2x+13=0$

$2x+0y+13=0$

$2x+13=0$

$2x=-13$

$x=\frac{-13}{2}$

$x=-6\cdot5$

On Cartesian plane, equation represents all points $y-axis$ for which $x=-6\cdot5$.

3. Solve the equation 3x + 2 = x − 8, and represent the solution on (i) the number line (ii) the Cartesian plane.

Sol:

(i)

$3x+2=x-8$

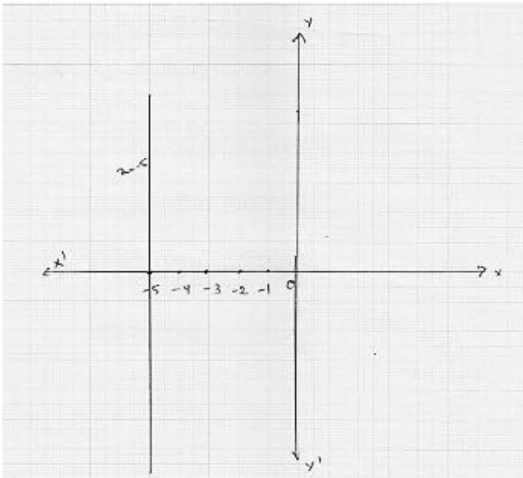
$\Rightarrow 3x-x=8-2$

$\Rightarrow 2x=-10$

$\Rightarrow x=-5$

Points A represents -5 on number line

(ii)



On Cartesian plane, equation represents all points on $y-axis$ for which $x=5$

4. Write the equation of the line that is parallel to x-axis and passing through the point
(i) (0, 3) (ii) (0, -4) (iii) (2, -5) (iv) (3, 4)

Sol:

(i) The equation of the line that is parallel to $x-axis$ and passing through the point $(0,3)$ is $y=3$.

(ii) The equation of the line that is parallel to $x-axis$ and passing through the point $(0,-4)$ is $y=4$.

(iii) The equation of the line that is parallel to $x-axis$ and passing through the point $(2,-5)$ is $y=-5$

(iv) The equation of the line that is parallel to $x-axis$ and passing through the point $(-4,-3)$ is $y=-3$

5. Write the equation of the line that is parallel to y-axis and passing through the point
(i) (4, 0) (ii) (−2, 0) (iii) (3, 5) (iv) (− 4, −3)

Sol:

(i) The equation of the line that is parallel to $y-axis$ and passing through $(4,0)$ will be $x=4$

(ii) The equation of the line that is parallel to $y-axis$ and passing through $(-2,0)$ will be $x=-2$

(iii) The equation of the line that is parallel to $y-axis$ and passing through $(3,5)$ will be $x=3$

(iv) The equation of the line that is parallel to $y-axis$ and passing through $(-4,-3)$ will be $x=-4$.

Exercise – 14.1

1. Three angles of a quadrilateral are respectively equal to 110°, 50° and 40°. Find its fourth angles.

Sol:

Given

Three angles are $110°, 50°$ and $40°$

Let fourth angle be x

We have,

Sum of all angles of a quadrilaterals $= 360°$

$110° + 50° + 40° + x° = 360°$

$\Rightarrow x = 360° - 200°$

$\Rightarrow x = 160°$

Required fourth angle $= 160°$.

2. In a quadrilateral ABCD, the angles A, B, C and D are in the ratio 1 : 2 : 4 : 5. Find the measure of each angles of the quadrilateral.

Sol:

Let the angles of the quadrilateral be

$A = x, B = 2x, C = 4x$ and $D = 5x$ then,

$A + B + C + D = 360°$

$\Rightarrow x + 2x + 4x + 5x = 360°$

$\Rightarrow 12x = 360°$

$\Rightarrow x = \frac{360°}{12}$

$\Rightarrow x = 30°$

$\therefore A = x = 30°$

$B = 2x = 60°$

$C = 4x = 30°(4) = 120°$

$D = 5x = 5(30°) = 150°$

3. In a quadrilateral ABCD, CO and DO are the bisectors of ∠C and ∠D respectively. Prove that ∠COD = $\frac{1}{2}(\angle A + \angle B)$.

Sol:

In ΔDOC

$\angle 1 + \angle COD + \angle 2 = 180°$ [Angle sum property of a triangle]

$\Rightarrow \angle COD = 180 - \angle 1 - \angle 2$

$\Rightarrow \angle COD = 180 - \angle 1 + \angle 2$

$\Rightarrow \angle COD = 180 - \left[\frac{1}{2}\angle C + \frac{1}{2}\angle D\right]$

[$\because$ OC and OD are bisectors of $\angle C$ and $\angle D$ represents]

$\Rightarrow \angle COD = 180 - \frac{1}{2}(\angle C + \angle D)]$ .....(1)

In quadrilateral $ABCD$

$\angle A + \angle B + \angle C + \angle D = 360°$

$\angle C + \angle D = 360 - \angle A + \angle B$ ......(2) [Angle sum property of quadrilateral]

Substituting (ii) in (i)

$\Rightarrow \angle COD = 180 - \frac{1}{2}(360 - (\angle A + \angle B))$

$\Rightarrow \angle COD = 180 - 180 + \frac{1}{2}(\angle A + \angle B)$

$\Rightarrow \angle COD = \frac{1}{2}(\angle A + \angle B)$

4. The angles of a quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral.

Sol:

Let the common ratio between the angle is 'x' so the angles will be $3x, 5x, 9x$ and $13x$ respectively

Since the sum of all interior angles of a quadrilateral is $360°$

$\therefore 3x + 5x + 9x + 13x = 360°$

$\Rightarrow 30x = 360°$

$\Rightarrow x = 12°$

Hence, the angles are

$3x = 3 \times 12 = 36°$

$5x = 5 \times 12 = 60°$

$9x = 9 \times 12 = 108°$

$13x = 13 \times 12 = 156°$

Exercise – 14.2

1. Two opposite angles of a parallelogram are (3x – 2)° and (50 – x)°. Find the measure of each angle of the parallelogram.

Sol:

We know that

Opposite sides of a parallelogram are equal

$\therefore 3x-2=50-x$

$\Rightarrow 3x+x=50+2$

$\Rightarrow 4x=52$

$\Rightarrow x=13°$

$\therefore (3x-2)°=(3\times13-2)=37°$

$(50-x)°=(50-13°)=37°$

Adjacent angles of a parallelogram are supplementary

$\therefore x+37=180°$

$\therefore x=180°-37°=143°$

Hence, four angles are : $37°,143°,37°,143°$

2. If an angle of a parallelogram is two-third of its adjacent angle, find the angles of the parallelogram.

Sol:

Let the measure of the angle be x

$\therefore$ The measure of the angle adjacent is $\frac{2x}{3}$

We know that the adjacent angle of a parallelogram is supplementary

Hence $x+\frac{2x}{3}=180°$

$2x+3x=540°$

$\Rightarrow 5x=540°$

$\Rightarrow x=108°$

Adjacent angles are supplementary

$\Rightarrow x+108°=180°$

$\Rightarrow x=180°-108°=72°$

$\Rightarrow x=72°$

Hence, four angles are : $180°,72°,108°,72°$

3. Find the measure of all the angles of a parallelogram, if one angle is 24° less than twice the smallest angle.

Sol:

Let the smallest angle be x

Then, the other angle is $(3x-24)$

Now, $x+2x-24=180°$

$3x-24=180°$

$\Rightarrow 3x=180+24$

$\Rightarrow 3x=204°$

$\Rightarrow x=\frac{204}{3}=68°$

$\Rightarrow x=68°$

$\Rightarrow 2x-24°=2\times 68°-24°=136°-24°=112°$

Hence four angles are $68°, 112°, 68°, 112°$.

4. The perimeter of a parallelogram is 22 cm. If the longer side measures 6.5 cm what is the measure of the shorter side?

Sol:

Let the shorter side be x

$\therefore$ Perimeter $=x+6.5+6\cdot 5+x$ [sum of all sides]

$22=2(x+6\cdot 5)$

$11=x+6\cdot 5$

$\Rightarrow x=11-6\cdot 5=4\cdot 5\,\text{cm}$

$\therefore$ Shorter side $=4\cdot 5\text{cm}$

5. In a parallelogram ABCD, ∠D = 135°, determine the measures of ∠A and ∠B.

Sol:

In a parallelogram $ABCD$

Adjacent angles are supplementary

So, $\angle D+\angle C=180°$

$135°+\angle C=180° \Rightarrow \angle C=180°-135°$

$\angle C=45°$

In a parallelogram opposite sides are equal

$\angle A=\angle C=45°$

$\angle B=\angle D=135°$

6. ABCD is a parallelogram in which ∠A = 70°. Compute ∠B, ∠C and ∠D.

Sol:

In a parallelogram ABCD.

$\angle A = 70°$

$\angle A = \angle B = 180°$ [∵ Adjacent angles supplementary]

$70° + \angle B = 180°$ $[\because \angle A = 70°]$

$\angle B = 180° - 70°$

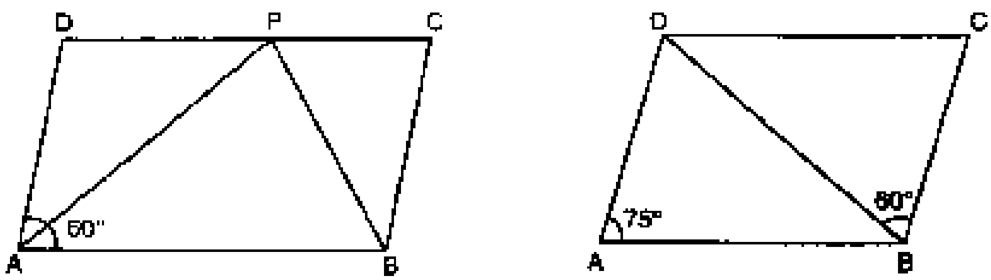
$= 110°$

In a parallelogram opposite sides are equal

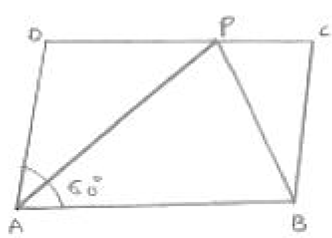
$\angle A = \angle C = 70°$

$\angle B = \angle D = 110°$

7. In Fig., below, ABCD is a parallelogram in which ∠A = 60°. If the bisectors of ∠A and ∠B meet at P, prove that AD = DP, PC = BC and DC = 2AD.



Sol:



AP bisects $\angle A$

Then, $\angle AP = \angle PAB = 30°$

Adjacent angles are supplementary

Then, $\angle A + \angle B = 180°$

$\angle B + 60° = 180°$ $\angle A = 60°$

$\angle B = 180° - 60°$

$\angle B = 120°$

BP bisects $\angle B$

Then, $\angle PBA \quad \angle PBC = 30°$

$\angle PAB = \angle APD = 30°$ [Alternative interior angles]

$\therefore AD = DP$ [∵ Sides opposite to equal angles are in equal length]

Similarly

$\angle PBA = \angle BPC = 60°$ [Alternative interior angle]

$\therefore PC = BC$

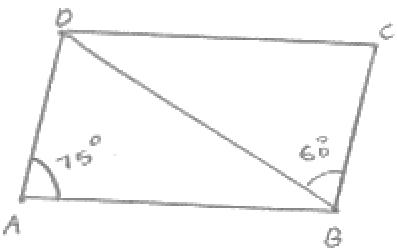
$DC = DP + PC$

$DC = AD + BC$ $[\because DP = AD, PC = BC]$

$DC = 2AD$ [$\because AD = BC$ Opposite sides of a parallelogram are equal].

8. In Fig. below, ABCD is a parallelogram in which ∠DAB = 75° and ∠DBC = 60°. Compute ∠CDB and ∠ADB.

Sol:



To find $\angle CDB$ and $\angle ADB$

$\angle CBD = \angle ABD = 60°$ [Alternative interior angle $AD \parallel BC$ and BD is the transversal]

In a parallelogram $ABCD$

$\angle A = \angle C = 75°$ [$\because$ Opposite side angles of a parallelogram are equal]

In $\angle BDC$

$\angle CBD + \angle C + \angle CDB = 180°$ [Angle sum property]

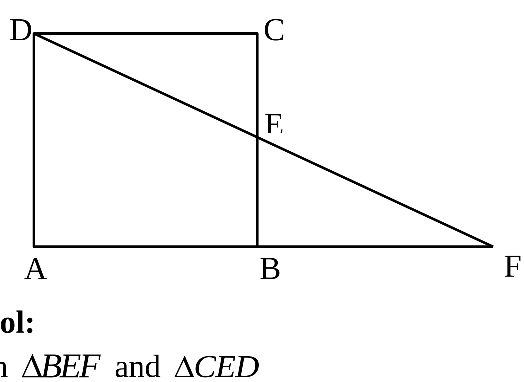
$\Rightarrow 60° + 75° + \angle CDB = 180°$

$\Rightarrow \angle CDB = 180° - (60° + 75°)$

$\Rightarrow \angle CDB = 45°$

Hence $\angle CDB = 45°, \angle ADB = 60°$

9. In below fig. ABCD is a parallelogram and E is the mid-point of side BC. If DE and AB when produced meet at F, prove that AF = 2AB.

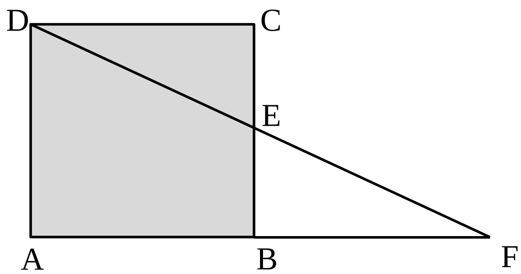


Sol:

In ΔBEF and ΔCED

$\angle BEF = \angle CED$ [Verified opposite angle]

$BE = CE$ [$\because$ E is the mid-point of BC]



$\angle EBF = \angle ECD$ [$\therefore$ Alternate interior angles are equal]

$\because \nabla BEF \cong \Delta CED$ [Angle side angle congruence]

$\because BF = CD$ [Corresponding Parts of Congruent Triangles]

$AF = AB + AF$

$AF = AB + AB$

$AF = 2AB$

10. Which of the following statements are true (T) and which are false (F)?

(i) In a parallelogram, the diagonals are equal.

(ii) In a parallelogram, the diagonals bisect each other.

(iii) In a parallelogram, the diagonals intersect each other at right angles.

(iv) In any quadrilateral, if a pair of opposite sides is equal, it is a parallelogram.

(v) If all the angles of a quadrilateral are equal, it is a parallelogram.

(vi) If three sides of a quadrilateral are equal, it is a parallelogram.

(vii) If three angles of a quadrilateral are equal, it is a parallelogram.

(viii) If all the sides of a quadrilateral are equal it is a parallelogram.

Sol:

(i) False

(ii) True

(iii) False

(iv) False

(v) True

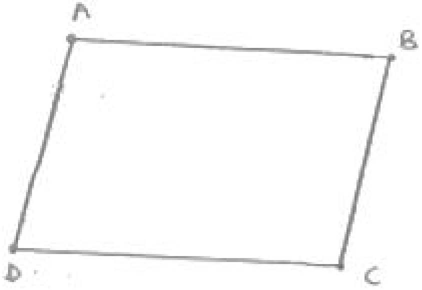
(vi) False

(vii) False

(viii) True

Exercise – 14.3

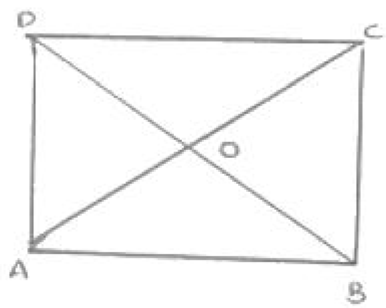
1. In a parallelogram ABCD, determine the sum of angles ∠C and ∠D.
Sol:



$\angle C$ and $\angle D$ are consecutive interior angles on the same side of the transversal CD
$\therefore \angle C + \angle D = 180°$

2. In a parallelogram ABCD, if ∠B = 135°, determine the measures of its other angles.
Sol:
Given $\angle B = 135°$
$ABCD$ is a parallelogram
$\therefore \angle A = \angle C, \angle B = \angle D$ and $\angle A + \angle B = 180°$
$\angle A + \angle B = 180°$
$\angle A = 45°$
$\Rightarrow \angle A = \angle C = 45°$ and $\angle B = \angle C = 135°$

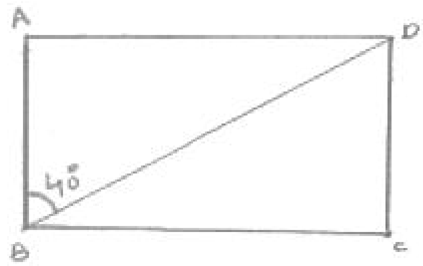
3. ABCD is a square. AC and BD intersect at O. State the measure of ∠AOB.
Sol:



Since, diagonals of square bisect each other at right angle
$\therefore \angle ADB = 90°$

4. ABCD is a rectangle with ∠ABD = 40°. Determine ∠DBC.

Sol:



We have,

$\angle ABC = 90°$

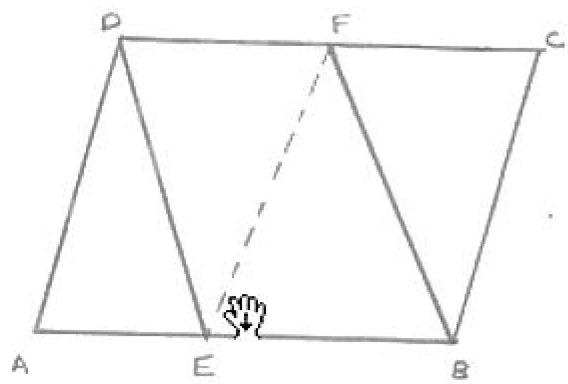
$\Rightarrow \angle ABD + \angle DBC = 90°$ $\quad [\because \angle ABD = 40°]$

$\Rightarrow 40° + \angle DBC = 90°$

$\therefore \angle DBC = 50°$

5. The sides AB and CD of a parallelogram ABCD are bisected at E and F. Prove that EBFD is a parallelogram.

Sol:



Since ABCD is a parallelogram

$\therefore AB \parallel DC$ and $AB = DC$

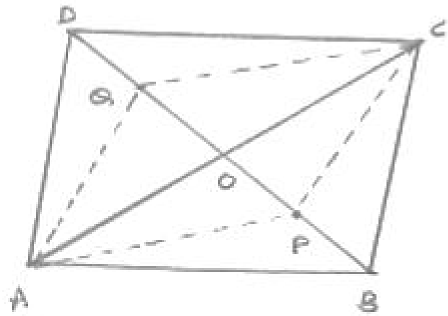
$\Rightarrow EB \parallel DF$ and $\frac{1}{2}AB = \frac{1}{2}DC$

$\Rightarrow EB \parallel DF$ and $EB = DF$

$EBFD$ is a parallelogram

6. P and Q are the points of trisection of the diagonal BD of a parallelogram ABCD. Prove that CQ is parallel to AP. Prove also that AC bisects PQ.

Sol:



We know that, diagonals of a parallelogram bisect each other

$\therefore OA = OC$ and $OB = OD$

Since P and Q are point of intersection of BD

$\therefore BP = PQ = QD$

Now, $OB = OD$ and $BP = QD$

$\Rightarrow OB - BP = OD - QD$

$\Rightarrow OP = OQ$

Thus in quadrilateral APCQ, we have

$OA = OC$ and $OP = OQ$

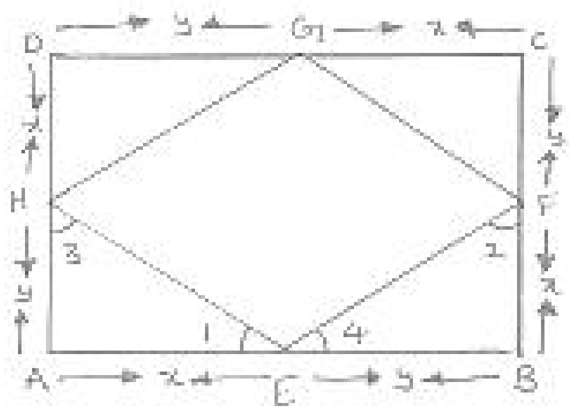
$\Rightarrow$ diagonals of quadrilateral APCQ bisect each other

$\therefore APCQ$ is a parallelogram

Hence $AP \parallel CQ$

7. ABCD is a square E, F, G and H are points on AB, BC, CD and DA respectively, such that AE = BF = CG = DH. Prove that EFGH is a square.

Sol:



We have

$AE = BF = CG = DH = x(\text{say})$

$\therefore BE = CF = DG = AH = y(\text{say})$

In $\Delta's\ AEH$ and BEF, we have

$AE = BF$

$\angle A = \angle B$

And $AH = BE$

So, by SAS configuration criterion, we have

$\Delta AEH \cong \Delta BFE$

$\Rightarrow \angle 1 = \angle 2$ and $\angle 3 = \angle 4$

But $\angle 1 + \angle 3 = 90°$ and $\angle 2 + \angle 4 = 90°$

$\Rightarrow \angle 1 + \angle 3 + \angle 2 + \angle 4 = 90° + 90°$

$\Rightarrow \angle 1 + \angle 4 + \angle 1 + \angle 4 = 180°$

$\Rightarrow 2(\angle 1 + \angle 4) = 180°$

$\Rightarrow \angle 1 + \angle 4 = 90°$

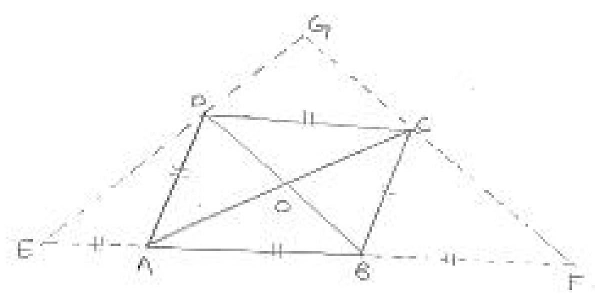
$HEF = 90°$

Similarly we have $\angle F = \angle G = \angle H = 90°$

Hence, $EFGH$ is a square

8. ABCD is a rhombus, EABF is a straight line such that EA = AB = BF. Prove that ED and FC when produced meet at right angles.

Sol:



We know that the diagonals of a rhombus are perpendicular bisector of each other

$\therefore OA = OC, OB = OD, \angle AOD = \angle COD = 90°$

And $\angle AOB = \angle COB = 90°$

In ΔBDE, A and O are mid points of BE and BD respectively

$OA \parallel DE$

$OC \parallel DG$

In ΔCFA, B and O are mid points of AF and AC respectively

$\therefore OB \parallel CF$

$OD \parallel GC$

Thus, in quadrilateral $DOCG$, we have

$OC \parallel DG$ and $OD \parallel GC$

$\Rightarrow DOCG$ is a parallelogram

$\angle DGC = \angle DOC$

$\angle DGC = 90°$

9. ABCD is a parallelogram, AD is produced to E so that DE = DC and EC produced meets AB produced in F. Prove that BF = BC.

Sol:

Draw a parallelogram $ABCD$ with AC and BD intersecting at O

Produce AD to E such that $DE = DC$

Join EC and produce it to meet AB produced at F.

In ΔDCE ,

$\therefore \angle DCE = \angle DEC$ ....CD [In a triangle, equal sides have equal angles opposite]

$AB \parallel CD$ (Opposite sides of the parallelogram are parallel)

$\therefore AE \parallel CD$ $(AB$ Lies on $AF)$

$AF \parallel CD$ and EF is the transversal.

$\therefore \angle DCE = \angle BFC$ .....(2) [Pair of corresponding angles]

From (1) and (2), we get

$\angle DEC = \angle BFC$

In ΔAFE,

$\angle AFE = \angle AEF$ $(\angle DEC = \angle BFC)$

$\therefore AE = AF$ (In a triangle, equal angles have equal sides opposite to them)

$\Rightarrow AD + DE = AB + BF$

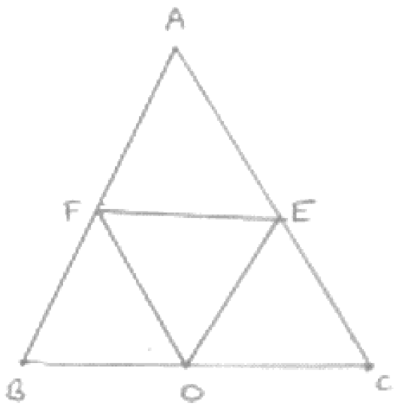
$\Rightarrow BC + AB = AB + BF$ $[\because AD = BC, DE = CD$ and $CD = AB, AB = DE]$

$\Rightarrow BC = BF$.

Exercise – 14.4

1. In a ΔABC, D, E and F are, respectively, the mid-points of BC, CA and AB. If the lengths of side AB, BC and CA are 7 cm, 8 cm and 9 cm, respectively, find the perimeter of ΔDEF.

Sol:



Given that

$AB = 7cm, BC = 8cm, AC = 9cm$.

In ΔABC

$\therefore F$ and E are the midpoint of AB and AC

$\therefore EF = \frac{1}{2} BC$ [Mid-points theorem]

Similarly

$DF = \frac{1}{2} AC, DE = \frac{1}{2} AB$

Perimeter of $\Delta DEF = DE + EF + DF$

$= \frac{1}{2} AB + \frac{1}{2} BC \frac{1}{2} AC$

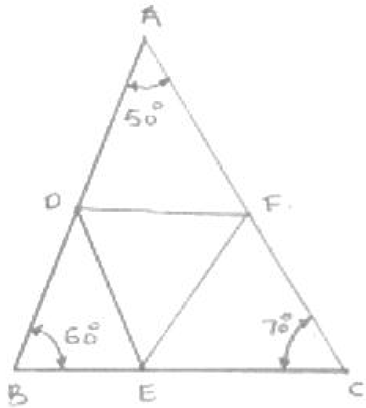
$= \frac{1}{2} \times 7 + \frac{1}{2} \times 8 + \frac{1}{2} \times 9$

$= 3 \cdot 5 + 4 + 4 \cdot 5 = 12cm$

$\therefore$ Perimeter of $\Delta DEF = 12cm$

2. In a triangle ∠ABC, ∠A = 50°, ∠B = 60° and ∠C = 70°. Find the measures of the angles of the triangle formed by joining the mid-points of the sides of this triangle.

Sol:



In ΔABC
D and E are midpoints of AB and BC
By midpoint theorem

$\therefore DE \parallel AC, DE = \frac{1}{2}AC.$

F is the midpoint of AC

Then, $DE = \frac{1}{2}AC = CF$

In a quadrilateral DECF

$DE \parallel AC, DE = CF$

Hence $DECF$ is a parallelogram

$\therefore \angle C = \angle D = 70°$ [Opposite sides of parallelogram]

Similarly

$BEFD$ is a parallelogram, $\angle B = \angle F = 60°$

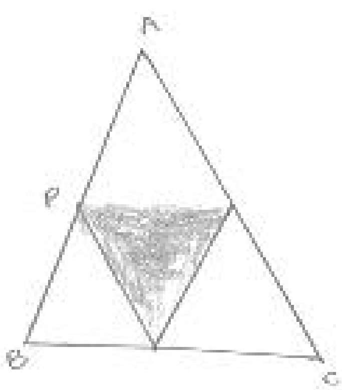
$ADEF$ is a parallelogram, $\angle A = \angle E = 50°$

$\therefore$ Angles of ΔDEF

$\angle D = 70°, \angle E = 50°, \angle F = 60°$

3. In a triangle, P, Q and R are the mid-points of sides BC, CA and AB respectively. If AC = 21 cm, BC = 29 cm and AB = 30 cm, find the perimeter of the quadrilateral ARPQ.

Sol:



In ΔABC
R and P are the midpoint of AB and BC

$\therefore RP \parallel AC, RP = \frac{1}{2}AC$ [By midpoint theorem]

In a quadrilateral
[A pair of side is parallel and equal]

$RP \parallel AQ, RP = AQ$

$\therefore RPQA$ is a parallelogram

$AR = \frac{1}{2}AB = \frac{1}{2}\times 30 = 15cm$

$AR = QP = 15$ [$\because$ Opposite sides are equal]

$\Rightarrow RP = \frac{1}{2}AC = \frac{1}{2}\times 21 = 10\cdot 5cm$ [$\because$ Opposite sides are equal]

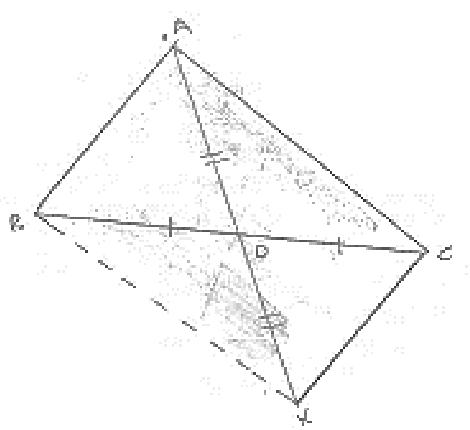
Now,
Perimeter of $ARPQ = AR + QP + RP + AQ$

$= 15 + 15 + 10\cdot 5 + 10\cdot 5$

$= 51cm$

4. In a ΔABC median AD is produced to X such that AD = DX. Prove that ABXC is a parallelogram.

Sol:



In a quadrilateral $ABXC$, we have

$AD = DX$ $[Given]$

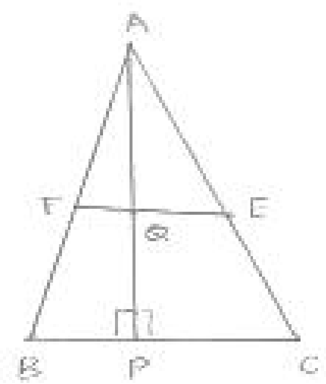
$BD = DC$ $[Given]$

So, diagonals AX and BC bisect each other

$\therefore ABXC$ is a parallelogram

5. In a ΔABC, E and F are the mid-points of AC and AB respectively. The altitude AP to BC intersects FE at Q. Prove that AQ = QP.

Sol:



In ΔABC

E and F are midpoints of AB and AC

$\therefore EF \parallel FE, \frac{1}{2}BC = FE$ [∵ By mid-point theorem]

In ΔABP

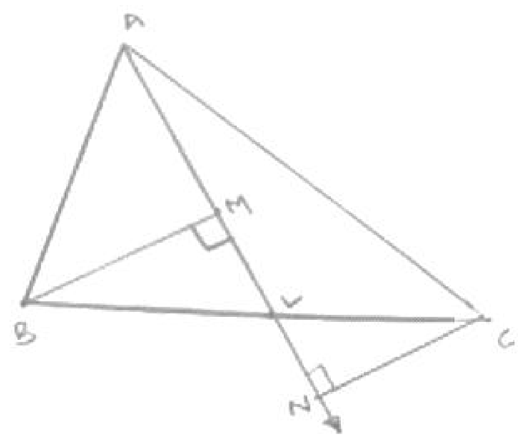
F is the midpoint of AB and $FQ \parallel BP$ $[\because EF \parallel BC]$

$\therefore Q$ is the midpoint of AP [By converse of midpoint theorem]

Hence, $AQ = QP$

6. In a ΔABC, BM and CN are perpendiculars from B and C respectively on any line passing through A. If L is the mid-point of BC, prove that ML = NL.

Sol:



In B

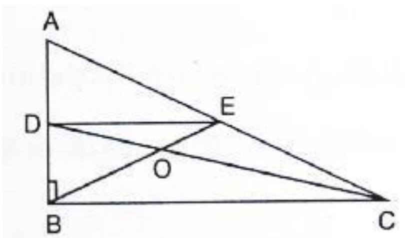
Given that

In ΔBLM and ΔCLN

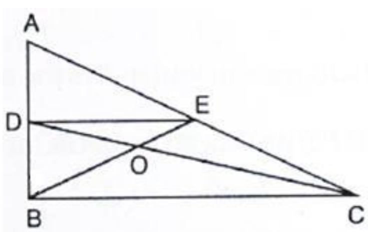
$\angle BML = \angle CNL = 90°$

$BL = CL$ [L is the midpoint of BC]
$\angle MLB = \angle NLC$ [vertically opposite angle]
$\therefore \Delta BLM = \Delta CLN$ $(A \cdot L \cdot A \cdot S)$
$\therefore LM = LN$ [Corresponding plats parts of congruent triangles]

7. In Fig. below, triangle ABC is right-angled at B. Given that AB = 9 cm, AC = 15 cm and D, E are the mid-points of the sides AB and AC respectively, calculate
(i) The length of BC (ii) The area of ΔADE.



Sol:



In right $\Delta ABC, \angle B = 90°$
By using Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$
$$\Rightarrow \quad 15^2 = 9^2 + BC^2$$
$$\Rightarrow \quad BC = \sqrt{15^2 - 9^2}$$
$$\Rightarrow \quad BC = \sqrt{225 - 81}$$
$$\Rightarrow \quad BC = \sqrt{144}$$
$$= 12cm$$

In ΔABC
D and E are midpoints of AB and AC

$\therefore DE \parallel BC, DE = \frac{1}{2}BC$ [By midpoint theorem]

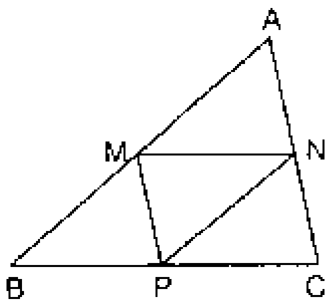
$AD = OB = \frac{AB}{2} = \frac{9}{2} = 4 \cdot 5cm$ [$\because D$ is the midpoint of AB]

$DE = \frac{BC}{2} = \frac{12}{2} = 6cm$

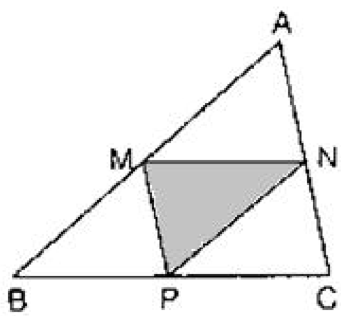
Area of $\Delta ADE = \frac{1}{2} \times AD \times DE$

$= \frac{1}{2} \times 4 \cdot 5 \times 6 = 13 \cdot 5 cm^2$

8. In Fig. below, M, N and P are the mid-points of AB, AC and BC respectively. If MN = 3 cm, NP = 3.5 cm and MP = 2.5 cm, calculate BC, AB and AC.



Sol:



Given $MN = 3cm, NP = 3 \cdot 5cm \text{ and } MP = 2 \cdot 5cm$

To find BC, AB and AC

In ΔABC

M and N are midpoints of AB and AC

$\therefore MN = \frac{1}{2} BC, MN \parallel BC$ [By midpoint theorem]

$\Rightarrow 3 = \frac{1}{2} BC$

$\Rightarrow 3 \times 2 = BC$

$\Rightarrow BC = 6cm$

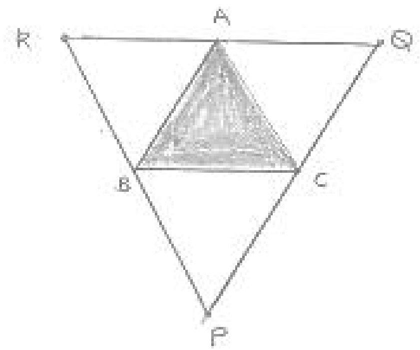
Similarly

$AC = 2MP = 2(2 \cdot 5) = 5cm$

$AB = 2NP = 2(3 \cdot 5) = 7cm$

9. ABC is a triangle and through A, B, C lines are drawn parallel to BC, CA and AB respectively intersecting at P, Q and R. Prove that the perimeter of ΔPQR is double the perimeter of ΔABC.

Sol:



Clearly ABCQ and ARBC are parallelograms.

$\therefore BC = AQ \text{ and } BC = AR$

$\Rightarrow AQ = AR$

$\Rightarrow A$ is the midpoint of QR.

Similarly B and C are the midpoints of PR and PQ respectively

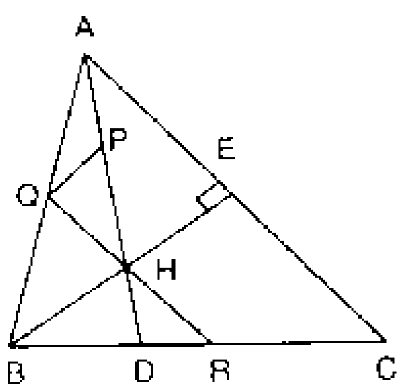
$\therefore AB = \frac{1}{2}PQ, BC = \frac{1}{2}QR, CA = \frac{1}{2}PR$

$\Rightarrow PQ = 2AB, QR = 2BC$ and $PR = 2CA$

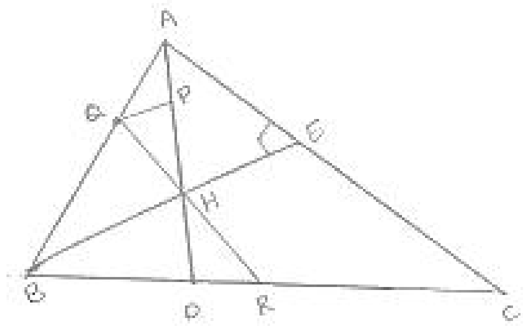
$\Rightarrow PQ + QR + RP = 2(AB + BC + CA)$

$\Rightarrow$ Perimeter of $\Delta PQR = 2$ [Perimeter of ΔABC]

10. In Fig. below, BE ⊥ AC. AD is any line from A to BC intersecting BE in H. P, Q and R are respectively the mid-points of AH, AB and BC. Prove that ∠PQR = 90°.



Sol:



Given

$BE \perp AC$ and P, Q and R are respectively midpoint of AH, AB and BC

To prove:

$\angle PQRD = 90°$

Proof: In $\Delta ABC, Q$ and R are midpoints of AB and BC respectively

$\therefore QR \parallel AC$ $......(i)$

In $\Delta ABH, Q$ and P are the midpoints of AB and AH respectively

$\therefore QP \parallel BH$

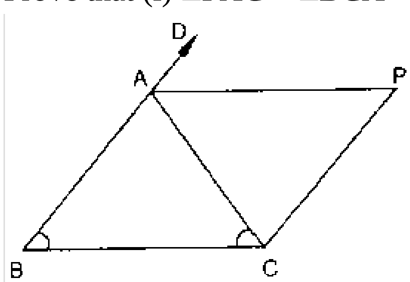
$\Rightarrow QP \parallel BE$ $......(ii)$

But, $AC \perp BE$ $\therefore$ from equation (i) and equation (ii) we have

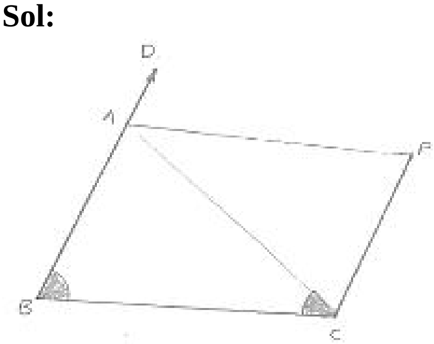
$QP \perp QR$

$\Rightarrow \angle PQR = 90°,$ hence proved.

11. In Fig. below, AB = AC and CP || BA and AP is the bisector of exterior ∠CAD of ΔABC. Prove that (i) ∠PAC = ∠BCA (ii) ABCP is a parallelogram.

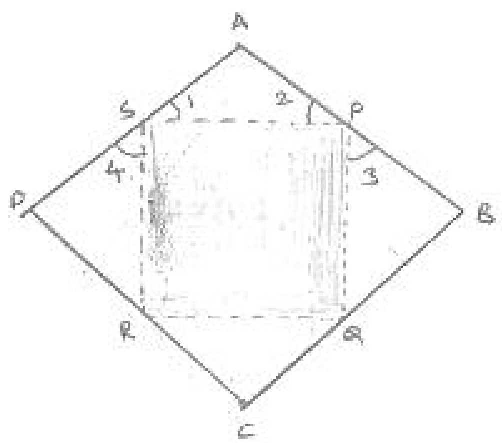


Sol:



Given
$AB = AC$ and $CD \parallel BA$ and AP is the bisector of exterior
$\angle CAD$ of ΔABC
To prove:
(i) $\angle PAC = \angle BCA$
(ii) $ABCD$ is a parallelogram
Proof:
(i) We have,
AB = AC
$\Rightarrow \angle ACB = \angle ABC$ [Opposite angles of equal sides of triangle are equal]
Now, $\angle CAD = \angle ABC + \angle ACB$
$\Rightarrow \angle PAC + \angle PAD = 2\angle ACB \ (\because \angle PAC = \angle PAD)$
$\Rightarrow 2\angle PAC = 2\angle ACB$
$\Rightarrow \angle PAC = \angle ACB$
(ii) Now,
$\angle PAC = \angle BCA$
$\Rightarrow AP \parallel BC$
And, $CP \parallel BA$ [Given]
$\therefore ABCD$ is a parallelogram

12. ABCD is a kite having AB = AD and BC = CD. Prove that the figure formed by joining the mid-points of the sides, in order, is a rectangle.
Sol:



Given,
A kite $ABCD$ having $AB = AD$ and $BC = CD \cdot P, Q, R, S$ are the midpoint of sides AB, BC, CD and DA respectively PQ, QR, RS and spare joined
To prove:
$PQRS$ is a rectangle

Proof:

In $\Delta ABC, P$ and Q are the midpoints of AB and BC respectively.

$\therefore PQ \parallel AC$ and $PQ = \frac{1}{2}AC$ ....(i)

In $\Delta ADC, R\ and\ S$ are the midpoint of CD and AD respectively.

$\therefore RS \parallel AC$ and $RS = \frac{1}{2}AC$ .....(ii)

From (i) and (ii), we have

$PQ \parallel RS$ and $PQ = RS$

Thus, in quadrilateral PQRS, a pair of opposite sides are equal and parallel. So PQRS is a parallelogram. Now, we shall prove that one angle of parallelogram PQRS it is a right angle

Since $AB = AD$

$\Rightarrow \frac{1}{2}AB = AD\left(\frac{1}{2}\right)$

$\Rightarrow AP = AS$ ...(iii) [$\because P$ and S are the midpoints of B and AD respectively]

$\Rightarrow \angle 1 = \angle 2$ ....(iv)

Now, in ΔPBQ and ΔSDR, we have

$PB = SD$ [$\because AD = AB \Rightarrow \frac{1}{2}AD = \frac{1}{2}AB$]

$BQ = DR$ $\therefore PB = SD$

And $PQ = SR$ [$\because PQRS$ is a parallelogram]

So by SSS criterion of congruence, we have

$\Delta PBQ \cong \Delta SOR$

$\Rightarrow \angle 3 = \angle 4$ $[CPCT]$

Now, $\angle 3 + \angle SPQ + \angle 2 = 180°$

And $\angle 1 + \angle PSR + \angle 4 = 180°$

$\therefore \angle 3 + \angle SPQ + \angle 2 = \angle 1 + \angle PSR + \angle 4$

$\Rightarrow \angle SPQ = \angle PSR$ $(\angle 1 = \angle 2$ and $\angle 3 = \angle 4)$

Now, transversal PS cuts parallel lines SR and PQ at S and P respectively. $\therefore \angle SPQ + \angle PSR = 180°$

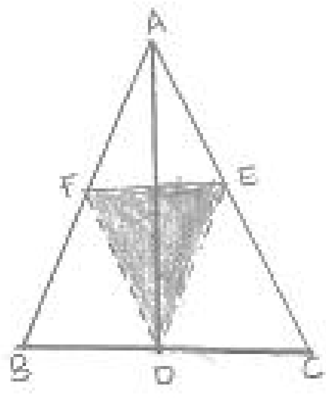
$\Rightarrow 2\angle SPQ = 180° = \angle SPQ = 90°$ $[\because \angle PSR = \angle SPQ]$

Thus, PQRS is a parallelogram such that $\angle SPQ = 90°$

Hence, PQRS is a parallelogram.

13. Let ABC be an isosceles triangle in which AB = AC. If D, E, F be the mid-points of the sides BC, CA and A B respectively, show that the segment AD and EF bisect each other at right angles.

Sol:



Since D, E and F are the midpoints of sides

BC, CA and AB respectively

$\therefore AB \parallel DF$ and $AC \parallel FD$

$AB \parallel DF$ and $AC \parallel FD$

$ABDF$ is a parallelogram

$AF = DE$ and $AE = DF$

$\frac{1}{2}AB = DE$ and $\frac{1}{2}AC = DF$

$DE = DF$ $(\because AB = AC)$

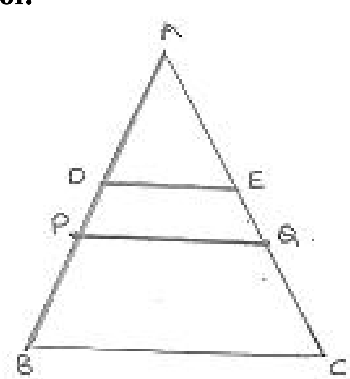
$AE = AF = DE = DF$

$ABDF$ is a rhombus

$\Rightarrow AD$ and FE bisect each other at right angle.

14. ABC is a triangle. D is a point on AB such that AD = $\frac{1}{4}$ AB and E is a point on AC such that AE = $\frac{1}{4}$ AC. Prove that DE = $\frac{1}{4}$ BC.

Sol:



Let P and Q be the midpoints of AB and AC respectively.
Then $PQ \parallel BC$ such that

$PQ = \frac{1}{2}BC$ ......(i)

In ΔAPQ, D and E are the midpoint of AP and AQ are respectively

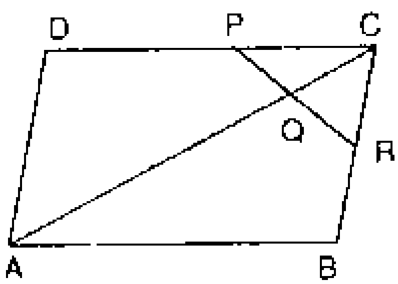
$\therefore DE \parallel PQ$ and $DE = \frac{1}{2}PQ$ ....(ii)

From (1) and (2) $DE = \frac{1}{2}PQ = \frac{1}{2}PQ = \frac{1}{2}\left(\frac{1}{2}BC\right)$ ....

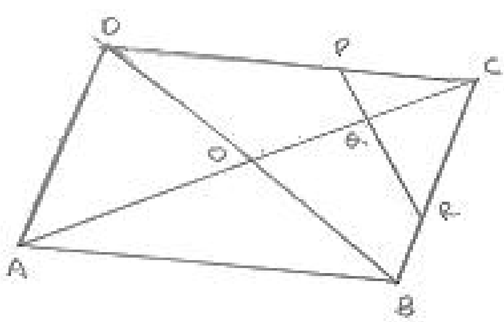
$DE = \frac{1}{4}BC$

Hence proved.

15. In below Fig, ABCD is a parallelogram in which P is the mid-point of DC and Q is a point on AC such that CQ = $\frac{1}{4}$ AC. If PQ produced meets BC at R, prove that R is a mid-point of BC.



Sol:



Join B and D, suppose AC and BD out at O.

Then $OC = \frac{1}{2}AC$

Now,

$CQ = \frac{1}{4}AC$

$\Rightarrow CQ = \frac{1}{2}\left[\frac{1}{2}AC\right]$

$= \frac{1}{2} \times OC$

In $\Delta DCO, P$ and Q are midpoints of DC and OC respectively

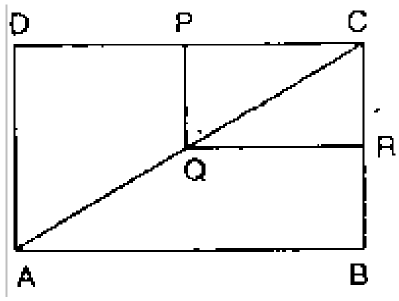
$\therefore PQ \parallel PO$

Also in $\Delta COB, Q$ is the midpoint of OC and $QR \parallel OB$

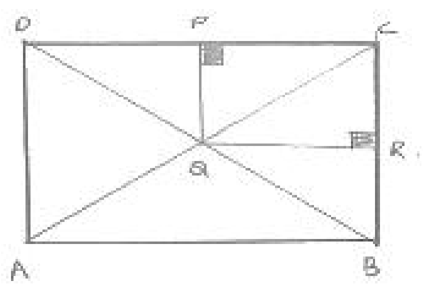
$\therefore R$ is the midpoint of BC

16. In the below Fig, ABCD and PQRC are rectangles and Q is the mid-point of AC. Prove that

(i) DP = PC (ii) PR = $\frac{1}{2}$ AC



Sol:



(i) In $\Delta ADC, Q$ is the midpoint of AC such that

$PQ \parallel AD$

$\therefore P$ is the midpoint of DC

$\Rightarrow DP = DC$ [Using converse of midpoint theorem]

(ii) Similarly, R is the midpoint of BC

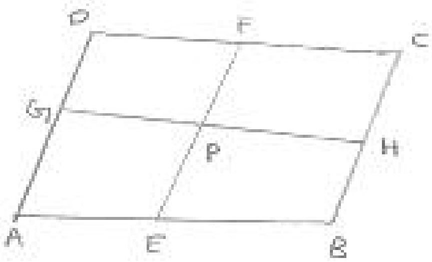
$\therefore PR = \frac{1}{2}BD$

[Diagonal of rectangle are equal $\therefore BD = AC$]

$PR = \frac{1}{2}AC$

17. ABCD is a parallelogram, E and F are the mid-points of AB and CD respectively. GH is any line intersecting AD, EF and BC at G, P and H respectively. Prove that GP = PH.

Sol:



Since E and F are midpoints of AB and CD respectively

$\therefore AE = BE = \frac{1}{2} AB$

And $CF = DF = \frac{1}{2} CD$

But, $AB = CD$

$\therefore \frac{1}{2} AB = \frac{1}{2} CD$

$\Rightarrow BE = CF$

Also, $BE \parallel CF$ $\quad [\because AB \parallel CD]$

$\therefore BEFC$ is a parallelogram

$\Rightarrow BC \parallel EF$ and $BF = PH$ $\quad ....(i)$

Now, $BC \parallel EF$

$\Rightarrow AD \parallel EF$ $\quad [\because BC \parallel AD$ as $ABCD$ is a parallel]

$\Rightarrow AEFD$ is parallelogram

$\Rightarrow AE = GP$

But is the midpoint of AB

$\therefore AE = BE$

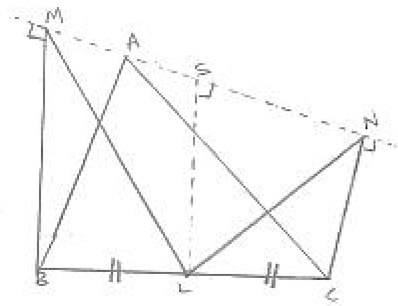
$\Rightarrow GP = PH$

18. BM and CN are perpendiculars to a line passing through the vertex A of a triangle ABC. If L is the mid-point of BC, prove that LM = LN.

Sol:

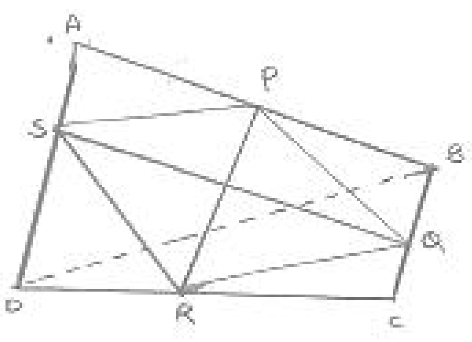
To prove $LM = LN$

Draw LS perpendicular to line MN



$\therefore$ The lines BM, LS and CN being the same perpendiculars, on line MN are parallel to each other.
According to intercept theorem,
If there are three or more parallel lines and the intercepts made by them on a transversal or equal. Then the corresponding intercepts on any other transversal are also equal.
In the drawn figure, MB and LS and NC are three parallel lines and the two transversal line are MN and BC
We have, $BL = LC$ (As L is the given midpoint of BC)
$\therefore$ using intercept theorem, we get
$MS = SN \quad ....(i)$
Now in ΔMLS *and* LSN
$MS = SN$ using ….(i)
$\angle LSM = \angle LSN = 90° LS \perp MN$ and $SL = LS$ common
$\therefore \Delta MLS \cong \Delta LSN$ (SAS congruency theorem)
$\therefore LM = LN \quad (CPCT)$

19. Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.
Sol:



Let ABCD is a quadrilateral in which P, Q, R and S are midpoints of sides AB, BC, CD and DA respectively join PQ, QR, RS, SP and BD
In ΔABD, S and P are the midpoints of AD and AB respectively.
So, by using midpoint theorem we can say that

$SP \parallel BD \ and \ SP = \frac{1}{2}BD \qquad ......(1)$

Similarly in ΔBCD

$QR \parallel BD$ and $QR = \frac{1}{2}BD \qquad .....(2)$

From equation (1) and (2) we have

$SP \parallel QR$ and $SP = QR$

As in quadrilateral SPQR one pair of opposite side are equal and parallel to each other.

So, SPQR is parallelogram

Since, diagonals of a parallelogram bisect each other.

Hence PR and QS bisect each other.

20. Fill in the blanks to make the following statements correct:

(i) The triangle formed by joining the mid-points of the sides of an isosceles triangle is ____

(ii) The triangle formed by joining the mid-points of the sides of a right triangle is ______

(iii) The figure formed by joining the mid-points of consecutive sides of a quadrilateral is _____

Sol:

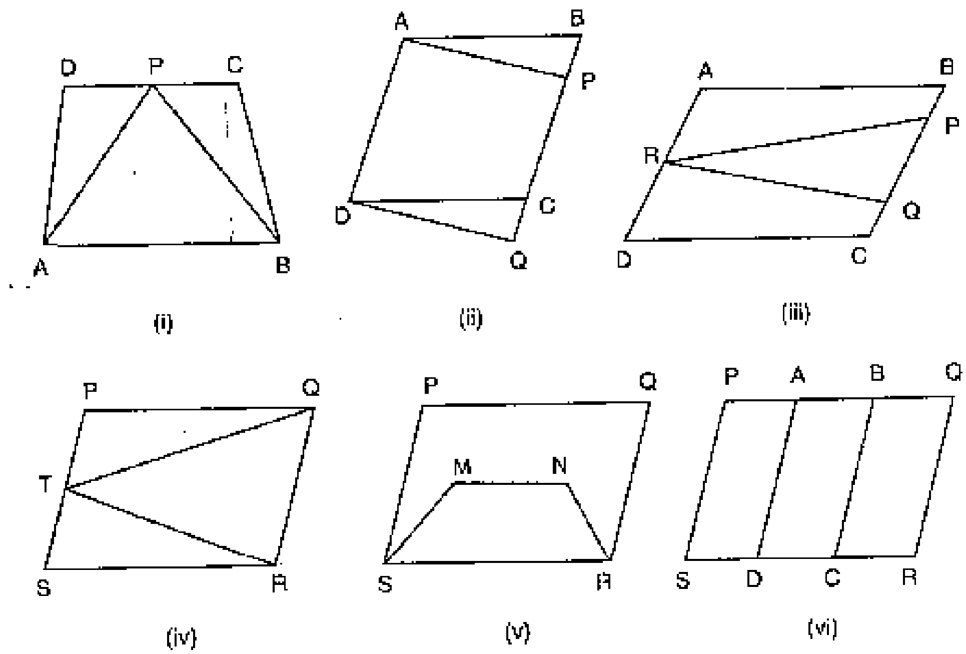
(i) Isosceles

(ii) Right triangle

(iii) Parallelogram

Exercise – 15.1

1. Which of the following figures lie on the same base and between the same parallels. In such a case, write the common base and two parallels.



Sol:

(i) ΔPCD and trapezium ABCD or on the same base CD and between the same parallels AB and DC.

(ii) Parallelogram ABCD and APQD are on the same base AD and between the same parallels AD and BQ.

(iii) Parallelogram ABCD and ΔPQR are between the same parallels AD and BC but they are not on the same base.

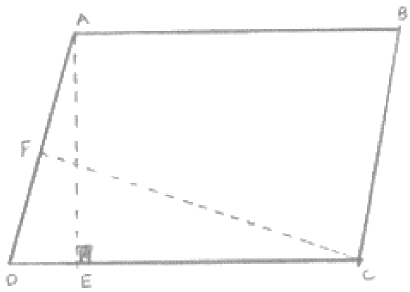
(iv) ΔQRT and parallelogram PQRS are on the same base QR and between the same parallels QR and PS

(v) Parallelogram PQRS and trapezium SMNR on the same base SR but they are not between the same parallels.

(vi) Parallelograms PQRS, AQRD, BCQR and between the same parallels also, parallelograms PQRS, BPSC and APSD are between the same parallels.

Exercise – 15.2

1. In fig below, ABCD is a parallelogram, AE ⊥ DC and CF ⊥ AD. If AB = 16 cm, AE = 8 cm and CF = 10 cm, find AD.



Sol:

Given that,

In a parallelogram $ABCD, CD = AB = 16cm$ [Opposite sides of a parallelogram are equal]

We know that,

Area of parallelogram = base $\times$corresponding attitude

Area of parallelogram $ABCD = CD \times AE = AD \times CF$

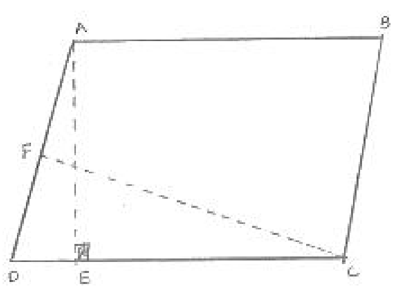
$16cm \times 8cm = AD \times 10cm$

$$AD = \frac{16 \times 8}{10} cm = 12 \cdot 8cm$$

Thus, the length of AD is $12 \cdot 8cm$

2. In Q. No 1, if AD = 6 cm, CF = 10 cm, and AE = 8cm, find AB.

Sol:



We know that,

Area of parallelogram ABCD = AD$\times$CF ……(1)

Again area of parallelogram $ABCD = DC \times AE$ ……(2)

Compare equation (1) and equation (2)

$AD \times CF = DC \times AE$

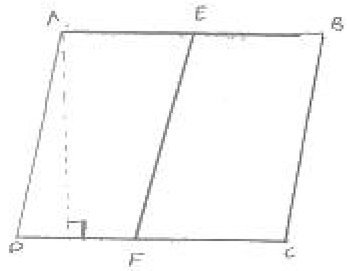
$\Rightarrow 6 \times 10 = D \times B$

$$\Rightarrow D = \frac{60}{8} = 7 \cdot 5cm$$

$\therefore AB = DC = 7 \cdot 5cm$ [$\therefore$ Opposite sides of a parallelogram are equal]

3. Let ABCD be a parallelogram of area 124 cm^2. If E and F are the mid-points of sides AB and CD respectively, then find the area of parallelogram AEFD.

Sol:



Given,

Area of parallelogram $ABCD = 124cm^2$

Construction: draw $AP \perp DC$

Proof:

Area of parallelogram $AFED = DF \times AP$ ....(1)

And area of parallelogram $EBCF = FC \times AP$ ......(2)

And $DF = FC$ .....(3) [F is the midpoint of DC]

Compare equation (1), (2) and (3)

Area of parallelogram $AEFD =$ Area of parallelogram $EBCF$

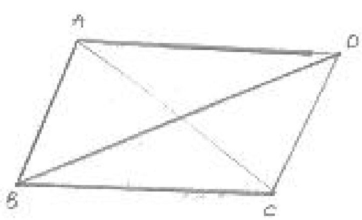
$\therefore$ Area of parallelogram $AEFD = \dfrac{\text{Area of parallelogram } ABCD}{2}$

$$= \frac{124}{2} = 62cm^2$$

4. If ABCD is a parallelogram, then prove that

$ar\ (\Delta ABD) = ar\ (\Delta BCD) = ar\ (\Delta ABC) = ar\ (\Delta ACD) = \frac{1}{2} ar\ (||^{gm}\ ABCD)$

Sol:



Given: $ABCD$ is a parallelogram

To prove: area $(\Delta ABD) = ar(\Delta ABC) = are\ (\Delta ACD)$

$= \frac{1}{2} ar\left(||^{gm}\ ABCD\right)$

Proof: we know that diagonals of a parallelogram divides it into two equilaterals.

Since, AC is the diagonal.

Then, $ar(\Delta ABC) = ar(\Delta ACD) = \frac{1}{2} ar(||^{gm}\ ABCD)$ ......(1)

Since, BD is the diagonal

Then, $ar(\Delta ABD) = ar(\Delta BCD) = \frac{1}{2} ar(||^{gm}\ ABCD)$ .....(2)

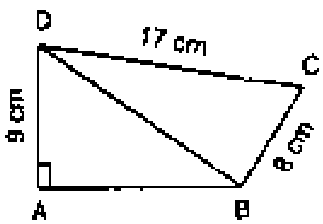
Compare equation (1) and (2)

$\therefore ar(\Delta ABC) = ar(\Delta ACD)$

$= ar(\Delta ABD) = ar(\Delta BCD) = \frac{1}{2} ar(||^{gm}\ ABCD)$

Exercise – 15.3

1. In the below figure, compute the area of quadrilateral ABCD.



Sol:

Given that

$DC = 17cm$

$AD = 9cm$ and $BC = 8cm$

In ΔBCD we have

$CD^2 = BD^2 + BC^2$

$\Rightarrow (17)^2 = BD^2 + (8)^2$

$\Rightarrow BD^2 = 289 - 64$

$\Rightarrow BD = 15$

In ΔABD, we have

$AB^2 + AD^2 = BD^2$

$\Rightarrow (15)^2 = AB^2 + (9)^2$

$\Rightarrow AB^2 = 225 - 81 = 144$

$\Rightarrow AB = 12$

$ar(\text{quad, } ABCD) = ar(\Delta ABD) + ar(\Delta BCD)$

$\Rightarrow ar(\text{quad, } ABCD) = \frac{1}{2}(12 \times 9) + \frac{1}{2}(8 \times 17) = 54 + 68$

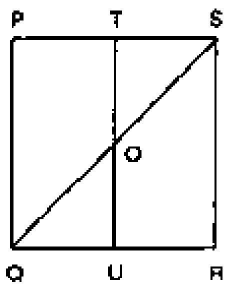
$= 112cm^2$

$\Rightarrow ar$ (quad, $ABCD = \frac{1}{2}(12\times 9)+\frac{1}{2}(8\times 15)$

$= 54+60cm^2$

$= 114cm^2$

2. In the below figure, PQRS is a square and T and U are respectively, the mid-points of PS and QR. Find the area of ΔOTS if PQ = 8 cm.



Sol:

From the figure

T and U are the midpoints of PS and QR respectively.

$\therefore TU \parallel PQ$

$\Rightarrow TO \parallel PQ$

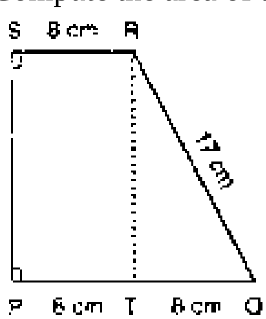
Thus, in $\Delta PQS, T$ is the midpoint of PS and $TO \parallel PQ$

$\therefore TO = \frac{1}{2}PQ = 4cm$

Also, $TS = \frac{1}{2}PS = 4cm$

$\therefore ar(\Delta OTS) = \frac{1}{2}(TO\times TS) = \frac{1}{2}(4\times 4)cm^2 = 8cm^2$

3. Compute the area of trapezium PQRS is Fig. below.



Sol:

We have

$ar(\text{trap } PQRS) = ar(\text{rect } PSRT) + \text{are } a(\Delta QRT)$

$\Rightarrow ar(\text{trap} \cdot PQRS) = PT \times RT + \frac{1}{2}(QT \times RT)$

$= 8 \times RT + \frac{1}{2}(8 \times RT) = 12 \times RT$

In ΔQRT, we have

$QR^2 = QT^2 + RT^2$

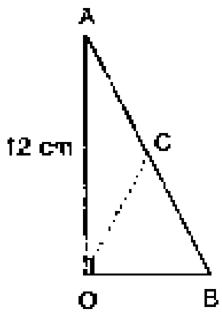
$\Rightarrow RT^2 = QR^2 - QT^2$

$\Rightarrow (RT)^2 = 17^2 - 8^2 = 225$

$\Rightarrow RT = 15$

Hence, $ar(\text{trap} \cdot PQRS) = 12 \times 15 cm^2 = 180 cm^2$

4. In the below fig. ∠AOB = 90°, AC = BC, OA = 12 cm and OC = 6.5 cm. Find the area of ΔAOB.



Sol:

Since, the midpoint of the hypotenuse of a right triangle is equidistant from the vertices

$\therefore CA = CB = OC$

$\Rightarrow CA = CB = 6 \cdot 5 cm$

$\Rightarrow AB = 13 cm$

In a right angle triangle OAB, we have

$AB^2 = OB^2 + OA^2$

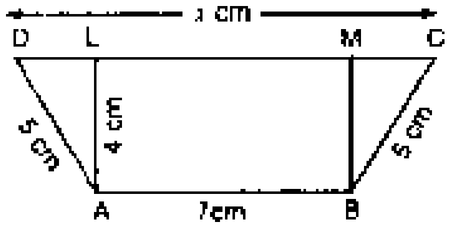
$\Rightarrow 13^2 = OB^2 + 12^2$

$\Rightarrow OB^2 = 13^2 - 12^2 = 169 - 144 = 25$

$\Rightarrow OB = 5$

$\therefore ar(\Delta AOB) = \frac{1}{2}(OA \times OB) = \frac{1}{2}(12 \times 5) = 30 cm^2$

5. In the below fig. ABCD is a trapezium in which AB = 7 cm, AD = BC = 5 cm, DC = x cm, and distance between AB and DC is 4cm. Find the value of x and area of trapezium ABCD.



Sol:

Draw $AL \perp DC, BM \perp DC$ Then,

$AL = BM = 4cm$ and $LM = 7cm$

In ΔADL, we have

$AD^2 = AL^2 + DL^2 \Rightarrow 25 = 16 + DL^2 \Rightarrow DL = 3cm$

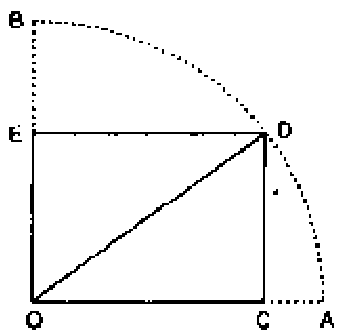
Similarly $MC = \sqrt{BC^2 - BM^2} = \sqrt{25-16} = 3cm$

$\therefore x = CD = CM + ML + CD = 3 + 7 + 3 = 13cm$

$ar(\text{trap} \cdot ABCD) = \frac{1}{2}(AB + CD) \times AL = \frac{1}{2}(7+13) \times 4cm^2$

$= 40cm^2$

6. In the below fig. OCDE is a rectangle inscribed in a quadrant of a circle of radius 10 cm. If OE = $2\sqrt{5}$, find the area of the rectangle.



Sol:

Given $OD = 10cm$ and $OE = 2\sqrt{5}cm$

By using Pythagoras theorem

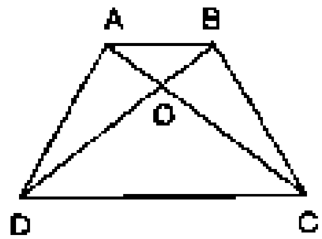
$\therefore OD^2 = OE^2 + DE^2$

$\Rightarrow DE = \sqrt{OD^2 - OF^2} = \sqrt{(10)^2 - (2\sqrt{5})^2} = 4\sqrt{5}cm$

$\therefore ar(\text{rect } DCDE) = OE \times DE = 2\sqrt{5} \times 4\sqrt{5}cm^2$

$= 40cm^2 \qquad \left[\because \sqrt{5} \times \sqrt{5} = 5\right]$

7. In the below fig. ABCD is a trapezium in which AB || DC. Prove that ar (ΔAOD) = ar(ΔBOC).



Sol:

Given: $ABCD$ is a trapezium with $AB \parallel DC$

To prove: $ar\left(\Delta AOD\right) = ar\left(BOC\right)$

Proof:

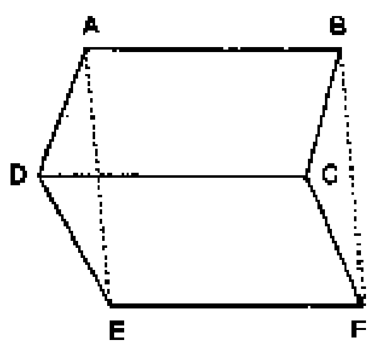
Since ΔADC and ΔBDC are on the same base DC and between same parallels AB and DC

Then, $ar\left(\Delta ADC = ar(\Delta BDC\right)$

$\Rightarrow ar\left(\Delta AOD\right) + ar\left(DOC\right) = ar\left(\Delta BOC\right) + ar\left(\Delta DOC\right)$

$\Rightarrow ar\left(\Delta AOD\right) = ar\left(\Delta BOC\right)$

8. In the given below fig. ABCD, ABFE and CDEF are parallelograms. Prove that ar (ΔADE) = ar (ΔBCF)



Sol:

Given that,

$ABCD$ is a parallelogram $\Rightarrow AD = BC$

$CDEF$ is a parallelogram $\Rightarrow DE = CF$

$ABFE$ is a parallelogram $\Rightarrow AE = BF$

Thus, in $\Delta s\ \ ADE\ and\ BCF$, we have

$AD = BC, DE = CF$ and $AE = BF$

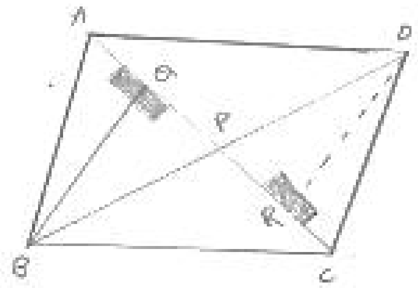
So, by SSS criterion of congruence, we have

$\Delta ADE \cong \Delta ABCF$

$\therefore ar\left(\Delta ADE\right) = ar\left(BCF\right)$

9. Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that:
ar(ΔAPB) × ar(ΔCPD) = ar(ΔAPD) × ar (ΔBPC)

Sol:



Construction: Draw $BQ \perp AC$ and $DR \perp AC$

Proof:

L.H.S

$= ar(\Delta APB) \times ar(\Delta CPD)$

$= \frac{1}{2}[(AP \times BQ)] \times \left(\frac{1}{2} \times PC \times DR\right)$

$= \left(\frac{1}{2} \times PC \times BQ\right) \times \left(\frac{1}{2} \times AP \times DR\right)$

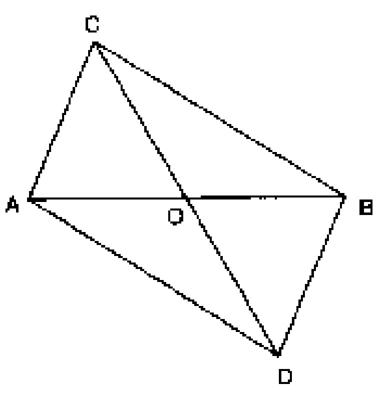
$= ar(\Delta BPC) \times ar(APD)$

$= RHS$

$\therefore LHS = RHS$

Hence proved.

10. In the below Fig, ABC and ABD are two triangles on the base AB. If line segment CD is bisected by AB at O, show that ar (Δ ABC) = ar (Δ ABD)



Sol:

Given that CD is bisected at O by AB

To prove: $ar(\Delta ABC) = ar(\Delta ABD)$

Construction: Draw CP $\perp$ AB and DQ $\perp$ AB

Proof:-

$ar(\Delta ABC) = \frac{1}{2} \times AB \times CP$ ....(i)

$ar(\Delta ABC) = \frac{1}{2} \times AB \times DQ$ ....(ii)

In $\angle CPO$ and ΔDQO

$\angle CPQ = \angle DQO$ $[Each\ 90°]$

Given that $CO = DO$

$\angle COP = \angle DOQ$ [vertically opposite angles are equal]

Then, $\Delta CPO \cong DQO$ [By AAS condition]

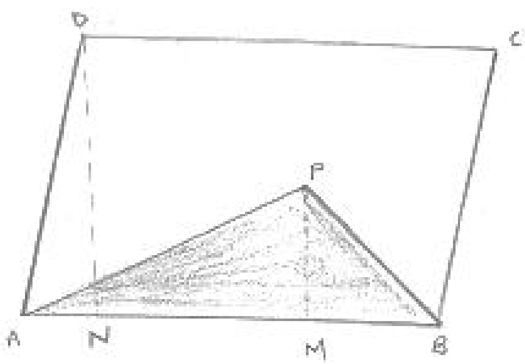
$\therefore CP = DQ$ ......(3) [CP.C.T]

Compare equation (1), (2) and (3)

Area $(\Delta ABC) = area\ of\ \ \Delta ABD$

11. If P is any point in the interior of a parallelogram ABCD, then prove that area of the triangle APB is less than half the area of parallelogram.

Sol:



Draw $DN \perp AB$ and $PM \perp AB$.

Now,

Area $(||^{gm}\ ABCD) = AB \times DN, ar(\Delta APB) = \frac{1}{2}(AB \times PM)$

Now, $PM < DN$

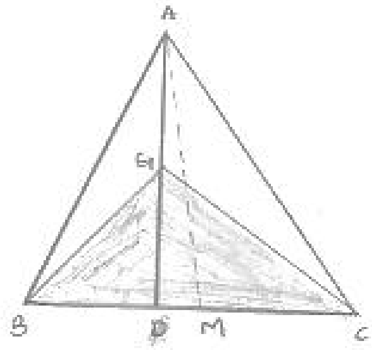
$\Rightarrow AB \times PM < AB \times DN$

$\Rightarrow \frac{1}{2}(AB \times PM) < \frac{1}{2}(AB \times DN)$

$\Rightarrow area(\Delta APB) < \frac{1}{2} ar(\text{Parragram } ABCD)$

12. If AD is a median of a triangle ABC, then prove that triangles ADB and ADC are equal in area. If G is the mid-point of median AD, prove that ar (Δ BGC) = 2 ar (Δ AGC).

Sol:



Draw $AM \perp BC$

Since, AD is the median of ΔABC

$\therefore BD = DC$

$\Rightarrow BD = AM = DC \times AM$

$\Rightarrow \frac{1}{2}(BD \times AM) = \frac{1}{2}(DC \times AM)$

$\Rightarrow ar(\Delta ABD) = ar(\Delta ACD)$ ....(i)

In $\Delta BGC, GD$ is the median

$\therefore ar(BGD) = area(OGD)$ ....(ii)

In $\Delta ACD, CG$ is the median

$\therefore$ area $(AGC) = area(\Delta CGD)$ ....(iii)

From (i) and (ii), we have

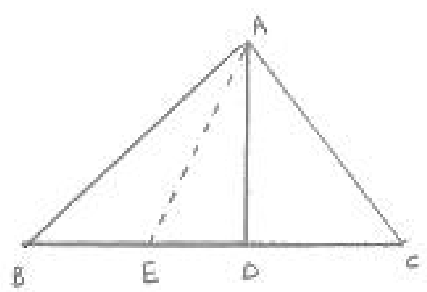
Area $(\Delta BGD) = ar(\Delta AGC)$

But, $ar(\Delta BGC) = 2ar(BGD)$

$\therefore ar(BGC) = 2ar(\Delta AGC)$

13. A point D is taken on the side BC of a ΔABC such that BD = 2DC. Prove that ar(Δ ABD) = 2ar (ΔADC).

Sol:



Given that,
In $\Delta ABC, BD = 2DC$

To prove: $ar(\Delta ABD) = 2ar(\Delta ADC)$

Construction: Take a point E on BD such that $BE = ED$
Proof: Since, $BE = ED$ and $BD = 2DC$
Then, $BE = ED = DC$
We know that median of Δ^{le} divides it into two equal Δ^{les}
$\therefore$ In $\Delta ABD, AE$ is a median

Then, area $(\Delta ABD) = 2ar(\Delta AED)$ .....(i)

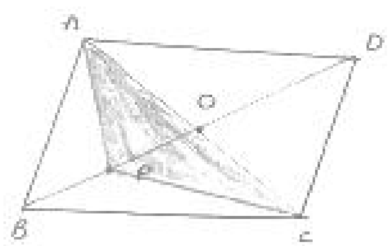
In $\Delta AEC, AD$ is a median

Then area $(\Delta AED) = area(\Delta ADC)$ …..(ii)

Compare equation (i) and (ii)
Area $(\Delta ABD) = 2ar(\Delta ADC)$.

14. ABCD is a parallelogram whose diagonals intersect at O. If P is any point on BO, prove that: (i) ar (ΔADO) = ar (ΔCDO) (ii) ar (ΔABP) = ar (ΔCBP)

Sol:



Given that $ABCD$ is a parallelogram
To prove: (i) $ar(\Delta ADO) = ar(\Delta CDO)$

(ii) $ar(\Delta ABP) = ar(\Delta CBP)$

Proof: We know that, diagonals of a parallelogram bisect each other
$\therefore AO = OC$ and $BO = OD$

(i) In $\Delta DAC,$ since DO is a median

Then area $(\Delta ADO) = area(\Delta CDO)$

(ii) In $\Delta BAC,$ Since BO is a median

Then; area $(\Delta BAO) = area(\Delta BCO)$ ......(1)

In a $\Delta PAC,$ Since PO is a median

Then, area $(\Delta PAO) = area(\Delta PCO)$ ......(2)

Subtract equation (2) from equation (1)

$$\Rightarrow area(\Delta BAO) - ar(\Delta PAO) = ar(\Delta BCO) - area(\Delta PCO)$$

$$\Rightarrow Area(\Delta ABP) = Area\ of\ \ \Delta CBP$$

15. ABCD is a parallelogram in which BC is produced to E such that CE = BC. AE intersects CD at F.

(i) Prove that ar (ΔADF) = ar (ΔECF)

(ii) If the area of ΔDFB = 3 cm^2, find the area of ||^{gm} ABCD.

Sol:

In triangles ADF and ECF, we have

$\angle ADF = \angle ECF$ [Alternative interior angles, Since $AD \parallel BE$]

$AD = EC$ [Since $AD = BC = CE$]

And $\angle DFA = \angle CFA$ [vertically opposite angles]

So, by AAS congruence criterion, we have

$\Delta ADF \cong ECF$

$\Rightarrow area(\Delta ADF) = area(\Delta ECF)\ \ and\ \ DF = CF.$

Now, $DF = CF$

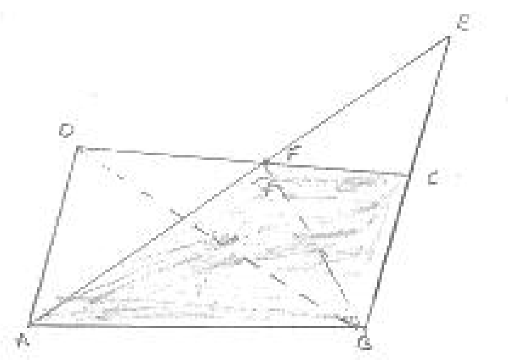
$\Rightarrow BF$ is a median in ΔBCD

$\Rightarrow area(\Delta BCD) = 2ar(\Delta BDF)$

$\Rightarrow area(\Delta BCD) = 2 \times 3cm^2 = 6cm^2$

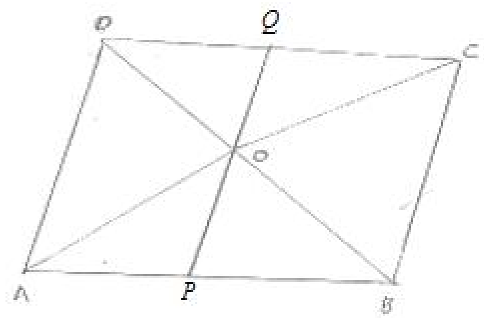
Hence, $ar(||^{gm}\ ABCD) = 2ar(\Delta BCD) = 2 \times 6cm^2$

$= 12cm^2$



16. ABCD is a parallelogram whose diagonals AC and BD intersect at O. A line through O intersects AB at P and DC at Q. Prove that ar (Δ POA) = ar (Δ QOC).

Sol:



In triangles POA and QOC, we have

$\angle AOP = \angle COQ$ [vertically opposite angles]

$OA = OC$ [Diagonals of a parallelogram bisect each other]

$\angle PAC = \angle QCA$ [$AB \| DC$; alternative angles]

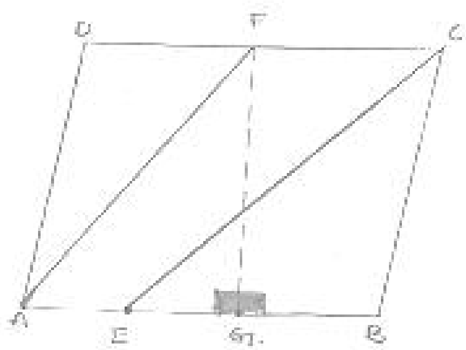
So, by ASA congruence criterion, we have

$\Delta POA \cong QOC$

Area $(\Delta POA) = area(\Delta QOC)$.

17. ABCD is a parallelogram. E is a point on BA such that BE = 2 EA and F is a point on DC such that DF = 2 FC. Prove that AE CF is a parallelogram whose area is one third of the area of parallelogram ABCD.

Sol:



Construction: Draw $FG \perp AB$

Proof: We have

$BE = 2EA$ and $DF = 2FC$

$\Rightarrow AB - AE = 2EA$ and $DC - FC = 2FC$

$\Rightarrow AB = 3EA$ and $DC = 3FC$

$\Rightarrow AE = \frac{1}{3}AB$ and $FC = \frac{1}{3}DC$ ......(1)

But $AB = DC$

Then, AE = DC [opposite sides of $\|^{gm}$]

Then, $AE = FC$.

Thus, AE = FC and AE || FC.

Then, $AECF$ is a parallelogram

Now $ar\left(\|^{gm}\ AECF\right) = AE \times FG$

$\Rightarrow ar\left(\|^{gm}\ AECF\right) = \frac{1}{3} AB \times FG$ from (1)

$\Rightarrow 3ar\left(\|^{gm}\ AECF\right) = AB \times FG$ ....(2)

and $area\left[\|^{gm}\ ABCD\right] = AB \times FG$ ....(3)

Compare equation (2) and (3)

$\Rightarrow 3\ ar\left(\|^{gm}\ AECF\right) = area\left(\|^{gm}\ ABCD\right)$

$\Rightarrow area\left(\|^{gm}\ AECF\right) = \frac{1}{3} area\left(\|^{gm}\ ABCD\right)$

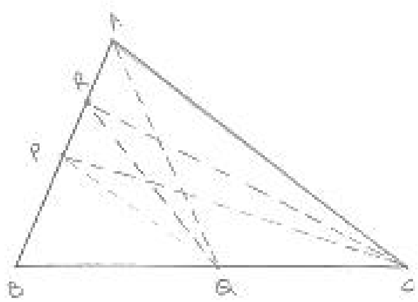
18. In a ΔABC, P and Q are respectively the mid-points of AB and BC and R is the mid-point of AP. Prove that :

(i) ar (Δ PBQ) = ar (Δ ARC)

(ii) ar (Δ PRQ) = $\frac{1}{2}$ ar (Δ ARC)

(iii) ar (Δ RQC) = $\frac{3}{8}$ ar (Δ ABC).

Sol:



(i) We know that each median of a Δ^{le} divides it into two triangles of equal area

Since, OR is a median of ΔCAP

$\therefore ar(\Delta CRA) = \frac{1}{2} ar(\Delta CAP)$ .....(i)

Also, CP is a median of ΔCAB

$\therefore ar(\Delta CAP) = ar(\Delta CPB)$ .....(ii)

From (i) and (ii) we get

$\therefore area(\Delta ARC) = \frac{1}{2} ar(CPB)$ ....(iii)

PQ is the median of ΔPBC

$\therefore area(\Delta CPB) = 2area(\Delta PBQ)$ .....(iv)

From (iii) and (iv) we get

$\therefore area(\Delta ARC) = area(PBQ)$ ......(v)

(ii) Since QP and QR medians of $\Delta^s QAB$ and QAP respectively.

$\therefore ar(\Delta QAP) = area(\Delta QBP)$ ....(vi)

And area $(\Delta QAP) = 2ar(\Delta QRP)$ ....(vii)

From (vi) and (vii) we have

Area $(\Delta PRQ) = \frac{1}{2}ar(\Delta PBQ)$ ....$(viii)$

From (v) and (viii) we get

Area $(\Delta PRQ) = \frac{1}{2}area(\Delta ARC)$

(iii) Since, $\angle R$ is a median of ΔCAP

$\therefore area(\Delta ARC) = \frac{1}{2}ar(\Delta CAP)$

$= \frac{1}{2} \times \frac{1}{2} \cdot ar(ABC)\Big]$

$= \frac{1}{4}area(ABC)$

Since RQ is a median of ΔRBC

$\therefore ar(\Delta RQC) = \frac{1}{2}ar(\Delta RBC)$

$= \frac{1}{2}[ar(\Delta ABC) - ar(ARC)]$

$= \frac{1}{2}\left[ar(\Delta ABC) - \frac{1}{4}(\Delta ABC)\right]$

$= \frac{3}{8}(\Delta ABC)$

19. ABCD is a parallelogram, G is the point on AB such that AG = 2 GB, E is a point of DC such that CE = 2DE and F is the point of BC such that BF = 2FC. Prove that:

(i) $ar(ADEG) = ar(GBCE)$

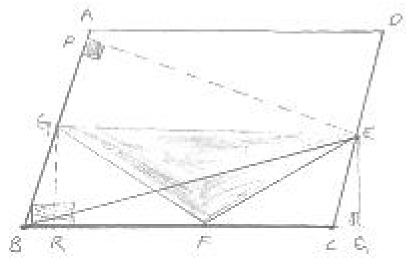
(ii) $ar(\Delta EGB) = \frac{1}{6}\ ar(ABCD)$

(iii) $ar(\Delta EFC) = \frac{1}{2}ar(\Delta EBF)$

(iv) $ar(\Delta EBG) = ar(\Delta EFC)$

(v) Find what portion of the area of parallelogram is the area of ΔEFG.

Sol:



Given,

$ABCD$ is a parallelogram

$AG = 2GB, CE = 2DE \ and \ BF = 2FC$

To prove:

(i) $ar(ADEG) = ar(GBCE)$

(ii) $ar(\Delta EGB) = \frac{1}{6} are(ABCD)$

(iii) $ar(\Delta EFC) = \frac{1}{2} area(\Delta EBF)$

(iv) area $(\Delta EBG) = \frac{3}{2} area(EFC)$

(v) Find what portion of the area of parallelogram is the area of ΔEFG.

Construction: draw $EP \perp AB$ and $EQ \perp BC$

Proof : we have,

$AG = 2GB$ and $CE = 2DE$ and $BF = 2FC$

$\Rightarrow AB - GB = 2GB$ and $CD - DE = 2DE$ and $BC - FC = 2FC$

$\Rightarrow AB - GB = 2GB$ and $CD - DE = 2DE$ and $BC - FC = 2FC$.

$\Rightarrow AB = 3GB$ and $CD = 3DE$ and $BC = 3FC$

$\Rightarrow GB = \frac{1}{3}AB$ and $DE = \frac{1}{3}CD$ and $FC = \frac{1}{3}BC$ ....(i)

(i) $ar(ADEG) = \frac{1}{2}(AG + DE) \times EP$

$\Rightarrow ar(ADEG) = \frac{1}{2}\left(\frac{2}{3}AB + \frac{1}{3}CD\right) \times EP$ [By using (1)]

$\Rightarrow ar(ADEG) = \frac{1}{2}\left(\frac{2}{3}AB + \frac{1}{3}AB\right) \times EP$ $[\because AB = CD]$

$\Rightarrow ar(ADEG) = \frac{1}{2} \times AB \times EP$ .....(2)

And $ar(GBCE) = \frac{1}{2}(GB + CE) \times EP$

$\Rightarrow ar\left(GBCE\right)=\frac{1}{2}\left[\frac{1}{3}AB+\frac{2}{3}CD\right]\times EP$ [By using (1)]

$\Rightarrow ar\left(GBCE\right)=\frac{1}{2}\left[\frac{1}{3}AB+\frac{2}{3}AB\right]\times EP$ $\left[\because AB=CD\right]$

$\Rightarrow ar\left(GBCE\right)=\frac{1}{2}\times AB\times EP$ .....(1)

Compare equation (2) and (3)

(ii) $ar\left(\Delta EGB\right)=\frac{1}{2}\times GB\times EP$

$=\frac{1}{6}\times AB\times EB$

$=\frac{1}{6}ar\left(1\right)^{9m}ABCD\Big]$.

(iii) Area $\left(\Delta EFC\right)=\frac{1}{2}\times FC\times EQ$ ......(4)

And area $\left(\Delta EBF\right)=\frac{1}{2}\times BF\times EQ$

$\Rightarrow ar\left(\Delta EBF\right)=\frac{1}{2}\times 2FC\times EQ$ $\left[BF=2FC\ \ given\right]$

$\Rightarrow ar\left(\Delta EBF\right)=FC\times EQ$ .....(5)

Compare equation 4 and 5

Area $\left(\Delta EFC\right)=\frac{1}{2}\times area\left(\Delta EBF\right)$

(iv) From (i) part

$ar\left(\Delta EGB\right)=\frac{1}{6}ar\left(11^{5m}ABCD\right)$ .....(6)

From (iii) part

$ar\left(\Delta EFC\right)=\frac{1}{2}ar\left(\Delta EBF\right)$

$\Rightarrow ar\left(\Delta EFC\right)=\frac{1}{3}ar\left(\Delta EBC\right)$

$\Rightarrow ar\left(\Delta EFC\right)=\frac{1}{3}\times\frac{1}{2}\times CE\times EP$

$=\frac{1}{2}\times\frac{1}{3}\times\frac{2}{3}CD\times EP$

$=\frac{1}{6}\times\frac{2}{3}\times ar\left(11^{gm}ABCD\right)$

$\Rightarrow ar\left(\Delta EFC\right)=\frac{2}{3}\times ar\left(\Delta EGB\right)$ [By using]

$\Rightarrow ar(\Delta EGB) = \frac{3}{2} ar(EFC)$.

(v) Area $(\Delta EFG) = ar(Trap \cdot BGEC) = -ar(\Delta BGF) \rightarrow (1)$

Now, area (trap BGEC) $= \frac{1}{2}(GB + EC) \times EP$

$= \frac{1}{2}\left(\frac{1}{3}AB + \frac{2}{3}CD\right) \times EP$

$= \frac{1}{2} AB \times EP$

$= \frac{1}{2} ar(11^{5m} ABCD)$

Area $(\Delta EFC) = \frac{1}{9} area(11^{5m} ABCD)$ [From iv part]

And area $(\Delta BGF) = \frac{1}{2} BF \times GR$

$= \frac{1}{2} \times \frac{2}{3} BC \times GR$

$= \frac{2}{3} \times \frac{1}{2} BC \times GR$

$= \frac{2}{3} \times ar(\Delta GBC)$

$= \frac{2}{3} \times \frac{1}{2} GB \times EP$

$= \frac{1}{3} \times \frac{1}{3} AB \times EP$

$= \frac{1}{9} AB \times EP$

$= \frac{1}{9} ar(11^{gm} ABCD)$ [From (1)]

$ar(\Delta EFG) = \frac{1}{2} ar(11^{gm} ABCD) = \frac{1}{9} ar(11^{gm} ABCD) = \frac{1}{9} ar(11^{gm} ABCD)$

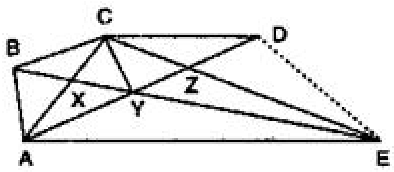
$= \frac{5}{18} ar(11^{gm} ABCD)$.

20. In Fig. below, CD || AE and CY || BA.

(i) Name a triangle equal in area of ΔCBX

(ii) Prove that ar (Δ ZDE) = ar (ΔCZA)

(iii) Prove that ar (BCZY) = ar (Δ EDZ)



Sol:

Since, ΔBCA *and* ΔBYA are on the same base BA and between same parallels BA and CY

Then area $(\Delta BCA) = ar(BYA)$

$\Rightarrow ar(\Delta CBX) + ar(\Delta BXA) = ar(\Delta BXA) + ar(\Delta AXY)$

$\Rightarrow ar(\Delta CBX) = ar(\Delta AXY)$ ......(1)

Since, ΔACE *and* ΔADE are on the same base AE and between same parallels CD and AE

Then, $ar(\Delta ACE) = ar(\Delta ADE)$

$\Rightarrow ar(\Delta CLA) + ar(\Delta AZE) = ar(\Delta AZE) + ar(\Delta DZE)$

$\Rightarrow ar(\Delta CZA) = (\Delta DZE)$ ....(2)

Since ΔCBY *and* ΔCAY are on the same base CY and between same parallels BA *and* CY

Then $ar(\Delta CBY) = ar(\Delta CAY)$

Adding $ar(\Delta CYG)$ on both sides, we get

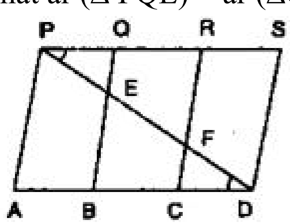
$\Rightarrow ar(\Delta CBX) + ar(\Delta CYZ) = ar(\Delta CAY) + ar(\Delta CYZ)$

$\Rightarrow ar(BCZX) = ar(\Delta CZA)$ ....(3)

Compare equation (2) and (3)

$ar(BCZY) = ar(\Delta DZE)$

21. In below fig., PSDA is a parallelogram in which PQ = QR = RS and AP || BQ || CR. Prove that ar (Δ PQE) = ar (ΔCFD).



Sol:

Given that PSDA is a parallelogram

Since, $AP \parallel BQ \parallel CR \parallel DS$ *and* $AD \parallel PS$

$\therefore PQ = CD$ .....(i)

In ΔBED, C is the midpoint of BD and $CF \parallel BE$

$\therefore F$ is the midpoint of ED

$\Rightarrow EF = PE$

Similarly

$EF = PE$

$\therefore PE = FD \qquad .....(2)$

In $\Delta SPQE \ and \ CFD$, we have

$PE = FD$

$\angle EDQ = \angle FDC,$ [Alternative angles]

And $PQ = CD$

So by SAS congruence criterion, we have $\Delta PQE \cong \Delta DCF$.

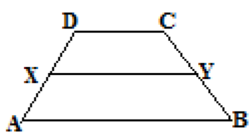
22. In the below fig. ABCD is a trapezium in which AB || DC and DC = 40 cm and AB = 60 cm. If X and Y are respectively, the mid-points of AD and BC, prove that:

(i) XY = 50 cm

(ii) DCYX is a trapezium

(iii) ar (trap. DCYX) = $\frac{9}{11}$ ar (trap. (XYBA))

Sol:



(i) Join DY and produce it to meet AB produced at P

In $\Delta's \ BYP$ and CYD we have

$\angle BYP = (\angle CYD)$ [Vertical opposite angles]

$\angle DCY = \angle PBY$ $[\because DC \parallel AP]$

And $BY = CY$

So, by ASA congruence criterion, we have

$\Delta BYP \cong CYD$

$\Rightarrow DY = YP \ and \ DC = BP$

$\Rightarrow y$ is the midpoint of DP

Also, x is the midpoint of AD

$\therefore XY \parallel AP \ and \ XY = \frac{1}{2} AD$

$\Rightarrow xy = \frac{1}{2}(AB + BD)$

$\Rightarrow xy = \frac{1}{2}(BA + DC) \Rightarrow xy = \frac{1}{2}(60 + 40)$

(ii) We have

$XY \parallel AP$

$\Rightarrow XY \parallel AB \text{ and } AB \parallel DC$ [As proved above]

$\Rightarrow XY \parallel DC$

$\Rightarrow DCY$ is a trapezium

(iii) Since *x* and *y* are the midpoint of DA and CB respectively

$\therefore$ Trapezium DCXY and ABYX are of the same height say hm

Now

$$ar(Trap\ DCXY) = \frac{1}{2}(DC + XY) \times h$$

$$= \frac{1}{2}(50 + 40)hcm^2 = 45hcm^2$$

$$\Rightarrow ar(trap\ ABXY) = \frac{1}{2}(AB + XY) \times h = \frac{1}{2}(60 + 50)hm^3$$

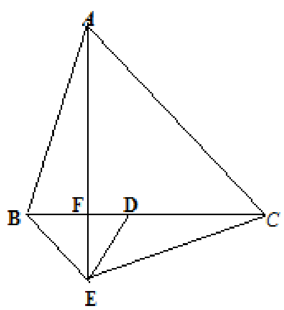
$$\Rightarrow ar(trap\ ABYX) = \frac{1}{2}(AB + XY) \times h = \frac{1}{2}(60 + 50)hcm^2$$

$$= 55cm^2$$

$$\frac{ar\ trap(YX)}{ar\ trap(ABYX)} = \frac{45h}{55h} = \frac{9}{11}$$

$$\Rightarrow ar(trap\ DCYX) = \frac{9}{11} ar(trap\ ABXY)$$

23. In Fig. below, ABC and BDE are two equilateral triangles such that D is the mid-point of BC. AE intersects BC in F. Prove that



(i) $ar(\Delta BDE) = \frac{1}{4} ar(\Delta ABC)$

(ii) $area(\Delta BDE) = \frac{1}{2} ar(\Delta BAE)$

(iii) $ar(BEF) = ar(\Delta AFD)$.

(iv) $area(\Delta ABC) = 2area(\Delta BEC)$

(v) $ar(\Delta FED) = \frac{1}{8} ar(\Delta AFC)$

(vi) $ar(\Delta BFE) = 2ar(\Delta EFD)$

Sol:

Given that,

ABC and BDE are two equilateral triangles.

Let $AB = BC = CA = x.$ Then $BD = \frac{x}{2} = DE = BE$

(i) We have

$$ar(\Delta ABC) = \frac{\sqrt{3}}{4}x^2$$

$$ar(\Delta ABC) = \frac{\sqrt{3}}{4}\left(\frac{x}{2}\right)^2 = \frac{1}{4}\times\frac{\sqrt{3}}{4}x^2$$

$$\Rightarrow ar(\Delta BDE) = \frac{\sqrt{3}}{4}\left(\frac{x}{2}\right)^2$$

(ii) It is given that triangles ABC and BED are equilateral triangles

$\angle ACB = \angle DBE = 60°$

$\Rightarrow BE \parallel AC$ (Since alternative angles are equal)

Triangles BAF and BEC are on the same base

BE and between the same parallel BE and AC

$\therefore ar(\Delta BAE) = area(\Delta BEC)$

$\Rightarrow ar(\Delta BAE) = 2ar(\Delta BDE)$

[$\because ED$ is a median of $\Delta EBC; ar(\Delta BEC) = 2ar(\Delta BDE)$]

$\Rightarrow area(\Delta BDE) = \frac{1}{2}ar(\Delta BAE)$

(iii) Since ΔABC and ΔBDE are equilateral triangles

$\therefore \angle ABC = 60°$ and $\angle BDE = 60°$

$\angle ABC = \angle BDE$

$\Rightarrow AB \parallel DE$ (Since alternative angles are equal)

Triangles BED and AED are on the same base ED and between the same parallels AB and DE.

$\therefore ar(\Delta BED) = area(\Delta AED)$

$\Rightarrow ar(\Delta BED) - area(\Delta EFD) = area(AED) - area(\Delta EFD)$

$\Rightarrow ar(BEF) = ar(\Delta AFD).$

(iv) Since ED is the median of ΔBEC

$\therefore area(\Delta BEC) = 2ar(\Delta BDE)$

$\Rightarrow ar(\Delta BEC) = 2\times\frac{1}{4}ar(\Delta ABC)$ [from (i)]

$\Rightarrow ar\left(\Delta BEC\right)=\frac{1}{2}area\left(\Delta ABC\right)$

$\Rightarrow area\left(\Delta ABC\right)=2area\left(\Delta BEC\right)$

(v) Let h be the height of vertex E, corresponding to the side BD on triangle BDE
Let H be the height of the vertex A corresponding to the side BC in triangle ABC
From part (i)

$ar\left(\Delta BDE\right)=\frac{1}{4}ar\left(\Delta ABC\right)$

$\Rightarrow \frac{1}{2}\times BD\times h=\frac{1}{4}ar\left(\Delta ABC\right)$

$\Rightarrow BD\times h=\frac{1}{4}\left(\frac{1}{2}\times BC\times H\right)$

$\Rightarrow h=\frac{1}{2}H$ ....(1)

From part .....(iii)

Area $\left(\Delta BFE\right)=ar\left(\Delta AFD\right)$

$=\frac{1}{2}\times FD\times H$

$=\frac{1}{2}\times FD\times H$

$=2\left(\frac{1}{2}\times FD\times 2h\right)$

$=2ar\left(\Delta EFD\right)$

(vi) $area\left(\Delta AFC\right)=area\left(AFD\right)+area\left(ADC\right)$

$\Rightarrow ar\left(\Delta BFE\right)+\frac{1}{2}ar\left(\Delta ABC\right)$

[using part (iii); and AD is the median ΔABC]

$=ar\left(\Delta BFE\right)+\frac{1}{2}\times 4ar\left(\Delta BDE\right)$ using part (i)]

$=ar\left(\Delta BFE\right)=2ar\left(\Delta FED\right)$ .....(3)

Area $\left(\Delta BDE\right)=ar\left(\Delta BFE\right)+ar\left(\Delta FED\right)$

$\Rightarrow R\ ar\left(\Delta FED\right)+ar\left(\Delta FED\right)$

$\Rightarrow 3\ ar\left(\Delta FED\right)$ ....(4)

From (2), (3) and (4) we get

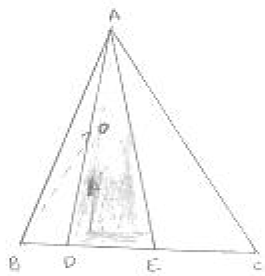
Area $\left(\Delta AFC\right)=2area\left(\Delta FED\right)+2\times 3ar\left(\Delta FED\right)$

$=8\ ar\left(\Delta FED\right)$

Hence, area $\left(\Delta FED\right)=\frac{1}{8}area\left(AFC\right)$

24. D is the mid-point of side BC of ΔABC and E is the mid-point of BD. if O is the mid-point of AE, prove that ar (ΔBOE) = $\frac{1}{8}$ ar (Δ ABC).

Sol:



Given that

D is the midpoint of side BC of ΔABC.

E is the midpoint of BD and

O is the midpoint of AE

Since AD and AE are the medians of ΔABC and ΔABD respectively

$\therefore ar(\Delta ABD) = \frac{1}{2} ar(\Delta ABC)$ ....(i)

$ar(\Delta ABE) = \frac{1}{2} ar(\Delta ABD)$ ....(ii)

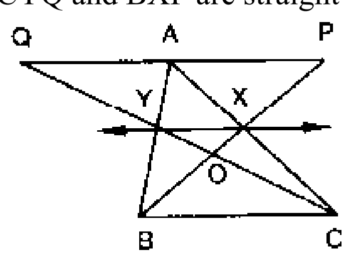
OB is a median of ΔABE

$\therefore ar(\Delta BOE) = \frac{1}{2} ar(\Delta ABE)$

From i, (ii) and (iii) we have

$ar(BOE) = \frac{1}{8} ar(\Delta ABC)$

25. In the below fig. X and Y are the mid-points of AC and AB respectively, QP || BC and CYQ and BXP are straight lines. Prove that ar (Δ ABP) = ar (ΔACQ).



Sol:

Since x and y are the midpoint AC and AB respectively

$\therefore XY \parallel BC$

Clearly, triangles BYC and BXC are on the same base BC and between the same parallels XY and BC

$\therefore area(\Delta BYC) = area(BXC)$

$\Rightarrow area(\Delta BYC) = ar(\Delta BOC) = ar(\Delta BXC) - ar(BOC)$

$\Rightarrow ar(\Delta BOY) = ar(\Delta COX)$

$\Rightarrow ar(BOY) + ar(XOY) = ar(\Delta COX) + ar(\Delta XOY)$

$\Rightarrow ar(\Delta BXY) = ar(\Delta CXY)$

We observe that the quadrilateral XYAP and XYAQ are on the same base XY and between the same parallel XY and PQ.

$\therefore area(quad\ \ XYAP) = ar(quad\ \ XYPA) \qquad ....(ii)$

Adding (i) and (ii), we get

$ar(\Delta BXY) + ar(quad\ \ XYAP) = ar(CXY) + ar(quad\ \ XYQA)$

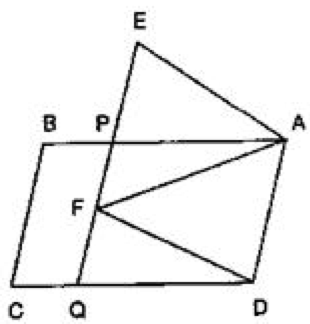
$\Rightarrow ar(\Delta ABP) = ar(\Delta ACQ)$

26. In the below fig. ABCD and AEFD are two parallelograms. Prove that

(i) PE = FQ

(ii) ar (Δ APE) : ar (ΔPFA) = ar Δ(QFD) : ar (Δ PFD)

(iii) $ar(\Delta PEA) = ar(\Delta QFD)$



Sol:

Given that, $ABCD\ and\ AEFD$ are two parallelograms

To prove: (i) $PE = FQ$

(ii) $\dfrac{ar(\Delta APE)}{ar(\Delta PFA)} = \dfrac{ar(\Delta QFD)}{ar(\Delta PFD)}$

(iii) $ar(\Delta PEA) = ar(\Delta QFD)$

Proof: (i) In $\Delta EPA\ and\ \Delta FQD$

$\angle PEA = \angle QFD$ [$\because$ Corresponding angles]

$\angle EPA = \angle FQD$ [Corresponding angles]

$PA = QD$ $\left[opp \cdot sides\ of\ 11^{gm}\right]$

Then, $\Delta EPA \cong \Delta FQD$ [By. AAS condition]

$\therefore EP = FQ \quad [c.p.c.t]$

(ii) Since, $\Delta PEA\ and\ \Delta QFD$ stand on the same base PE and FQ lie between the same parallels EQ and AD

$\therefore ar(\Delta PEA) = ar(\Delta QFD) \to (1)$

$AD \quad \therefore ar(\Delta PFA) = ar(PFD) \quad .....(2)$

Divide the equation (i) by equation (2)

$$\frac{area\ of\ (\Delta PEA)}{area\ of\ (\Delta PFA)} = \frac{ar\Delta(QFD)}{ar\Delta(PFD)}$$

(iii) From (i) part $\Delta EPA \cong FQD$

Then, $ar(\Delta EDA) = ar(\Delta FQD)$

27. In the below figure, ABCD is parallelogram. O is any point on AC. PQ || AB and LM || AD. Prove that ar (||gm DLOP) = ar (||gm BMOQ)

Sol:

Since, a diagonal of a parallelogram divides it into two triangles of equal area

$\therefore area(\Delta ADC) = area(\Delta ABC)$

$\Rightarrow area(\Delta APO) + area(11^{gm}\ DLOP) + area(\Delta OLC)$

$\Rightarrow area(\Delta AOM) + ar(11gmDLOP) + area(\Delta OQC) \quad .....(i)$

Since, AO and OC are diagonals of parallelograms AMOP and OQCL respectively.

$\therefore area(\Delta APO) = area(\Delta AMO) \quad .....(ii)$

And, area $(\Delta OLC) = Area(\Delta OQC) \quad ...(iii)$

Subtracting (ii) and (iii) from (i), we get

Area $(11^{gm}\ \ DLOP) = area(11^{gm}\ BMOQ)$

28. In a ΔABC, if L and M are points on AB and AC respectively such that LM || BC. Prove that:

(i) $ar(\Delta LCM) = ar(\Delta LBM)$

(ii) $ar\ (\Delta LBC) = ar(\Delta MBC)$

(iii) $ar(\Delta ABM) = ar(\Delta ACL)$

(iv) $ar(\Delta LOB) = ar(\Delta MOC)$

Sol:

(i) Clearly Triangles $LMB\ and\ LMC$ are on the same base LM and between the same parallels $LM\ and\ BC$.

$\therefore ar(\Delta LMB) = ar(\Delta LMC)$ ......(i)

(ii) We observe that triangles LBC and MBC area on the same base BC and between the same parallels LM and BC

$\therefore arc\ \Delta LBC = ar(MBC)$ .....(ii)

(iii) We have

$ar(\Delta LMB) = ar(\Delta LMC)$ [from (1)]

$\Rightarrow ar(\Delta ALM) + ar(\Delta LMB) = ar(\Delta ALM) + ar(LMC)$

$\Rightarrow ar(\Delta ABM) = ar(\Delta ACL)$

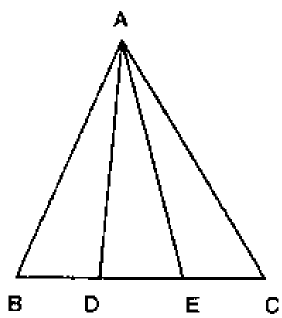
(iv) We have

$ar(\Delta CBC) = ar(\Delta MBC)$ $\therefore$ [from (1)]

$\Rightarrow ar(\Delta LBC) = ar(\Delta BOC) = a(\Delta MBC) - ar(BOC)$

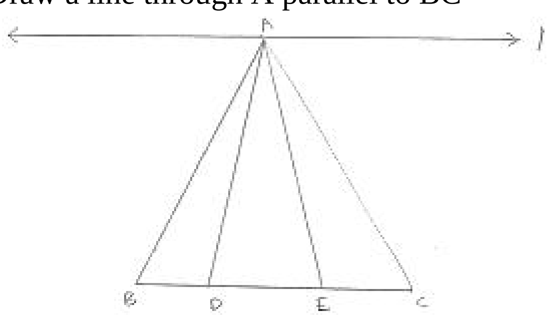
$\Rightarrow ar(\Delta LOB) = ar(\Delta MOC)$

29. In the below fig. D and E are two points on BC such that BD = DE = EC. Show that ar (ΔABD) = ar (ΔADE) = ar (ΔAEC).



Sol:

Draw a line through A parallel to BC



Given that, $BD = DE = EC$

We observe that the triangles ABD and AEC are on the equal bases and between the same parallels C and BC. Therefore, Their areas are equal.

Hence, $ar(ABD) = ar(\Delta ADE) = ar(\Delta ACDE)$

30. If below fig. ABC is a right triangle right angled at A, BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment AX ⊥ DE meets BC at Y. Show that:

(i) $\Delta MBC \cong \Delta ABD$

(ii) $ar(BYXD) = 2\,ar(\Delta MBC)$

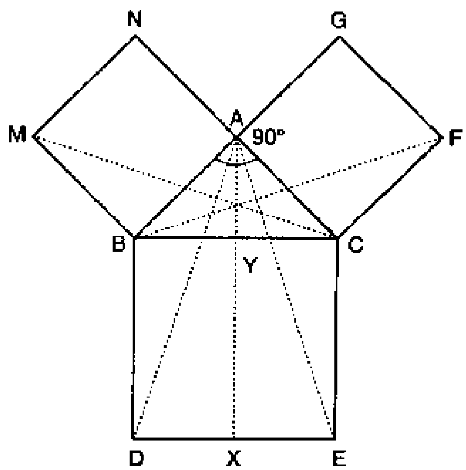
(iii) ar (BYXD) = ar (Δ ABMN)

(iv) ΔFCB ≅ Δ ACE

(v) ar (CYXE) = 2 ar (Δ FCB)

(vi) ar (CYXE) = ar (ACFG)

(vii) ar (BCED) = ar (ABMN) + ar (ACFG)



Sol:

(i) In ΔMBC and ΔABD, we have

$MB = AB$

$BC = BD$

And $\angle MBC = \angle ABD$

[$\because \angle MBC$ and $\angle ABC$ are obtained by adding $\angle ABC$ to a right angle]

So, by SAS congruence criterion, We have

$\Delta MBC \cong \Delta ABD$

$\Rightarrow ar(\Delta MBC) = ar(\Delta ABD)$ ....(1)

(ii) Clearly, ΔABC and $BYXD$ are on the same base BD and between the same parallels AX and BD

$$\therefore Area(\Delta ABD) = \frac{1}{2} Area(rect\ BYXD)$$

$$\Rightarrow ar(rect \cdot BYXD) = 2ar(\Delta ABD)$$

$$\Rightarrow are(rect \cdot BYXD) = 2ar(\Delta MBC) \qquad ......(2)$$

$$\left[\because ar(\Delta ABD) = ar(\Delta MBC) \qquad ......from(i)\right.$$

(iii) Since triangle $M \cdot BC$ and square $MBAN$ are on the same Base MB and between the same parallels MB and NC

$\therefore 2ar(\Delta MBC) = ar(MBAN)$ .....(3)

From (2) and (3) we have

$ar(sq \cdot MBAN) = ar(rect\ \ BYXD)$.

(iv) In triangles FCB and ACE we have

$FC = AC$

$CB = CF$

And $\angle FCB = \angle ACE$

[$\because \angle FCB\ \ and\ \ \angle ACE$ are obtained by adding $\angle ACB$ to a right angle]

So, by SAS congruence criterion, we have

$\Delta FCB \cong \Delta ACE$

(v) We have

$\Delta FCB \cong \Delta ACE$

$\Rightarrow ar(\Delta FCB) = ar(\Delta ECA)$

Clearly, ΔACE and rectangle CYXE are on the same base CE and between the same parallels CE and AX

$\therefore 2ar(\Delta ACE) = ar(CYXE)$ ....(4)

(vi) Clearly, ΔFCB and rectangle $FCAG$ are on the same base FC and between the same parallels FC and BG

$\therefore 2ar(\Delta FCB) = ar(FCAG)$ ......(5)

From (4) and (5), we get

Area $(CYXE) = ar(ACFG)$

(vii) Applying Pythagoras theorem in ΔACB, we have

$BC^2 = AB^2 + AC^2$

$\Rightarrow BC \times BD = AB \times MB + AC \times FC$

$\Rightarrow area(BCED) = area(ABMN) + ar(ACFG)$

Exercise – 16.1

1. Fill in the blanks:

(i) All points lying inside/outside a circle are called .... points/ .... points.
(ii) Circles having the same centre and different radii are called .... circles.
(iii) A point whose distance from the centre of a circle is greater than its radius lies in .... of the circle.
(iv) A continuous piece of a circle is .... of the circle.
(v) The longest chord of a circle is a . . . . of the circle.
(vi) An arc is a . . . . when its ends are the ends of a diameter.
(vii) Segment of a circle is the region between an arc and . . . . of the circle.
(viii) A circle divides the plane, on which it lies, in . .. parts.

Sol:

(i) Interior/exterior
(ii) Concentric
(iii) The exterior
(iv) Arc
(v) Diameter
(vi) Semi-circle
(vii) Centre
(viii) Three

2. Write the truth value (T/F) of the following with suitable reasons:

(i) A circle is a plane figure.
(ii) Line segment joining the centre to any point on the circle is a radius of the circle,
(iii) If a circle is divided into three equal arcs each is a major arc.
(iv) A circle has only finite number of equal chords.
(v) A chord of a circle, which is twice as long is its radius is a diameter of the circle.
(vi) Sector is the region between the chord and its corresponding arc.
(vii)The degree measure of an arc is the complement of the central angle containing the arc.
(viii) The degree measure of a semi-circle is 180°.

Sol:

(i) True
(ii) True
(iii) True
(iv) False
(v) True
(vi) True
(vii) False
(viii) True

Exercise – 16.2

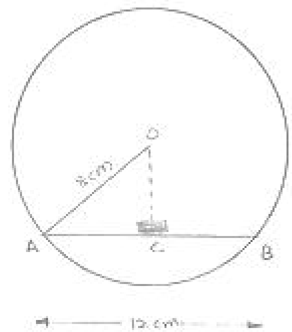
1. The radius of a circle is 8 cm and the length of one of its chords is 12 cm. Find the distance of the chord from the centre.

Sol:

Given that

Radius of circles $(OA) = 8cm$

Chord $(AB) = 12cm$



Draw $OC \perp AB$.

WKT,

The perpendicular from center to chord bisects the chord

$\therefore AC = BC = \frac{12}{2} = 6cm$

Now in ΔOCA, by Pythagoras theorem

$AC^2 + OC^2 = OA^2$

$\Rightarrow 6^2 + OC^2 = 8^2$

$\Rightarrow 36 + OC^2 = 64$

$\Rightarrow OC^2 = 64 - 36$

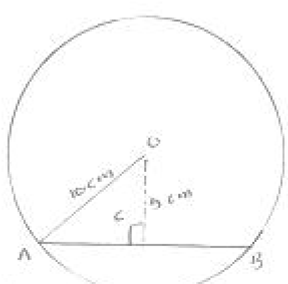
$\Rightarrow OC^2 = 28$

$\Rightarrow OC = \sqrt{28}$

$\Rightarrow OC = 5 \cdot 291\ cm$

2. Find the length of a chord which is at a distance of 5 cm from the centre of a circle ofradius 10 cm.

Sol:



Given that

Distance $(OC) = 5cm$

Radius of circle $(OA) = 10cm$

In ΔOCA by Pythagoras theorem

$AC^2 + OC^2 = OA^2$

$\Rightarrow AC^2 + 5^2 = 10^2$

$\Rightarrow AC^2 = 100 - 25$

$\Rightarrow AC = \sqrt{75} = 8 \cdot 66 cm$

WRK, the perpendicular from center to chord bisects the chord

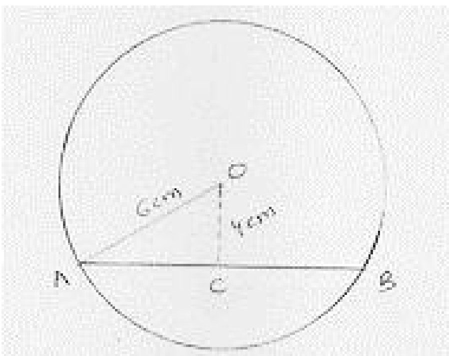
$\therefore AC = BC = 8 \cdot 66 cm$

Then chord $AB = 8 \cdot 66 + 8 \cdot 66$

$= 17 \cdot 32 cm$

3. Find the length of a chord which is at a distance of 4 cm from the centre of the circle of radius 6 cm.

Sol:



Radius of circle $(OA) = 6cm$

Distant $(OC) = 4cm$

In ΔOCA by Pythagoras theorem

$AC^2 + OC^2 = OA^2$

$\Rightarrow AC^2 + 4^2 = 6^2$

$\Rightarrow AC^2 = 36 - 16$

$\Rightarrow AC = \sqrt{20} = 4 \cdot 47 cm.$

WKT, the perpendicular distance from center to chord bisects the chord.

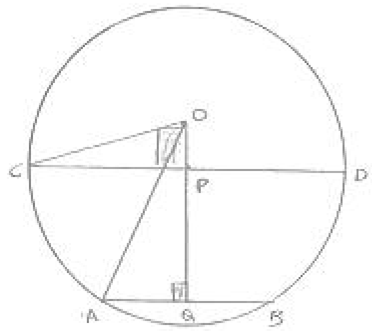
$AC = BC = 4 \cdot 47 cm$

Then, $AB = 4 \cdot 47 + 4 \cdot 47$

$= 8 \cdot 94 cm.$

4. Two chords AB, CD of lengths 5 cm, 11 cm respectively of a circle are parallel. If the distance between AB and CD is 3 cm, find the radius of the circle.

Sol:



Construction: Draw $OP \perp CD$

Chord $AB = 5cm$

Chord $CD = 11cm$

Distance $PQ = 3cm$

Let $OP = x\,cm$

And $OC = OA = rcm$

WKT perpendicular from center to chord bisects it

$\therefore CP = PD = \frac{11}{2}cm$

And $AQ = BQ = \frac{5}{2}cm$

In ΔOCP, by Pythagoras theorem

$OC^2 = OP^2 + CP^2$

$\Rightarrow r^2 = x^2 + \left(\frac{11}{2}\right)^2 \qquad ....(1)$

In ΔOQA, by Pythagoras theorem

$OA^2 = OQ^2 + AQ^2$

$\Rightarrow r^2 = (x+3)^2 + \left(\frac{5}{2}\right)^2 \qquad .....(2)$

Compare equation (1) and (2)

$(x+3)^2 + \left(\frac{5}{2}\right)^2 = x^2 + \left(\frac{11}{2}\right)^2$

$\Rightarrow x^2 + 9 + 6x + \frac{25}{4} = x^2 + \left(\frac{121}{4}\right)$

$\Rightarrow x^2 + 6x - x^2 = \frac{121}{4} - \frac{25}{4} - 9$

$\Rightarrow 6x = 15$

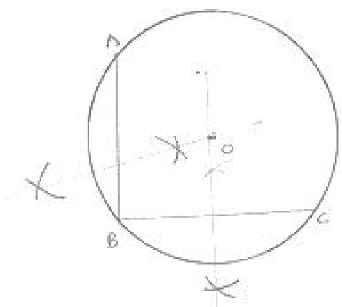
$\Rightarrow x = \frac{15}{6} = \frac{5}{2}$

5. Give a method to find the centre of a given circle.

Sol:

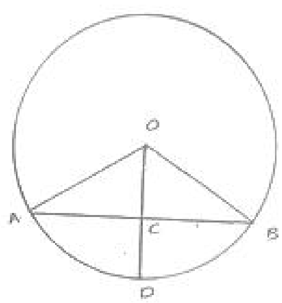
Steps of construction

(1) Take three point A,B and C on the given circle
(2) Join AB and BC
(3) Draw thee perpendicular bisectors of chord AB and BC which intersect each other at O
(4) Point will be required circle because WKT, perpendicular bisector of chord always passes through center



6. Prove that the line joining the mid-point of a chord to the centre of the circle passes through the mid-point of the corresponding minor arc.

Sol:



Given: C is the midpoint of chord AB

To prove: D is the midpoint of arc AB

Proof:

In ΔOAC *and* ΔOBC

$OA = OB$ [Radius of circle]

$OC = OC$ [Common]

$AC = BC$ [C is the midpoint of AB]

Then, $\Delta OAC \cong \Delta OBC$ [By SSS condition]

$\therefore \angle AOC = \angle BOC$ [$c \cdot p \cdot c \cdot t$]

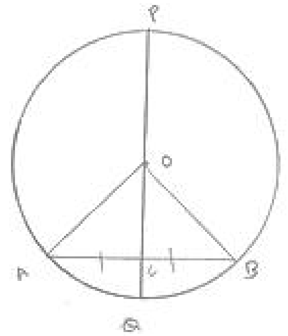
$\Rightarrow m\left(\overline{AD}\right) = m\left(\overline{BD}\right)$

$\Rightarrow \overline{AD} \cong \overline{BD}$

Here, D is the midpoint of arc AB

7. Prove that a diameter of a circle which bisects a chord of the circle also bisects the angle subtended by the chord at the centre of the circle.

Sol:



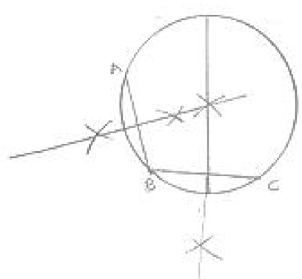
Given: PQ is a diameter of circle which Bisects
Chord AB at C
To prove: PQ bisects $\angle AOB$
Proof:
In ΔAOC *and* ΔBOC

$OA = OB$	[Radius of circle]
$OC = OC$	[Common]
$AC = BC$	[Given]
Then $\Delta AOC \cong BOC$	[by SSS condition]
$\therefore \angle AOC = \angle BOC$	[$c \cdot p \cdot c \cdot t$]

Hence PQ bisects $\angle AOB$

8. Given an arc of a circle, show how to complete the circle.

Sol:

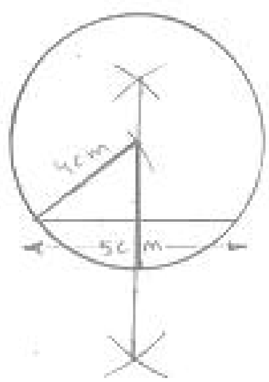


Steps of construction:

(i) Take three point A, B and C on the given Arc
(ii) Join AB and BC
(iii) Draw the perpendicular bisectors of chords AB and BC which interest each other at point O, then O will required center of the required circle
(iv) Join OA
(v) With center O and radius OA, complete the circle

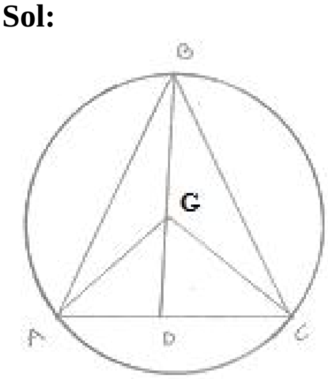
9. Prove that two different circles cannot intersect each other at more than two points.
Sol:
Suppose two circles intersect in three points A,B,C,
Then A,B,C are non-collinear. So, a unique circle passes through these three points. This is contradiction to the face that two given circles are passing through A,B,C. Hence, two circles cannot intersect each other at more than two points.

10. A line segment AB is of length 5 cm. Draw a circle of radius 4 cm passing through A and B. Can you draw a circle of radius 2 cm passing through A and B? Give reason in support of your answer.
Sol:



(i) Draw a line segment AB of 5cm
(ii) Draw the perpendicular bisectors of AB
(iii) Draw an arc which intersects the perpendicular bisectors at point O will be required center.
(iv) With center O and radius OA draw a circle. No, we cannot draw a circle of radius 2cm passing through A and B because when we draw an arc of radius 2cm with center A, the arc will not interest the perpendicular bisector and we will not find the center

11. An equilateral triangle of side 9cm is inscribed in a circle. Find the radius of the circle.
Sol:



Let ABC be an equilateral triangle of side 9cm and let AD one of its medians. Let G be the centroid of ΔABC. Then $AG:GD = 2:1$

WKT in an equilateral Δ^{le} centroid coincides with the circum center

Therefore, G is the center of the circumference with circum radius GA

Also G is the center and $GD \perp BC$. Therefore,

In right triangle ADB, we have

$AB^2 = AD^2 + DB^2$

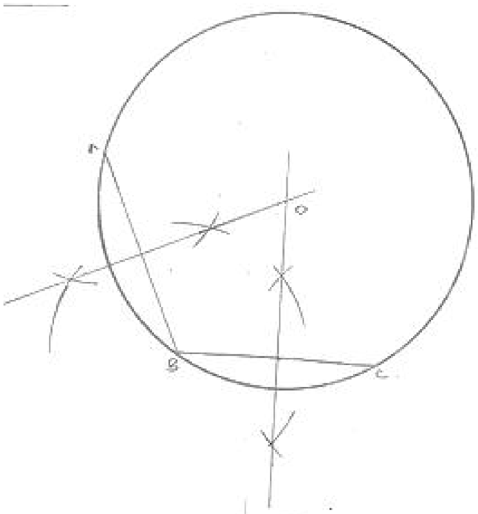
$\Rightarrow 9^2 = AB^2 + DB^2$

$\Rightarrow AD = \sqrt{81 - \frac{81}{4}} = \frac{9\sqrt{3}}{2} cm$

$\therefore$ Radius $= AG = \frac{2}{3} AD = 3\sqrt{3} cm.$

12. Given an arc of a circle, complete the circle.

Sol:



Steps of construction:

(i) Take three point A, B, C on the given Arc

(ii) Join AB and BC

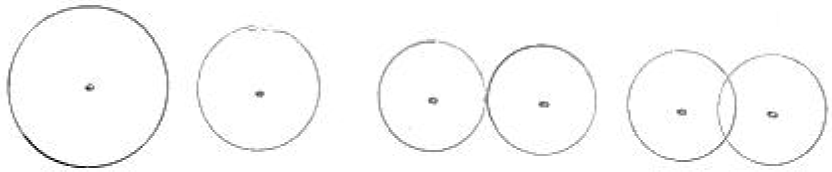
(iii) Draw the perpendicular bisectors of chords AB and BC which interest each other at point O, then O will required center of the required circle

(iv) Join OA

(v) With center O and radius OA, complete the circle

13. Draw different pairs of circles. How many points does each pair have in common? What is the maximum number of common points?

Sol:

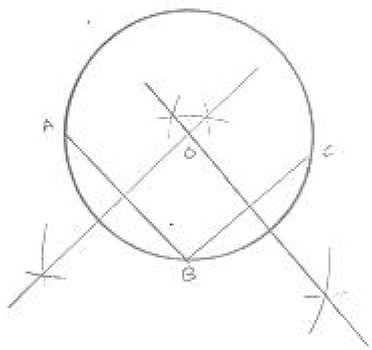


Each pair of circles have 0,1 or 2 points in common

The maximum number of points in common is ‘2’

14. Suppose you are given a circle. Give a construction to find its centre.

Sol:



Steps of constructions:

(1) Take three point A, B and C the given circle

(2) Join AB and BC

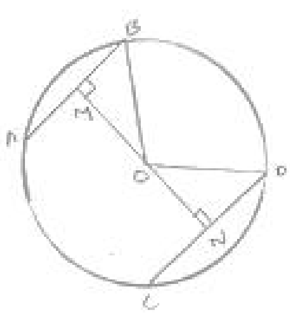
(3) Draw the perpendicular bisectors of chord AB and BC which intersect each other at O.

(4) Point O will be the required center of the circle because we know that the perpendicular bisector of the cord always passes through the center

15. Two chords AB and CD of lengths 5 cm and 11 cm respectively of a circle are parallel to each other and are opposite side of its center. If the distance between AB and CD is 6 cm. Find the radius of the circle.

Sol:

Draw $OM \perp AB$ *and* $ON \perp CD$. Join OB and OD



$BM = \frac{AB}{2} = \frac{5}{2}$ (Perpendicular from center bisects the chord)

$ND = \frac{CD}{2} = \frac{11}{2}$

Let ON be x, So OM will be $6-x$ in ΔMOB

$OM^2 + MB^2 = OB^2$

$(6-x)^2 + \left(\frac{5}{2}\right)^2 = OB^2$

$36 + x^2 - 12x + \frac{25}{4} = OB^2 \qquad .....(1)$

In ΔNOD

$ON^2 + ND^2 = OD^2$

$x^2 + \left(\frac{11}{2}\right)^2 = OD^2$

$x^2 + \frac{121}{4} = OD^2 \qquad ......(2)$

We have $OB = OD.$ (radii of same circle)

So, from equation (1) and (2).

$36 + x^2 - 12x + \frac{25}{4} = x^2 + \frac{121}{4}$

$\Rightarrow 12x = 36 + \frac{25}{4} - \frac{121}{4}$

$= \frac{144+25-121}{4} = \frac{48}{4} = 12$

$x = 1.$

From equation (2)

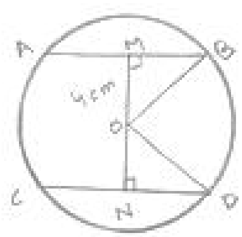
$(1)^2 + \left(\frac{121}{4}\right) = OD^2$

$OD^2 = 1 + \frac{121}{4} = \frac{121}{4}$

$OD = \frac{5\sqrt{5}}{2}$

So, radius of circle is found to be $\frac{5\sqrt{5}}{2}$ cm

16. The lengths of two parallel chords of a circle are 6 cm and 8 cm. if the smaller chord is at a distance of 4 cm from the centre, what is the distance of the other chord from the centre?
Sol:



Distance of smaller chord AB from center of circle $=4cm\ \ OM=4cm$

$MB=\frac{AB}{2}=\frac{6}{2}=3cm$

In ΔOMB

$OM^2+MB^2=OB^2$

$(4)^2+3^2=OB^2$

$16+9=OB^2$

$OB=\sqrt{25}$

$OB=5cm$

In ΔOND

$OD=OB=5cm$ [radii of same circle]

$ND=\frac{CD}{2}=\frac{8}{2}=4cm$

$ON^2+ND^2=OD^2$

$ON^2+(4)^2=(5)^2$

$ON^2=25-16=9$

$ON=3$

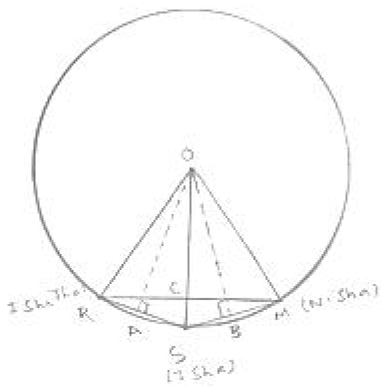
So, distance of bigger chord from circle is 3cm.

Exercise – 16.3

1. Three girls Ishita, Isha and Nisha are playing a game by standing on a circle of radius 20 m drawn in a park. Ishita throws a ball o Isha, Isha to Nisha and Nisha to Ishita. If the distance between Ishita and Isha and between Isha and Nisha is 24 m each, what is the distance between Ishita and Nisha.

Sol:

Let R, S and M be the position of Ishita, Isha and Nasha respectively



$AR = AS = \frac{24}{2} = 12cm$

$OR = OS = OM = 20m$ (radii of circle)

In OAR

$OA^2 + AR^2 = OR^2$

$OA^2 + (112m)^2 = (20m)^2$

$OD^2 = (400 - 144)m^2 = 256m^2$

$OA = 16m$

WKT, in an isosceles triangle altitude divides the base, So in $\Delta RSM \angle RCS$ will be $90°$ and $RC = CM$.

Area of $\Delta ORS = \frac{1}{2} \times OA \times RS$

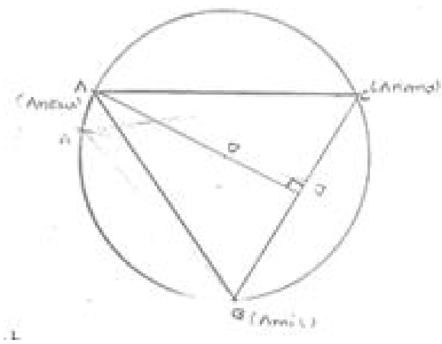
$\Rightarrow \frac{1}{2} \times RC \times OS = \frac{1}{2} \times 16 \times 24$

$\Rightarrow RC \times 20 = 16 \times 24 \Rightarrow RC = 192 \Rightarrow RM = 2(192) = 38 \cdot 4m$

So, distance between ishita and Nisha is $384m$.

2. A circular park of radius 40 m is situated in a colony. Three boys Ankur, Amit and Anand are sitting at equal distance on its boundary each having a toy telephone in his hands to talk to each other. Find the length of the string of each phone.

Sol:



Given hat $AB = BC = CA$

So, ABC is an equilateral triangle

OA(radius) $= 40m.$

Medians of equilaterals triangles pass through the circum center (0) of the equilaterals triangles ABC

We also know that median intersect each other at the $2:1$As AD is the median of equilaterals triangle ABC, we can write:

$$\frac{OA}{OD} = \frac{2}{7}$$

$$\Rightarrow \frac{40m}{OD} = \frac{2}{7}$$

$$\Rightarrow OD = 20m.$$

$$\therefore AD = OA + OD = (40 + 20)m$$

$$= 60m$$

In ΔADC

By using Pythagoras theorem

$$AC^2 = AD^2 + DC^2$$

$$AC^2 = AD^2 + DC^2$$

$$AC^2 = (60)^2 + \left(\frac{AC}{2}\right)^2$$

$$AC^2 = 3600 + \frac{AC^2}{4}$$

$$\Rightarrow \frac{3}{4}AC^2 = 3600$$

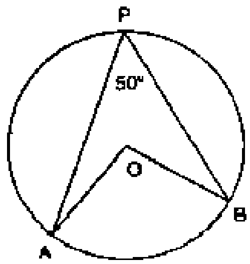
$$\Rightarrow AC^2 = 4800$$

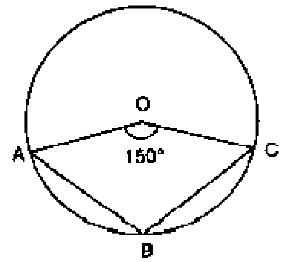
$$\Rightarrow AC = 40\sqrt{3}m$$

So, length of string of each phone will be $40\sqrt{3}m$

Exercise – 16.4

1. In the below fig. O is the centre of the circle. If ∠APB = 50°, find ∠AOB and ∠OAB.





Sol:

$\angle APB = 50°$

By degree measure theorem

$\angle AOB = 2\angle APB$

$\Rightarrow \angle APB = 2\times 50° = 100°$

Since $OA = OB$ [Radius of circle]

Then $\angle OAB = \angle OBA$ [Angle's opposite to equal sides]

Let $\angle OAB = x$

In $\Delta OAB \cdot$ by angle sum property

$\angle OAB + \angle OBA + \angle AOB = 180°$

$\Rightarrow x + x + 100 = 180°$

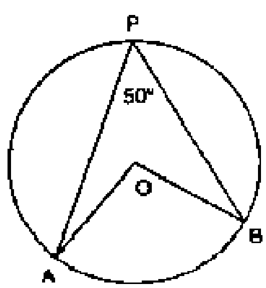
$\Rightarrow 2x + 100 = 180°$

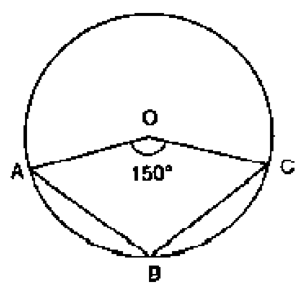
$\Rightarrow 2x = 80°$

$\Rightarrow x = 40°$

$\angle OAB = \angle OBA = 40°$

2. In the fig below, it is given that O is the centre of the circle and ∠AOC = 150°. Find ∠ABC.





Sol:

We have $\angle AOC = 150°$

$\therefore \angle AOC + reflex\ \angle AOC = 360°$ [complex angle]

$\Rightarrow 150° + reflex\ \angle AOC = 360°$

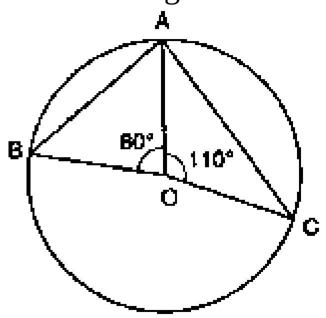
$\Rightarrow reflex\ \angle AOC = 360° - 150°$

$\Rightarrow reflex\ \angle AOC = 210°$

$\Rightarrow 2\angle ABC = 210°$ [By degree measure theorem]

$\Rightarrow \angle ABC = \frac{210}{2} = 105°$

3. In the below fig. O is the centre of the circle. Find ∠BAC.



Sol:

We have $\angle AOB = 80°$

And $\angle AOC = 110°$

$\therefore \angle AOB + \angle AOC + \angle BOC = 360°$ [Complete angle]

$\Rightarrow 80° + 110° + \angle BOC = 360°$

$\Rightarrow \angle BOC = 360° - 80° - 110°$

$\Rightarrow \angle BOC = 170°$

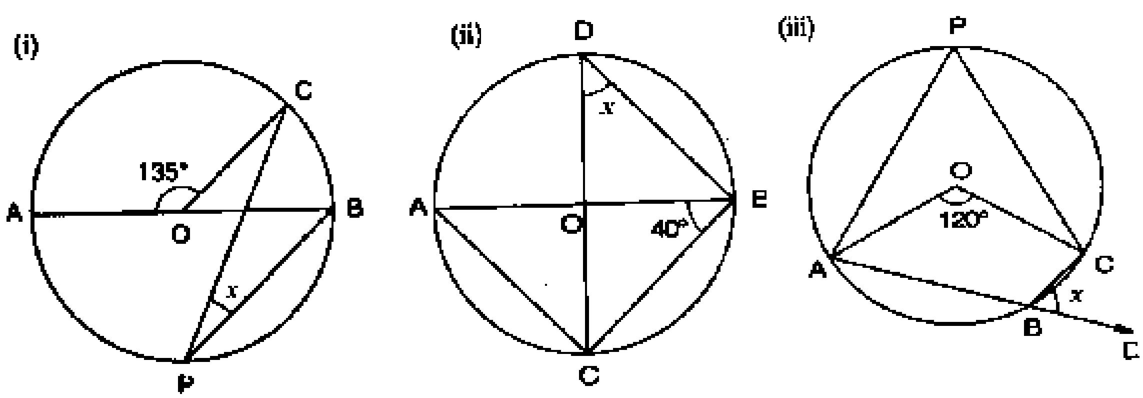
By degree measure theorem

$\angle BOC = 2\angle BAC$

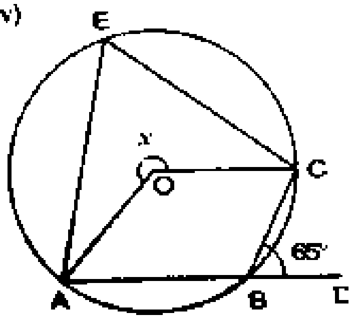
$\Rightarrow 170° = 2\angle BAC$

$\Rightarrow \angle BAC = \frac{170°}{2} = 85°$

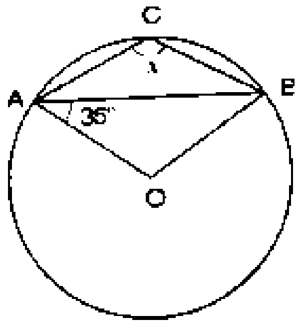
4. If O is the centre of the circle, find the value of x in each of the following figures:



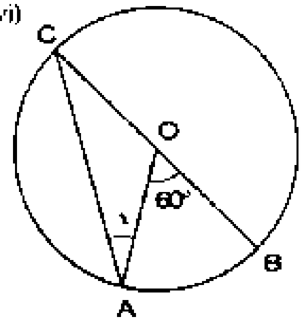
(iv)



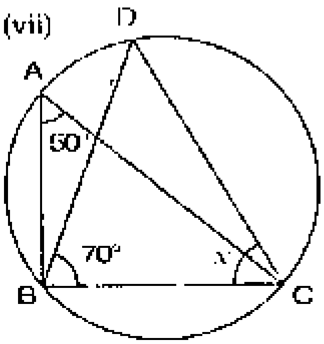
(v)



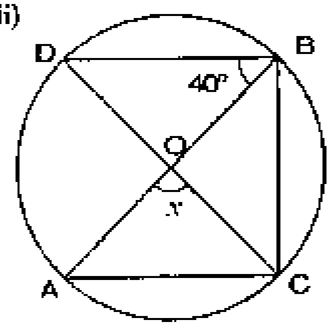
(vi)



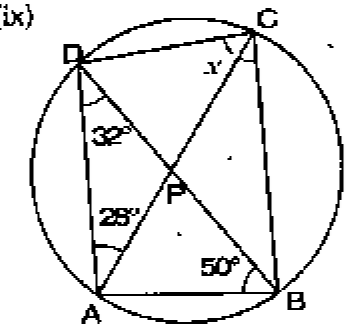
(vii)



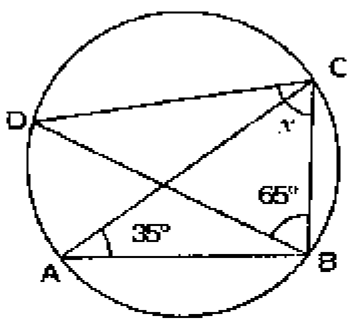
(viii)



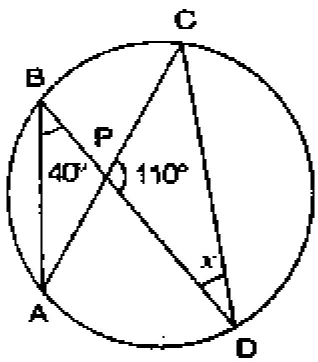
(ix)



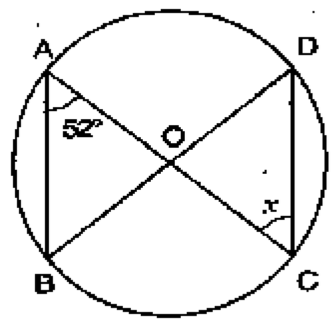
(x)



(xi)



(xii)



Sol:

(i) $\angle AOC = 135°$

$\therefore \angle AOC + \angle BOC = 180°$ [Linear pair of angles]

$\Rightarrow 135° + \angle BOC = 180°$

$\Rightarrow \angle BOC = 180° - 135° = 45°$

By degree measures theorem

$\angle BOC = 2\angle CDB$

$\Rightarrow 45° = 2x$

$\Rightarrow x = \frac{45°}{2} = 22\frac{1°}{2}.$

(ii) We have

$\angle ABC = 40°$

$\angle ACB = 90°$ [Angle in semicircle]

In $\Delta ABC,$ by angle sum property

$\angle CAB+\angle ACB+\angle ABC=180°$

$\Rightarrow \angle CAB+90°+40°=180°$

$\Rightarrow \angle CAB=180°-90°$

$\Rightarrow \angle CAB=50°$

Now,

$\angle COB=\angle CAB$ [Angle is same segment]

$\Rightarrow x=50°$

(iii) We have

$\angle AOC=120°$

By degree measure theorem

$\angle AOC=2\angle APC$

$\Rightarrow 120°=2\angle APC$

$\Rightarrow \angle APC=\frac{120°}{2}=60°$

$\therefore \angle APC+\angle ABC=180°$ [Opposite angles of cyclic quadrilateral]

$\Rightarrow 60°+\angle ABC=180°$

$\Rightarrow -60°+180°=\angle ABC$

$\Rightarrow \angle ABC=120°$

$\therefore \angle ABC+\angle DBC=180°$ [Linear pair of angles]

$\Rightarrow 120+x=180°$

$\Rightarrow x=180°-120°=60°$

(iv) We have

$\angle CBD=65°$

$\therefore \angle ABC+\angle CBD=180°$ [Linear pair of angles]

$\Rightarrow \angle ABC+65°=180°$

$\Rightarrow \angle ABC=180°-65°=115°$

$\therefore$ Reflex $\angle AOC=2\angle ABC$ [By degree measure theorem]

$\Rightarrow x=2\times 115°$

$\Rightarrow x=230°$

(v) We have

$\angle OAB=35°$

Then, $\angle OBA=\angle OAB=35°$ [Angles opposite to equal radii]

In $\Delta AOB,$ by angle sum property

$\angle AOB+\angle OAB+\angle OBA=180°$

$\Rightarrow \angle AOB+35°+35°=180°$

$\Rightarrow \angle AOB=180°-35°-35°=110°$

$\therefore \angle AOB+reflex\ \angle AOB=360°$ [comple angle]

$\Rightarrow 110° + reflex\ \angle AOB = 360°$

$\Rightarrow reflex\ \angle AOB = 360° - 110° = 250°$

By degree measure theorem reflex $\angle AOB = 2\angle ACB$

$\Rightarrow 250° = 2x$

$\Rightarrow x = \frac{250°}{2} = 125°$

(vi) We have

$\angle AOB = 60°$

By degree measure theorem

$\angle AOB = 2\angle ACB$

$\Rightarrow 60° = 2\angle ACB$

$\Rightarrow \angle ACB = \frac{60°}{2} = 30°$ [Angles opposite to equal radii]

$\Rightarrow x = 30°$

(vii) We have

$\angle BAC = 50°$

And $\angle DBC = 70°$

$\therefore \angle BDC = \angle BAC = 50°$ [Angle in same segment]

In ΔBDC, by angles sum property

$\angle BDC + \angle BCD + \angle DBC = 180°$

$\Rightarrow 50° + x + 70 = 180°$

$\Rightarrow x = 180° - 70° - 50° = 60°$

(viii) We have

$\angle DBO = 40°$

$\angle DBC = 90°$ [Angle in semi circle]

$\Rightarrow \angle DBO + \angle OBC = 90°$

$\Rightarrow 40° + \angle OBC = 90°$

$\Rightarrow \angle OBC = 90° - 40° = 50°$

By degree measure theorem

$\angle AOC = 2\angle OBC$

$\Rightarrow x = 2 \times 50° = 100°$

(ix) In ΔDAB, by angle sum property

$\angle ADB + \angle DAB + \angle ABD = 180°$

$\Rightarrow 32° + \angle DAB + 50° = 180°$

$\Rightarrow \angle OAB = 180° - 32° - 50°$

$\Rightarrow \angle DAB = 95°$

Now,

$\angle OAB + \angle DCB = 180°$ [Opposite angles of cyclic quadrilateral]

$\Rightarrow 98 + x = 180°$

$\Rightarrow x = 180 - 98° = 82°$

(x) We have

$\angle BAC = 35°$

$\angle BAC = \angle BAC = 35°$ [Angle in same segment]

In ΔBCD by angle sum property

$\angle BDC + \angle BCD + \angle DBC = 180°$

$\Rightarrow 35 + x + 65° = 180°$

$\Rightarrow x = 180° - 35° - 66° = 80°$

(xi) We have

$\angle ABD = 40°$

$\therefore \angle ACD = \angle ABD = 40°$ [Angle in same segment]

In ΔPCD, By angle sum property

$\angle PCD + \angle CPO + \angle PDC = 180°$

$\Rightarrow 40° + 110° + x° = 180°$

$\Rightarrow x° = 180° - 150°$

$\Rightarrow x = 30°$

(xii) Given that $\angle BAC = 52°$

Then, $\angle BDC = \angle BAC = 52°$ [Angle in same segment]

Since $OD = OC$

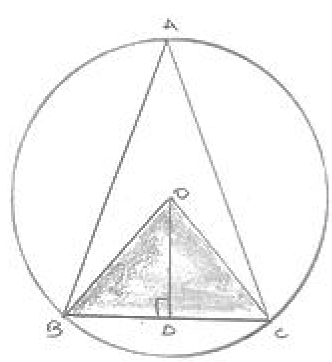
Then, $OD = OC$

Then, $\angle ODC = \angle OCD$ [Opposite angles to equal radii]

$\Rightarrow x = 52°$

5. O is the circumcentre of the triangle ABC and OD is perpendicular on BC. Prove that ∠BOD = ∠A.

Sol:



Given O is the circum center of ΔABC and $OD \perp BC$

To prove $\angle BOD = 2\angle A$

Proof:

In $\Delta OBA\ and\ \Delta OCA$

$\angle ODB = \angle ODC$ [Each $90°$]

$OB = OC$ [Radii of circle]

$OD = OD$ [Common]

Then, $\Delta OBD \cong \Delta OCD$ [By RHS condition]

$\therefore \angle BOD = \angle COD$ ....(1) $(P \cdot C \cdot T)$

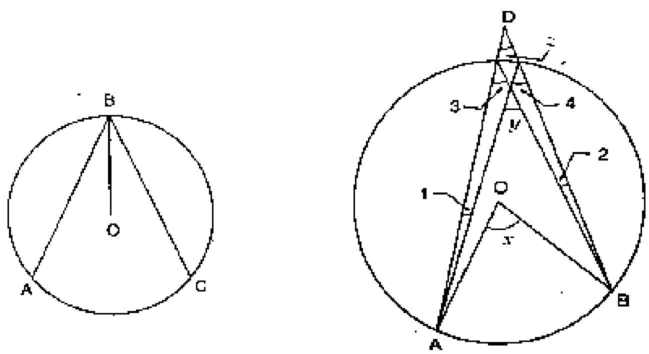
By degree measure theorem

$\angle BOC = 2\angle BAC$

$\Rightarrow 2\angle BOD = 2\angle BAC$ [By using (1)]

$\Rightarrow \angle BOD = \angle BAC$

6. In the fig. below, O is the centre of the circle, BO is the bisector of ∠ABC. Show that AB = AC.



Sol:

Given, BO is the bisector of $\angle ABC$

To prove $AB = BC$

Proof:

Since, BO is the bisector of $\angle ABC$

Then, $\angle ABO = \angle DAB$ ....(2) [Opposite angles to equal sides]

Since $OB = OC$ [Radius of circle]

The $\angle CBO = \angle OCB$ ....(3) [Opposite angles to equal sides]

Compare equation (1), (2) and (3)

$\angle OAB = \angle OCB$ ....(4)

In $\angle OAB = \angle OCB$ [from (4)]

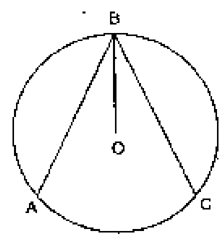
$\angle OBA = \angle OBC$ [Given]

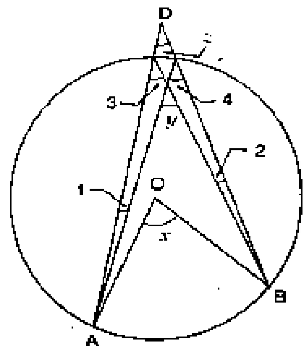
$OB = OB$ [Common]

Then, $\Delta OAB \cong OCB$ [By AAS condition]

$\therefore AB = BC$ $[c \cdot p \cdot c \cdot t]$

7. In the below fig. O is the centre of the circle, prove that ∠x = ∠y + ∠z.





Sol:

We have, $\angle 3 = \angle 4$ [Angles in same segment]

$\therefore \angle x = 2\angle 3$ [By degree measure theorem]

$\Rightarrow \angle x = \angle 3 + \angle 8$

$\Rightarrow \angle x = \angle 3 + \angle 4$ ....$(1)[\angle 3 = \angle 4]$

But $\angle y = \angle 3 + \angle 1$ [by exterior angle prop]

$\Rightarrow \angle 3 = \angle y - \angle 1$ ....(2)

From (1) and (2)

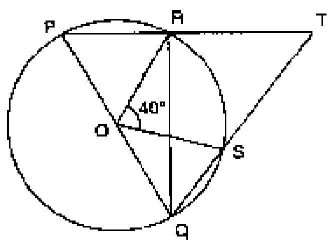
$\angle x = \angle y - \angle 1 + \angle 4$

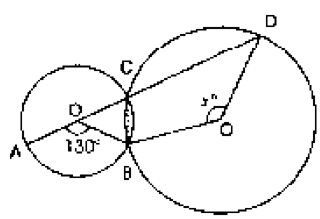
$\Rightarrow \angle x = \angle y + \angle y - \angle 1$

$\Rightarrow \angle x = \angle y + \angle z$ (By exterior angle prop)

$\Rightarrow \angle x = \angle y + \angle z$

8. In the below fig. O and O' are centres of two circles intersecting at B and C, ACD is a straight line, find x.





Sol:

By degree measure theorem

$\angle AOB = 2\angle ACB$

$\Rightarrow 130° = 2\angle ACB$

$\Rightarrow \angle ACB = \frac{130°}{2} = 65°$

$\therefore \angle ACB + \angle BCD = 180°$ [Linear pair of angle]

$\Rightarrow 65° + \angle BCD = 180°$

$\Rightarrow \angle BCD = 180° - 65° = 115°$

By degree measure theorem

Reflex $\angle BOD = 2\angle BCA$

$\Rightarrow$ Reflex $\angle BOD = 2 \times 115° = 230°$

Now, reflex $\angle BOD + \angle BOD = 360°$ [Complex angle]

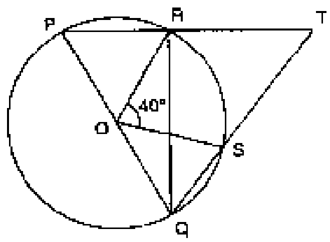
$\Rightarrow 230° + x = 360°$

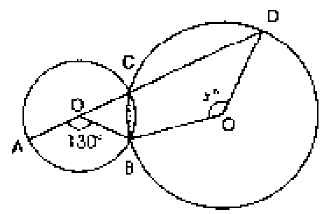
$\Rightarrow x = 360° - 230°$

$\Rightarrow 130°$

$x = 130°$

9. In the below fig. O is the centre and PQ is a diameter. If ∠ROS = 40°, find ∠RTS.





Sol:

Since PQ is diameter

Then, $\angle PRO = 90°$ [Angle in semi-circle]

$\therefore \angle PRQ + \angle TRQ = 180°$ [Linear pair of angle]

$\angle 90° + \angle TRQ = 180°$

$\angle TRQ = 180° - 90° = 90°$

By degree measure theorem

$\angle ROS = 2\angle RQS$

$\Rightarrow 40° = 2\angle RQS$

$\Rightarrow \angle RQS = \frac{40°}{2} = 20°$

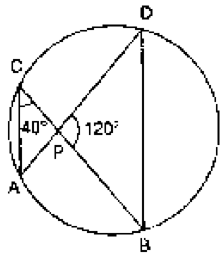
In ΔRQT, By angle sum property

$\angle RQT + QRT + \angle RTS = 180°$

$\Rightarrow 20° + 90° + \angle R + \angle S = 180°$

$\Rightarrow \angle RTS = 180° - 20° - 90° = 70°$

10. In the below fig. if ∠ACB = 40°, ∠DPB = 120°, find ∠CBD.



Sol:

We have

$\angle ACB = 40°, \angle DPB = 120°$

$\therefore \angle ADB = \angle ACB = 40°$ [Angle in same segment]

In $\Delta POB,$ by angle sum property

$\angle PDB + \angle PBD + \angle BPP = 180°$

$\Rightarrow 40° + \angle PBD + 120° = 180°$

$\Rightarrow \angle PBD = 180° - 40° - 120°$

$\Rightarrow \angle PBD = 20°$

$\therefore \angle CBD = 20°$

11. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

Sol:

We have

Radius $OA = chord\ \ AB$

$\Rightarrow OA = OB = AB$

Then ΔOAB is an equilateral triangle

$\therefore \angle AOB = 60°$ [one angle of equilateral]

By degree measure theorem

$\angle AOB = 2\angle APB$

$\Rightarrow 60° = 2\angle APB$

$\Rightarrow 60° = 2\angle APB$

$\Rightarrow \angle APB = \frac{60°}{2} = 30°$

Now, $\angle APB + \angle AQB = 180°$ [opposite angles of cyclic quadrilaterals]

$\Rightarrow 30° + \angle AQB = 180°$

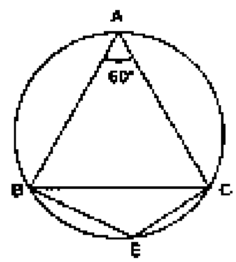
$\Rightarrow \angle AQB = 180° - 30° = 150°$

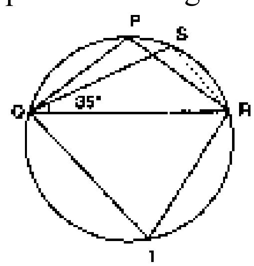
$\therefore$ Angle by chord AB at minor arc $= 150°$

Angle by chord AB at major arc $= 30°$

Exercise – 16.5

1. In the below fig. ΔABC is an equilateral triangle. Find m ∠BEC.





Sol:

Since, ΔABC is an equilateral triangles

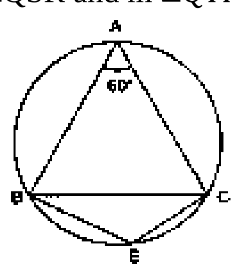
Then, $\angle BAC = 60°$

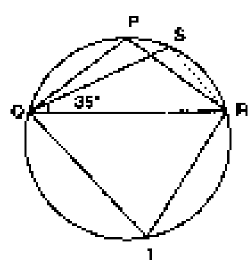
$\therefore \angle BAC + \angle BEC = 180°$ [Opposite angles of a quadrilaterals]

$\Rightarrow 60° + \angle BEC = 180° \Rightarrow \angle BEC = 180° - 60°$

$\Rightarrow \angle BEC = 180°$

2. In the below fig. ΔPQR is an isosceles triangle with PQ = PR and m ∠PQR = 35°. Find m ∠QSR and m ∠QTR.





Sol:

We have $\angle PQR = 35°$

Since, ΔPQR is an isosceles triangle with $PQ = RR$

Then $\angle PQR = \angle PRQ = 35°$

In ΔPQR by angle sum property

$\angle P + \angle PQR + \angle PRQ = 180°$

$\Rightarrow \angle P + 35° + 35° = 180°$

$\Rightarrow \angle P = 180° - 35° = 35°$

$\Rightarrow \angle P = 110°$ [Angles in same segment]

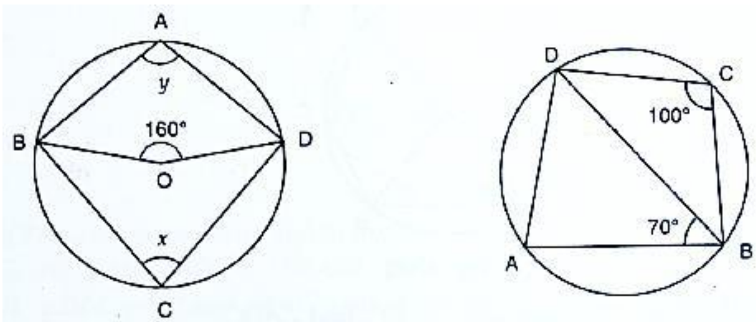
Now, $\angle QSR + \angle QTR = 180°$

$\Rightarrow 110° + \angle QTR = 180°$

$\Rightarrow \angle QTR = 180° - 110°$

$\Rightarrow \angle QTR = 70°$

3. In the below fig., O is the centre of the circle. If ∠BOD = 160°, find the values of x and y.



Sol:

Given that O is the center of the circle

We have, $\angle BOD = 160°$

By degree measure theorem

$\angle BOD = 2\angle BCD$

$\Rightarrow 160° = 2 \times x$

$\Rightarrow x = \frac{160°}{2} = 80°$

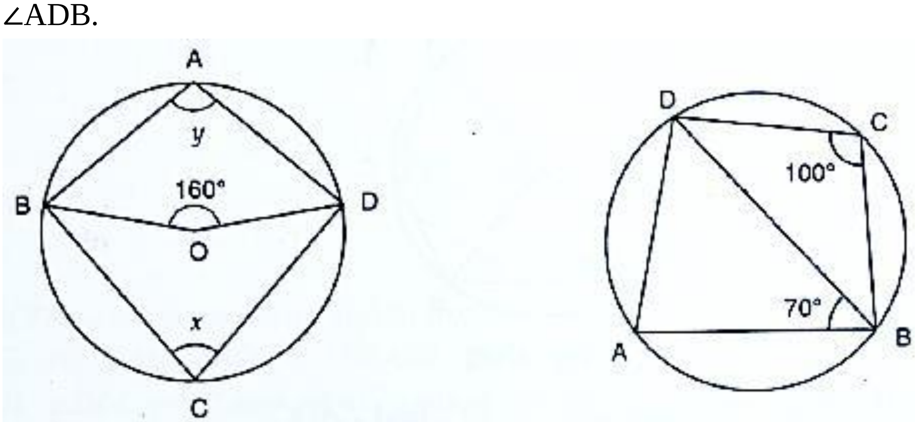
$\therefore \angle BAD + \angle BCD = 180°$ [Opposite angles of cyclic quadrilaterals]

$\Rightarrow y + x = 180°$

$\Rightarrow y + 80° = 180°$

$\Rightarrow y = 180° - 80° = 100°$

4. In the below fig. ABCD is a cyclic quadrilateral. If ∠BCD = 100° and ∠ABD = 70°, find ∠ADB.



Sol:

We have

$\angle BCD = 100°$ *and* $\angle ABD = 70°$

$\therefore \angle DAB + \angle BCD = 180°$ [Opposite angles of cyclic quadrilaterals]

$\Rightarrow \angle DAB + 100° = 180°$

$\Rightarrow \angle DAB = 180° - 100° = 80°$

$\Rightarrow \angle PAB = 80°$

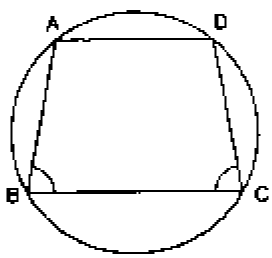
In $\Delta DAB,$ by angle sum property

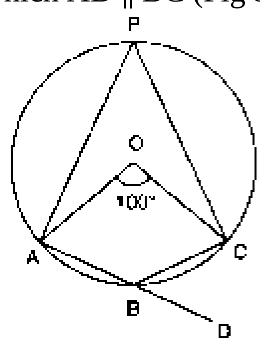
$\angle ADB + \angle DAB + \angle DBD = 180°$

$\Rightarrow \angle ABD + 80° + 70° = 180°$

$\Rightarrow \angle ABD = 180° - 150° = 30°$

5. If ABCD is a cyclic quadrilateral in which AD || BC (Fig below). Prove that ∠B = ∠C.





Sol:

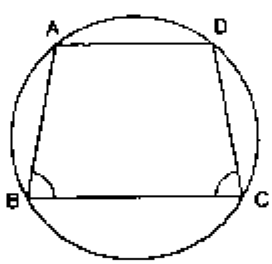
Since $ABCD$ is a cyclic quadrilateral with $AD \parallel BC$.

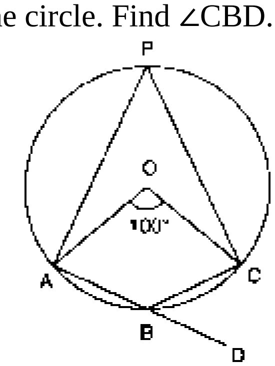
Then $\angle A + \angle C = 180°$ .....(1) [Opposite angles of cyclic quadrilaterals]

And, $\angle A + \angle B = 180°$ ......(2) [Co interior angles]

Compare (1) and (2) equations $\angle B = \angle C$

6. In the below fig. O is the centre of the circle. Find ∠CBD.





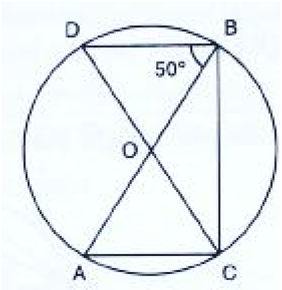
Sol:

Given that $\angle BOC = 100°$

By degree measure theorem

$\angle AOC = 2\angle APC$
$\Rightarrow 100° = 2\angle APC$
$\Rightarrow \angle APC = \frac{100°}{2} = 90°$
$\therefore \angle APC + \angle ABC = 180°$ [Opposite angles of cyclic quadrilaterals]
$\Rightarrow 50° + \angle ABC = 180°$
$\Rightarrow \angle ABC = 180° - 50°$
$= 130°$
$\therefore \angle ABC + \angle CBD = 180°$ [Linear pair of angles]
$\Rightarrow 130° + \angle CBD = 180°$
$\Rightarrow \angle CBD = 50°$

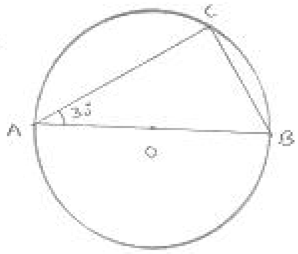
7. In the below fig. AB and CD are diameters of a circle with centre O. if ∠OBD = 50°, find ∠AOC.



Sol:
Given that,
$\angle OBD = 50°$
Since, AB *and* CD are the diameter of circle then O is the center of the circle
$\therefore \angle PBC = 90°$ [Angle in semicircle]
$\Rightarrow \angle OBD + \angle DBC = 90°$
$\Rightarrow 50° + \angle DBC = 90°$
$\Rightarrow \angle DBC = 90° - 50° = 40°$
By degree measure theorem
$\angle AOC = 2\angle ABC$
$\Rightarrow \angle AOC = 2 \times 40° = 80°$

8. On a semi-circle with AB as diameter, a point C is a taken, so that m (∠CAB) = 30°. Find m (∠ACB) and m (∠ABC).

Sol:



We have, $\angle CAB = 30°$

$\angle ACB = 90°$ [Angle in semicircle]

In $\Delta ABC,$ by angle sum property

$\angle CAB + \angle ACB + \angle ABC = 180°$

$\Rightarrow 30° + 90° + \angle ABC = 180°$

$\Rightarrow \angle ABC = 180° - 120°$

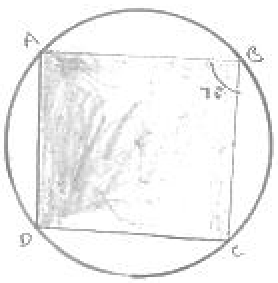
$= 60°$

9. In a cyclic quadrilateral ABCD if AB || CD and ∠B = 70°, find the remaining angles.

Sol:

Given that $\angle B = 70° = 70°$

Since $ABCD$ is a cyclic quadrilaterals



Then, $\angle B + \angle D = 180°$

$\Rightarrow 70° + \angle D = 180°$

$\Rightarrow \angle D = 180° - 70° = 110°$

Since $AB \parallel DC$

Then $\angle B + \angle C = 180°$

$\Rightarrow 70° + \angle C = 180°$ [Cointerior angles]

$\Rightarrow \angle C = 180° - 70°$

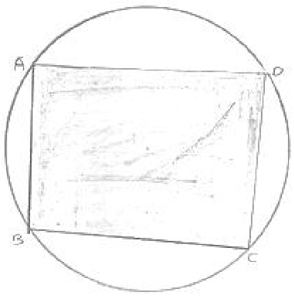
$= 110°$

Now, $\angle A + \angle C = 180°$ [Opposite angles of cyclic quadrilateral]

$\Rightarrow \angle A + 110° = 180°$

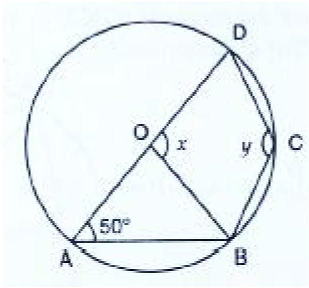
$\Rightarrow \angle A = 180° - 110°$
$\Rightarrow \angle A = 70°$

10. In a quadrilateral ABCD, if m ∠A = 3 (m ∠ C). Find m ∠A.
Sol:



We have, $\angle A = 3\angle C$
Let $\angle C = x$
Then $A = 3x$
$\therefore \angle A + \angle C = 180°$ [Opposite angles of cyclic quadrilaterals]
$\Rightarrow 3x + x = 180°$
$\Rightarrow 4x = 180° \Rightarrow x = \frac{180}{4} = 45°$
$\therefore \angle A = 3x$
$= 3 \times 45°$
$= 135°$
$\therefore \angle A = 135°$

11. In the below fig. O is the centre of the circle and ∠DAB = 50°. Calculate the values of x and y.



Sol:
We have $\angle DAB = 50°$
By degree measure theorem
$\angle BOD = 2\angle BAD$
$\Rightarrow x = 2 \times 50° = 100°$
Since, ABCD is a cyclic quadrilateral

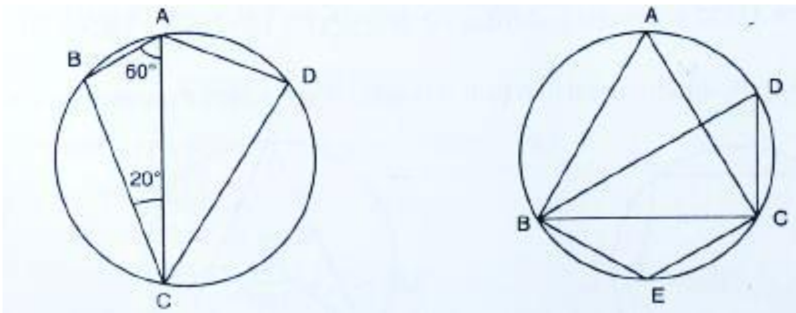
Then $\angle A + \angle C = 180°$

$\Rightarrow 50 + y = 180°$

$\Rightarrow y = 180° - 50°$

$= 130°$

12. In the below fig. if $\angle BAC = 60°$ and $\angle BCA = 20°$, find $\angle ADC$.



Sol:

By using angle sum property in ΔABC

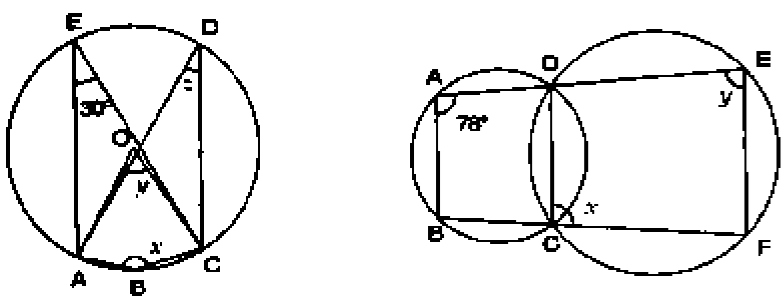
$\angle B = 180° - (60° + 120°) = 100°$

In cyclic quadrilaterals ABCD, we have:

$\angle B + \angle D = 180°$

$\angle D = 180° - 100° = 80°$

13. In the below fig. if ABC is an equilateral triangle. Find $\angle BDC$ and $\angle BEC$.



Sol:

Since ΔABC is an equilateral triangle

Then, $\angle BAC = 60°$

$\therefore \angle BDC = \angle BAC = 60°$ [Angles in same segment]

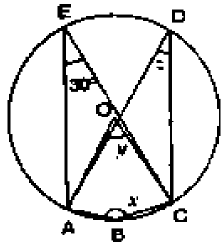
Since, quadrilaterals $ABEC$ is a cyclic quadrilaterals

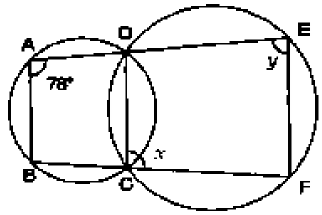
Then $\angle BAC + \angle BEC = 180°$

$\Rightarrow 60° + \angle BEC = 180°$

$\Rightarrow \angle BEC = 180° - 60° = 120°$

14. In the below fig. O is the centre of the circle, if ∠CEA = 30°, find the values of x, y and z.





Sol:

We have, $\angle AEC = 30°$

Since, quadrilateral $ABCE$ is a cyclic quadrilaterals

Then, $\angle ABC + \angle AEC = 180°$

$x + 30° = 180°$

$\Rightarrow x = 180° - 30° = 150°$

By degree measure theorem

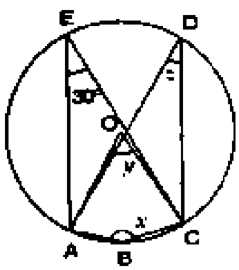
$\angle AOC = 2\angle AEC$

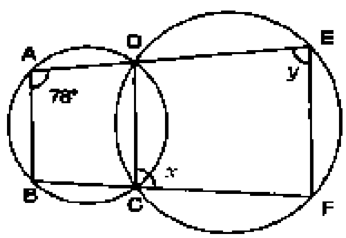
$\Rightarrow y = 2 \times 30° = 60°$

$\Rightarrow \therefore \angle ADC = \angle AEC$ [Angles in same segment]

$\Rightarrow z = 30°$

15. In the below fig. ∠BAD = 78°, ∠DCF = x° and ∠DEF y°. find the values of x, and y.





Sol:

We have, $\angle BAD = 78° \cdot \angle DCF = x°\ and\ \angle DEF = y°$

Since, ABCD is a cyclic quadrilateral

Then, $\angle BAD + \angle BCD = 180°$

$\Rightarrow 78° + \angle BCD = 180°$

$\Rightarrow \angle BCD = 180° - 78° = 102°$

Now, $\angle BCD + \angle DCF = 180°$ [Linear pair of angles]

$\Rightarrow 102° = x - 180°$

$\Rightarrow x = 180° - 102° = 78°$

Since, DCEF is a cyclic quadrilateral

Then, $x + y = 180°$

$\Rightarrow 78° + y = 180°$

$\Rightarrow y = 180° - 78° = 102°$

$\therefore y = 102°$

16. In a cyclic quadrilateral ABCD, if ∠A − ∠C = 60°, prove that the smaller of two is 60°.

Sol:

We have

$\angle A - \angle C = 60°$ ....(1)

Since, ABCD is a cyclic quadrilaterals

Then $\angle A + \angle C = 180°$ ....(2)

Add equations (1) and (2)

$\angle A - \angle C + \angle A + \angle C = 60° + 180°$

$\Rightarrow 2\angle A = 240°$

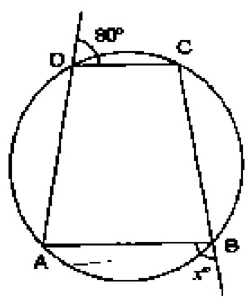
$\Rightarrow \angle A = \frac{240°}{2} = 120°$

Put value of $\angle A$ in equation (2)

$120° + \angle C = 180°$

$\Rightarrow \angle A = 180° - 120° = 60°$

17. In the below fig. ABCD is a cyclic quadrilateral. Find the value of x.



Sol:

$\angle EDC + \angle CDA = 180°$ [Linear pair of angles]

$\Rightarrow 80° + \angle CDA = 180°$

$\Rightarrow \angle CDA = 180° - 60° = 100°$

Since, ABCD is a cyclic quadrilateral

$\angle ADC + \angle ABC = 180°$

$\Rightarrow 100° + \angle ABC = 180°$

$\Rightarrow \angle ABC = 180° - 100° = 80°$

Now, $\angle ABC + \angle ABF = 180°$ [Linear pair of angles]

$\Rightarrow 80 + x° = 180°$

$\Rightarrow x = 180° - 80° = 100°$

18. ABCD is a quadrilateral in which:

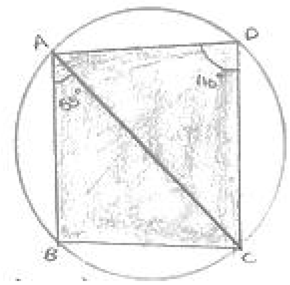
(i) BC || AD, ∠ADC = 110° and ∠BAC = 50°. Find ∠DAC.

(ii) ∠DBC = 80° and ∠BAC = 40°, find ∠BCD.

(iii) ∠BCD = 100° and ∠ABD = 70°, find ∠ADB.

Sol:

(i) Since, ABCD is a cyclic quadrilateral



Then, $\angle ABC + 110° = 180°$

$\Rightarrow \angle ABC + 110° = 180°$

$\Rightarrow \angle ABC = 180° - 110°$

$= 70°$

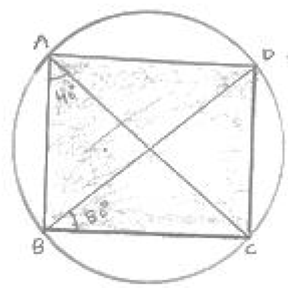
Since $AD \parallel BC$

Then, $\angle DAB + \angle ABC = 180°$ [Co-interior angle]

$\Rightarrow \angle DAC + 50° + 70° = 180°$

$\Rightarrow \angle DAC = 180° - 120° = 60°$

(ii) $\angle BAC = \angle BDC = 40°$ [Angle in same segment]



In ΔBDC, by angle sum property

$\angle DBC + \angle BCD + \angle BDC = 180°$

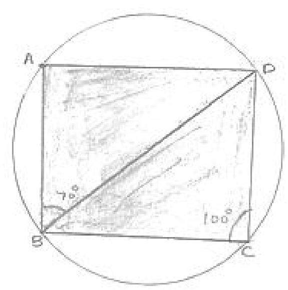
$\Rightarrow 80° + \angle BCD + 40° = 180°$

$\Rightarrow \angle BCD = 180° - 40° - 80°$

$\Rightarrow \angle BCD = 60°$

(iii) Given that $ABCD$ is a cyclic quadrilaterals

Ten $\angle BAD + \angle BCD = 180°$



$\Rightarrow \angle BAD + 100° = 180°$

$\Rightarrow \angle BAD = 180° - 100°$

$\Rightarrow \angle BAD = 80°$

In ΔABD, by angle sum property

$\angle ABD + \angle ADB + \angle BAD = 180°$

$\Rightarrow 70° + \angle ADB + 80° = 180°$

$\Rightarrow \angle ADB = 180° - 150°$

$\Rightarrow \angle ADB = 30°$

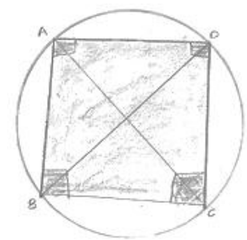
19. Prove that the perpendicular bisectors of the sides of a cyclic quadrilateral are concurrent.

Sol:

Let ABCD be a cyclic quadrilateral, and let O be the center of the corresponding circle
Then, each side of the equilateral ABCD is a chord of the circle and the perpendicular bisector of a chord always passes through the center of the circle
So, right bisectors of the sides of quadrilaterals ABCD, will pass through the circle O of the corresponding circle

20. Prove that the centre of the circle circumscribing the cyclic rectangle ABCD is the point of intersection of its diagonals.

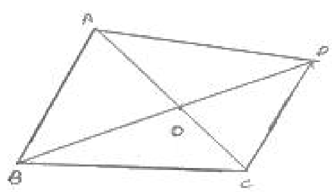
Sol:



Let O be the circle circumscribing the cycle rectangle ABCD. Since $\angle ABC = 90°$ and AC is a chord of the circle, so AC is a diameter of a circle. Similarly BD is a diameter
Hence, point of intersection of AC and BD is the center of the circle

21. Prove that the circles described on the four sides of a rhombus as diameters, pass through the point of intersection of its diagonals.

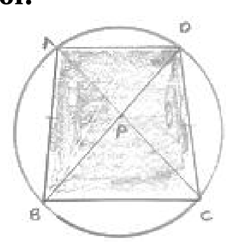
Sol:



Let ABCD be a rhombus such that its diagonals AC and BD interest at O
Since, the diagonals of a rhombus intersect at right angle
$\therefore \angle ACB = \angle BOC = \angle COD = \angle DOA = 90°$
Now, $\angle AOB = 90° \Rightarrow$ circle described on BC, AD and CD as diameter pass through O.

22. If the two sides of a pair of opposite sides of a cyclic quadrilateral are equal, prove that its diagonals are equal.

Sol:



Given $ABCD$ is a cyclic quadrilateral in which $AB = DC$
To prove: $AC = BD$
Proof: In $\Delta PAB \ and \ \Delta PDC$

Given that $AB = DC$

$\angle BAD = \angle CDP$ [Angles in the same segment]

$\angle PBA = \angle PCD$ [Angles in same segment]

Then $\Delta PAB = \Delta PDC$ ......(1) $[c \cdot p \cdot c \cdot t]$

$PC = PB$ ......$(2)[c \cdot p \cdot c \cdot t]$

Add equation (1) and (2)

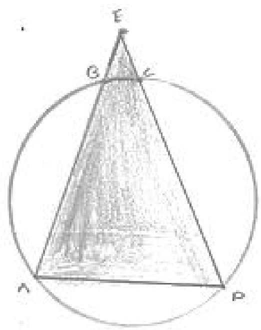
$PA + PC = PD + PB$

$\Rightarrow AC = BD$

23. ABCD is a cyclic quadrilateral in which BA and CD when produced meet in E and EA = ED. Prove that:

(i) AD || BC (ii) EB = EC

Sol:



Given ABCD is a cyclic quadrilateral in which $EA = ED$

To prove: (i) $AD \| BC$ (ii) $EB = EC$

Proof: (i) Since $EA = ED$

Then $\angle EAD = \angle EDA$ [Opposite angles to equal sides]

Since, ABCD is a cyclic quadrilaterals

Then, $\angle ABC + \angle ADC = 180°$

But $\angle ABC + \angle EBC = 180°$ [Linear pair of angles]

Then $\angle ADC = EBC$ ......(2)

Compare equations (1) and (2)

$\angle EAD = \angle CBA$ ......(3)

Since, corresponding angle are equal

Then $BC \| AD$

(ii) From equation (2)

$\angle EAD = \angle EBC$ .....(3)

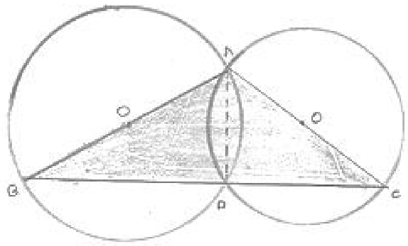
Similarly $\angle EDA = \angle ECB$ ....(4)

Compare equation (1), (3) and (4) $\angle EBC = \angle ECD$

$\Rightarrow EB = EC$ (Opposite angles to equal sides)

24. Circles are described on the sides of a triangle as diameters. Prove that the circles on any two sides intersect each other on the third side (or third side produced).

Sol:



Since AB is a diameter

Then $\angle ADB = 90°$ ......(1) [Angle in semicircle]

Since AC is a diameter

Then $\angle ADC = 90°$ ....(2) [Angle in semicircle]

Add equation (1) and (2)

$\angle ADB + \angle ADC = 90° + 90°$

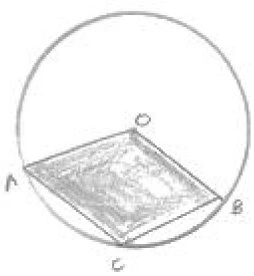
$\Rightarrow \angle BDC = 180°$

Then, BDC is a line

Hence, the circles on any two sides intersect each other on the third side

25. Prove that the angle in a segment shorter than a semicircle is greater than a right angle.

Sol:



Given: $\angle ACB$ is an angle in mirror segment

To prove: $\angle ACB > 90°$

Proof: By degree measure theorem

Reflex $\angle AOB > 180°$

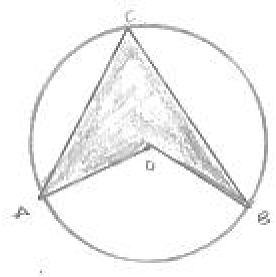
And reflex $\angle AOB > 180°$

Then, $2\angle ACB > 180°$

$\angle ACB > \frac{180°}{2}$

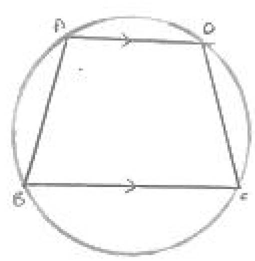
$\Rightarrow \angle ACB > 90°$

26. Prove that the angle in a segment greater than a semi-circle is less than a right angle.
Sol:



Given:
$\angle ACB$ is an angle in major segment
To prove $\angle ACB < 90°$
Proof: by degree measure theorem
$\angle AOB = 2\angle ACB$
And $\angle AOB < 180°$
Then, $2\angle ACB < 180°$
$\angle ACB < 90°$

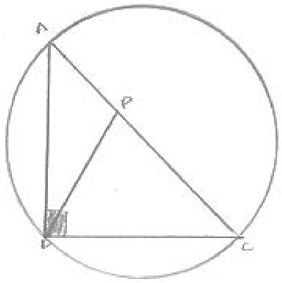
27. ABCD is a cyclic trapezium with AD || BC. If ∠B = 70°, determine other three angles of the trapezium.
Sol:



Given that
ABCD is a cyclic trapezium with $AD \parallel BC$ *and* $\angle B = 70°$
Since, $ABCD$ is a quadrilateral
Then $\angle B + \angle D = 180°$
$\Rightarrow 70° + \angle D = 180°$
$\Rightarrow \angle D = 180° - 70° = 110°$
Since $AD \parallel BC$
Then $\angle A + \angle B = 180° \Rightarrow \angle A + 70° = 180°$ [Cointerior angles]
$\Rightarrow \angle A = 110°$
Since $ABCD$ is a cyclic quadrilateral then $\angle A + \angle c = 180°$
$\Rightarrow 110° + \angle C = 180°$
$\Rightarrow \angle C = 180° - 110° = 70°$

28. Prove that the line segment joining the mid-point of the hypotenuse of a right triangle to its opposite vertex is half of the hypotenuse.

Sol:



Let ΔABC be a right angle triangle at angle B.

Let P be the midpoint of trypotenuse AC.

Draw a circle with center P and AC as a diameter

Since, $\angle ABC = 90°$, therefore the circle passes through B

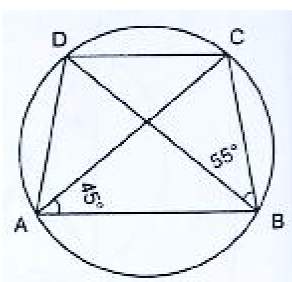
$\therefore BP = radius$

Also $AD = CP = Radius$

$\therefore AP = BP = CP$

Hence, $BP = \frac{1}{2} AC$.

29. In Fig. below, ABCD is a cyclic quadrilateral in which AC and BD are its diagonals. If ∠DBC = 55° and ∠BAC = 45°, find ∠BCD.



Sol:

Since angles in the same segment of a circle are equal

$\therefore \angle CAD = \angle DBC = 65°$

$\therefore \angle DAB = \angle CAD + \angle BAC = 55° + 45° = 100°$

But, $\angle DAB + \angle BCD = 180°$ [Opposite angles of a cyclic]

$\therefore \angle BCD = 180° - 100°$

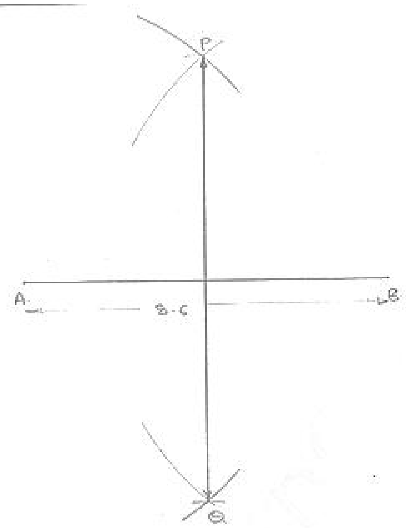
$= 80°$

$\therefore \angle BCD = 80°$

Exercise – 17.1

1. Draw a line segment of length 8.6 cm. Bisect it and measure the length of each part.

Sol:



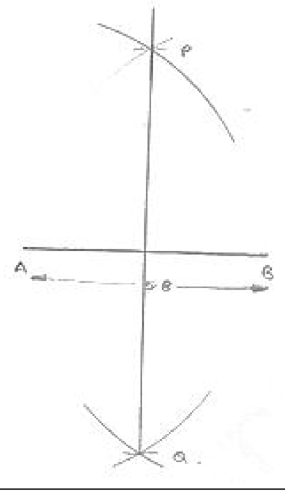
Steps of construction:

1. Draw a line segment AB of $8\cdot6cm$
2. With center A and radius more than $n\frac{1}{2}AB,$ draw arcs, one on each side of AB
3. With center B and same radius, draw arcs cutting the previous arcs at P and Q respectively
4. Join PQ

$\therefore AC = BC = 4\cdot3cm$

2. Draw a line segment AB of length 5.8 cm. Draw the perpendicular bisector of this line segment.

Sol:



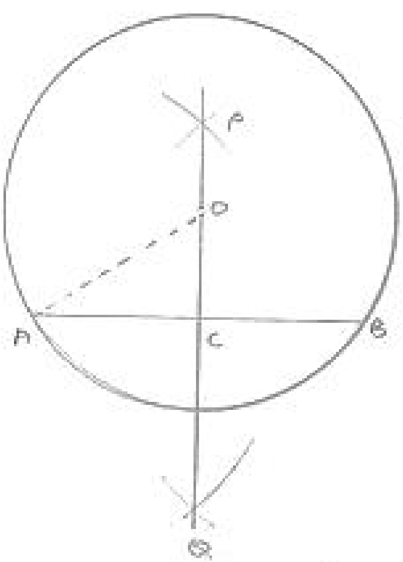
Steps of construction:

1. Draw a line segment AB of $5 \cdot 8 cm$
2. With center A and radius more than $\frac{1}{2}AB,$ draw arcs with one on each side of AB
3. With center B and same radius draw arcs cutting the previous arcs at P and Q respectively.
4. Join PQ

Hence, PQ is the perpendicular bisector of AB.

3. Draw a circle with centre at point O and radius 5 cm. Draw its chord AB, draw the perpendicular bisector of line segment AB. Does it pass through the centre of the circle?

Sol:



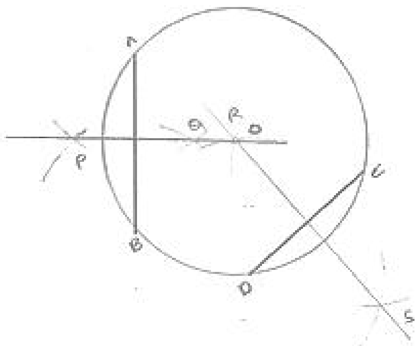
Steps of construction:

1. With center O and radius 5cm draw a circle
2. Draw a chord AB.
3. With center A and radius more than $\frac{1}{2}AB,$ draw arcs one on each side of
4. With center B and same radius draw arcs cutting previous arcs at P and Q respectively.
5. Join PQ

$\therefore$ yes perpendicular bisector PQ of AB passes through center of the circle.

4. Draw a circle with centre at point O. Draw its two chords AB and CD such that AB is not parallel to CD. Draw the perpendicular bisectors of AB and CD. At what point do they intersect?

Sol:



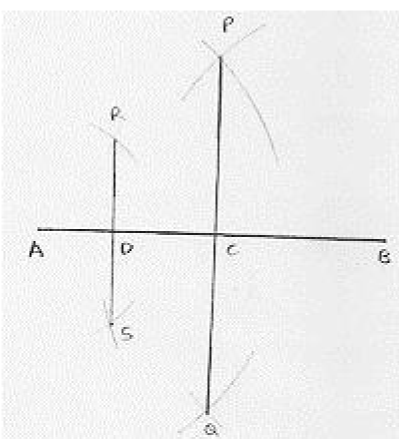
Steps of construction:

1. With center O and any radius, draw a circle
2. Draw two chords AB and CD.
3. With center A and radius more than $\frac{1}{2}AB,$ draw arcs, one on each side of AB
4. With center B and same radius draw arcs cutting previous arcs at P and Q respectively.
5. Join PQ
6. With center D and radius more than $\frac{1}{2}DC.$ draw arcs, one on each side of DC
7. With center C and same radius, draw arcs cutting previous arcs at R and S respectively
8. Join RS

Both perpendicular bisector PQ and RS intersect each other at the center O of he circle.

5. Draw a line segment of length 10 cm and bisect it. Further bisect one of the equal parts and measure its length.

Sol:



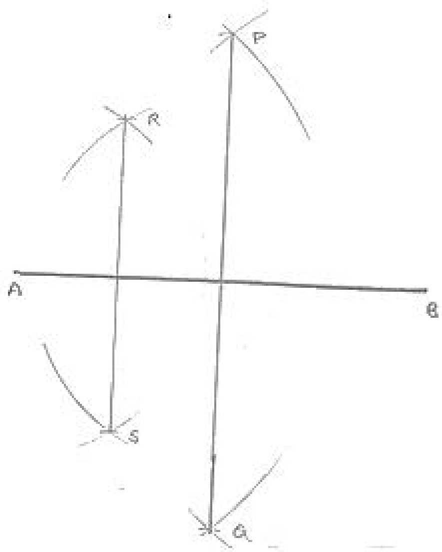
Steps of construction:

1. Draw a line segment AB of $10cm$
2. With center A and radius more than $\frac{1}{2}AB,$ draw arcs one on each side of AB
3. With center B and same radius draw arcs cutting previous arcs at P and Q respectively.
4. Join PQ and which intersect AB at C
5. With center A and radius more than $\frac{1}{2}AC,$ drawing on each side of AC.
6. With center C and same radius, draw arcs cutting previous arcs at R and S respectively.
7. Join RS and which intersect AC at b.

$\therefore AD = 2 \cdot 5cm.$

6. Draw a line segment AB and bisect it. Bisect one of the equal parts to obtain a line segment of length $\frac{1}{2}$ (AB).

Sol:



Steps of construction:

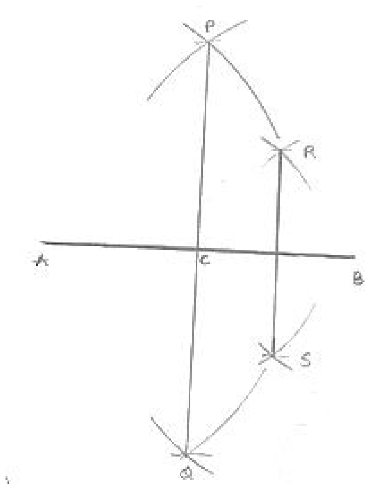
1. Draw a line segment AB
2. With center A and radius more than $\frac{1}{2}AB,$ draw arcs one on each side of AB
3. With center B and same radius draw arcs cutting previous arcs at P and Q respectively.
4. Join PQ and which intersect AB at C
5. With center A and radius more than $\frac{1}{2}AC,$ draw arcs, one on each side of AC.

6. With center C and same radius, draw arcs cutting previous arcs at R and S respectively.
7. Join RS and which intersect AC at D

$\therefore AD = \frac{1}{4} AB.$

7. Draw a line segment AB and by ruler and compasses1 obtain a line segment of length $\frac{3}{4} AB$.

Sol:



Steps of construction:

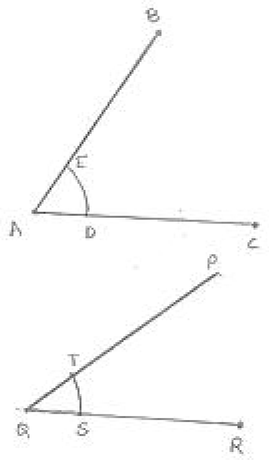
1. Draw a line segment AB
2. With center A and radius more than $\frac{1}{2} AB,$ draw arcs one on each side of AB.
3. With center B and same radius draw arcs cutting previous arcs at P and Q respectively.
4. Join PQ and which intersect AB at C
5. With center C and radius more than $\frac{1}{2} CB,$ draw arcs, one on each side of CB.
6. With center B and same radius, draw arcs cutting previous arcs at R and S respectively.
7. Join RS and which intersect CB at D

$\therefore AD = \frac{3}{4} AB.$

Exercise – 17.2

1. Draw an angle and label it as ∠BAC. Construct another angle, equal to ∠BAC.

Sol:



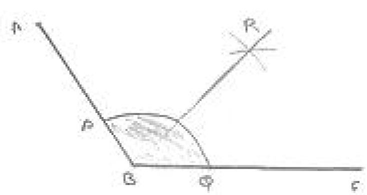
Steps of construction:

1. Draw an angle ABO and a Line segment QR
2. With center A and any radius, draw an arc which intersects $\angle BAC$ at E and O
3. With center Q and same radius draw arc which intersect QR at S.
4. With center S and radius equal to DE, draw an arc which intersect previous arc at T
5. Draw a line segment joining Q and T

$\therefore \angle PQR = \angle BAC$

2. Draw an obtuse angle, Bisect it. Measure each of the angles so obtained.

Sol:



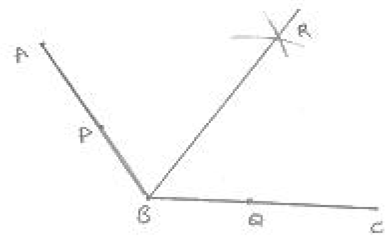
Steps of construction:

1. Draw angle ABC of $120°$
2. With center B and any radius, draw an arc which intersects AB at P and BC at Q
3. With center P and Q and radius more than $\frac{1}{2}PQ$, draw two arcs, with intersect each other at R.
4. Join BR

$\therefore \angle ABR = \angle RBC = 60°$

3. Using your protractor, draw an angle of measure 108°. With this angle as given, draw an angle of 54°.

Sol:



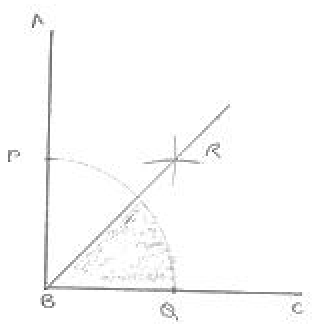
Steps of construction:

1. Draw an angle ABC of $108°$
2. With center B and any radius, draw an arc which intersects AB at P and BC at Q
3. With center P and Q and radius more than $\frac{1}{2}PQ$, draw two arcs, which intersect each other at R.
4. Join BR

$\therefore \angle RBC = 54°$

4. Using protractor, draw a right angle. Bisect it to get an angle of measure 45°.

Sol:



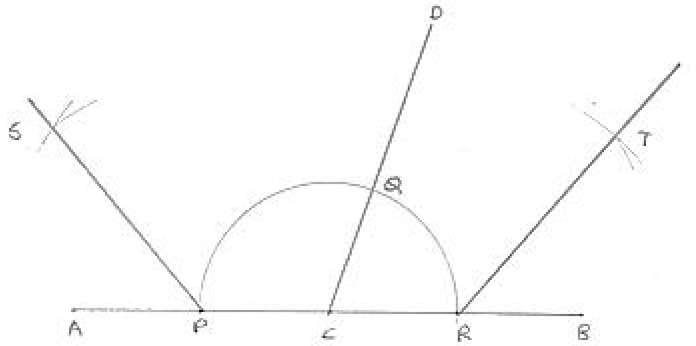
Steps of construction:

1. Draw an angle ABC of $90°$
2. With center B and any radius, draw an arc which intersects AB at P and BC at Q
3. With center P and Q and radius more than $\frac{1}{2}PQ$, draw two arcs, which intersect each other at R.
4. Join RB

$\therefore \angle RBC = 45°$

5. Draw a linear pair of angles. Bisect each of the two angles. Verify that the two bisecting rays are perpendicular to each other.

Sol:



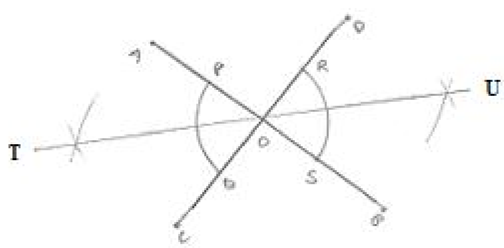
Steps of construction:

1. Draw two angle DCA and DCB forming Linear pair
2. With center C and any radius, draw an arc which intersects AC at P, CD at Q and CB at R.
3. With center P and Q and any radius draw two arcs which interest each other at S
4. Join SC
5. With center Q and R any radius draw two arcs, which intersect each other at T.
6. Join TC

$\angle SCT = 90°$ [By using protractor]

6. Draw a pair of vertically opposite angles. Bisect each of the two angles. Verify that the bisecting rays are in the same line.

Sol:



Steps of construction:

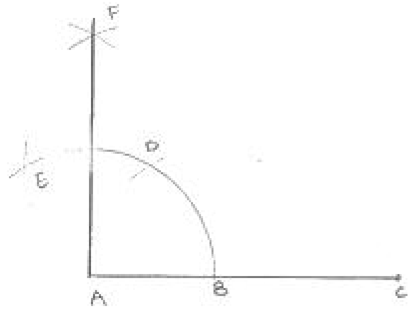
1. Draw a pair of vertically opposite angle AOC and DOB
2. With center O and any radius drawn two arcs which intersect OA at P, $Q \cdot OB$ at S and OD at R.
3. With center P and Q and radius more than $\frac{1}{2}PQ$, draw two arcs which intersect each other at 7.
4. Join to

5. With center R and S radius more than $\frac{1}{2}RS$, draw two arcs which intersect each other at U.
6. Join OU.

$\therefore TOU$ is a straight line

7. Using ruler and compasses only, draw a right angle.

Sol:



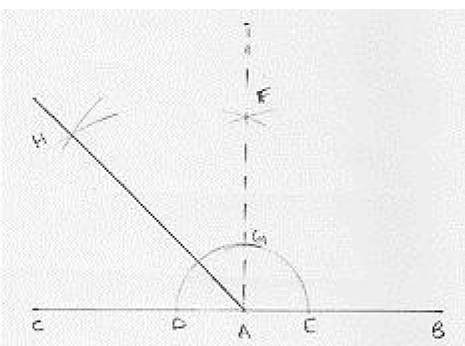
Steps of construction:

1. Draw a line segment AB
2. With center A and any radius draw arc which intersect AB at C.
3. With center C and same radius draw an arc which intersects AB at C.
4. With center D and same radius draw arc which intersect arc in (2) at E.
5. With centers E and C and any radius, draw two arcs which intersect each other at F.
6. Join FA

$\angle FAB = 90°$

8. Using ruler and compasses only, draw an angle of measure 135°.

Sol:



Steps of construction:

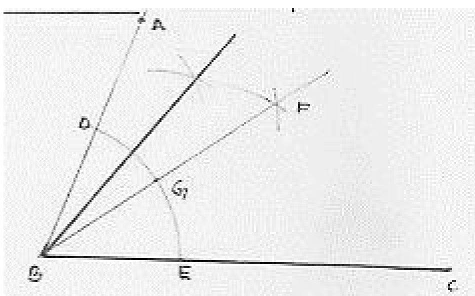
1. Draw a line segment AB and produce BA to point C.
2. With center A and any radius draw arc which intersect AC at D and AB at E.

3. With center D and E and radius more than $\frac{1}{2}DE,$ draw two arcs which intersect each other at F.
4. Join FA which intersect the arc in (2) at G.
5. With centers G and D and radius more than $\frac{1}{2}GD,$ draw two arcs which intersect each other at H.
6. Join HA

$\therefore \angle HAB = 135°$

9. Using a protractor, draw an angle of measure 72°. With this angle as given, draw angles of measure 36° and 54°.

Sol:



Steps of construction:

1. Draw an angle ABC of $72°$ with the help of protractor.
2. With center B and any radius, draw an arc which intersect AB at D and BC at E.
3. With center D and E and radius more than $\frac{1}{2}DE,$ draw two arcs which intersect each other at F.
4. Join FB which intersect the arc in (2) at G.
5. With centers D and G and radius more than $\frac{1}{2}DE,$ draw two arcs which intersect each other at F.
6. With centers D and G and radius more than $n\frac{1}{2}DG$ draw two arcs which intersect each other at H
7. Join HB

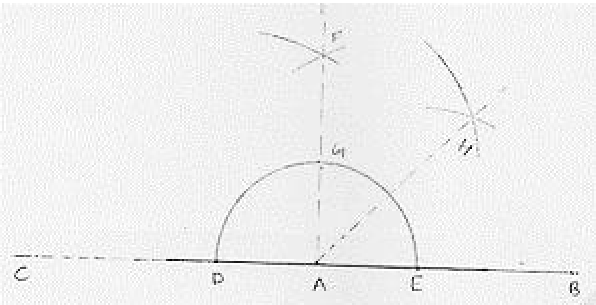
$\therefore \angle HBC = 54°$

$\angle FBC = 36°$

10. Construct the following angles at the initial point of a given ray and justify the construction:
(i) 45° (ii) 90°

Sol:

(i)

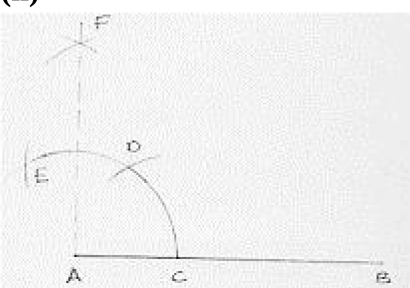


Steps of construction:

1. Draw a line segment AB and produce BA to point C.
2. With center A and any radius drawn an arc which intersect AC at D and AB at E.
3. With center D and E and radius more than $\frac{1}{2}DE,$ draw arcs cutting each other at F.
4. Join FA which intersect arc in (2) at G.
5. With centers G and E and radius more than $\frac{1}{2}GE,$ draw arcs cutting each other at H.
6. Join HA

   $\therefore \angle HAB = 45°$

(ii)



Steps of construction:

1. Draw a line segment AB.
2. With center A and any radius draw in arc which intersect AB at C.
3. With center C and same radius draw an arc which intersects previous arc at D.
4. With centers D same radius draw an arc which intersects are in (2) at E.
5. With centers E and D same radius more than $\frac{1}{2}$ ED draw an arc cutting each other at F.
6. Join FA

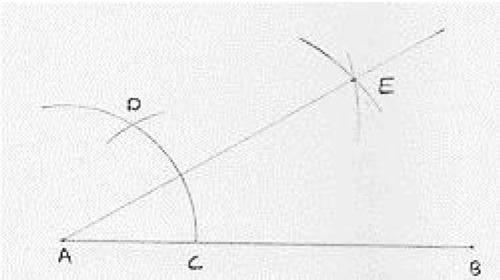
$\angle FAB = 90°$

11. Construct the angles of the following measurements:

(i) 30° (ii) 75° (iii) 105° (iv) 135° (v) 15° (vi) $22\frac{1°}{2}$

Sol:

(i)

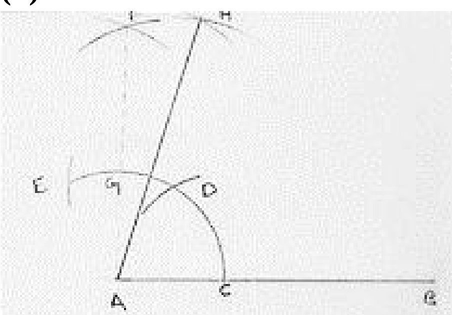


Steps of construction:

1. Draw a line segment AB.
2. With center A and any radius, draw an arc which intersect AB at C.
3. With center C and same radius, draw an arc which intersects previous arc at D.
4. With centers D and C and radius more than $\frac{1}{2}DC$, draw arcs intersecting each other at E
5. Join EA

$\therefore \angle EAB = 30°$

(ii)

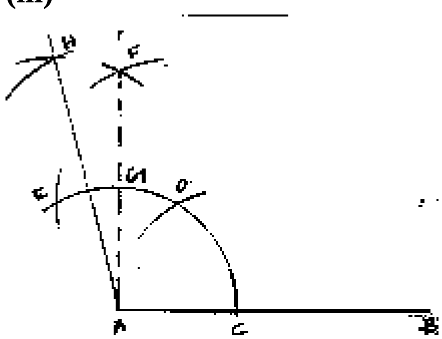


Steps of construction:

1. Draw a line segment AB.
2. With center A any radius, draw an arc which intersect AB at C.
3. With center C and same radius, draw an arc which intersects previous arc at D.
4. With center D and same radius, draw an arc which interest are in (2) at E
5. With centers E and D and radius more than $\frac{1}{2}ED$, draw arcs intersecting each other at F.
6. Join FA which intersects arc in (2) at G

7. With centers G and D, and radius more than $\frac{1}{2}GD,$ draw arcs intersecting each other at H.
8. Join HA
   $\therefore \angle HAB = 75°$

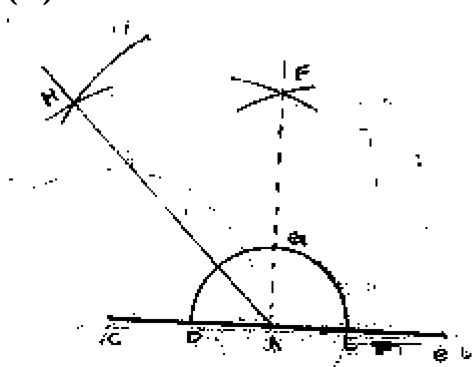
(iii)



Steps of construction:

1. Draw a line segment AB.
2. With center A and any radius, draw an arc which intersect AB at C.
3. With center C and same radius, draw an arc which intersects previous arc at D.
4. With center D and same radius, draw an arc which interest are in (2) at E
5. With centers E and D and radius more than $\frac{1}{2}ED,$ draw arcs intersecting each other at F.
6. Join FA which intersects arc in (2) at E
7. With centers E and G, and radius more than half of EG, draw arcs intersecting each other at H.
8. Join HA
   $\angle HAB = 105°$

(iv)

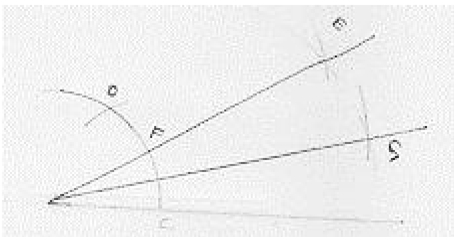


Steps of construction:

1. Draw a line segment AB and produce BA to pint C
2. With center A and any radius, draw an arc which intersect AC to D and AB at E.

3. With center D and E and radius more than half of DE, draw two arcs which intersects each other at F.
4. Join FA which intersect the arc in (2) at G
5. With center G and D radius more than $\frac{1}{2}GD,$ draw two arcs which intersect each other at H
6. Join HA
   $\angle HAB = 135°$

(v)



Steps of construction:

Step 1: Draw a line segment AB

Step 2: with center A and any radius, draw an arc which intersects previous arc at C

Step 3: with center C and same radius, draw an arc which intersect previous arc at D

Step 4: with center D and C radius more than half of DC draw arcs intersecting each other at E

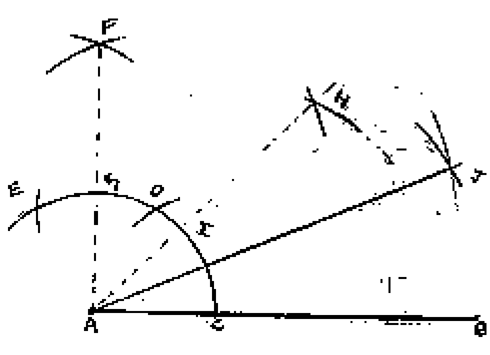
Step 5: Join EA which intersects arc in (2) at F.

Step 6: With centers F and C and radius more than $\frac{1}{2}FC,$ draw arcs intersecting each other

Step 7: Join GA

$\therefore \angle GAB = 15°$

(vi)



Steps of construction:

Step 1: Draw a line segment AB

Step 2: with center A and any radius, draw an arc which intersects AB at C

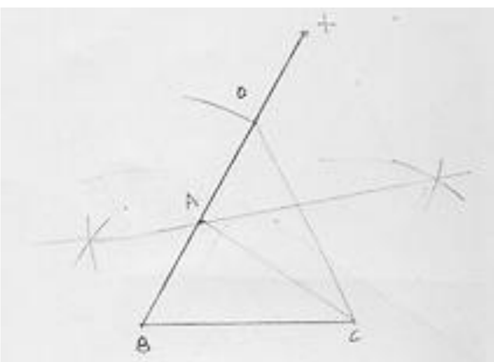
Step 3: with center C and same radius, draw an arc which intersect previous arc at D

Step 4: with center D and same radius, draw an arc which intersects arc in (2) at E.

Step 5: with center E and D and radius more than half of ED, draw arcs intersecting each other at F.
Step 6: Join FA which intersects arc in (2) at G
Step 7: with center G and C and radius more than half of GC, draw arcs intersecting each other at H
Step 8: Join HA which intersects arc in (2) at 1.
Step 9: with centers I and C and radius more than half of IC, draw arcs intersecting each other
Step 10: Join JA
$\therefore \angle JAB = 22\frac{1^\circ}{2}.$

Exercise – 17.3

1. Construct a ΔABC in which BC = 3.6 cm, AB + AC = 4.8 cm and ∠B = 60°.
Sol:

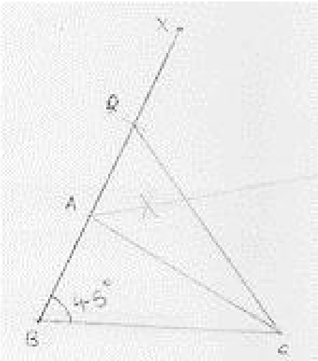


Steps of construction:
1. Draw a line segment BC of $3\cdot 6cm.$
2. At the point B, draw $\angle x\ BC$ of 60°
3. With center B and radius $4\cdot 8cm,$ draw an arc which intersects XB at D.
4. Join DC
5. Draw the perpendicular bisector of DC which intersects DB at A.
6. Join AC

Hence ΔABC is the required triangle

2. Construct a ΔABC in which AB + AC = 5.6 cm, BC = 4.5 cm, AB − AC = 1.5 cm and ∠B = 45°.

Sol:



Steps of construction:

Step 1: Draw a line segment BC of $4\cdot 5cm.$

Step 2: At B, draw an angle XBC of $45°$

Step 3: With center B and radius $5\cdot 6cm$, draw an arc which intersects BX at D.

Step 4: Join DC

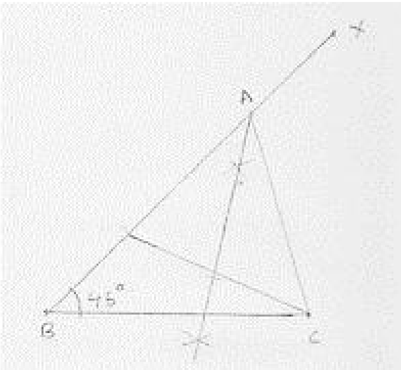
Step 5: Draw the perpendicular bisector of DC which intersects BD at A.

Step 6: Join AC

$\therefore \Delta ABC$ is a required triangle

3. Construct a ΔABC in which BC = 3.4 cm, AB − AC = 1.5 cm and ∠B = 45°.

Sol:



Steps of construction:

Step 1: Draw a line segment BC of $3\cdot 5cm.$

Step 2: At B, draw an angle XBC of $45°$

Step 3: With center B and radius $1\cdot 5cm$, draw an arc which intersects BX at D.

Step 4: Join DC

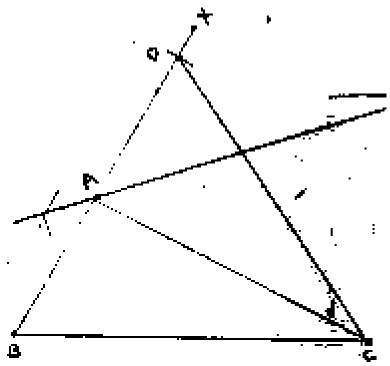
Step 5: Draw the perpendicular bisector of DC which intersects BD produced at A.

Step 6: Join AC

$\therefore \Delta ABC$ is the required triangle

4. Using ruler and compasses only, construct a ΔABC, given base BC = 7cm, ∠ABC = 60° and AB + AC = 12 cm.

Sol:



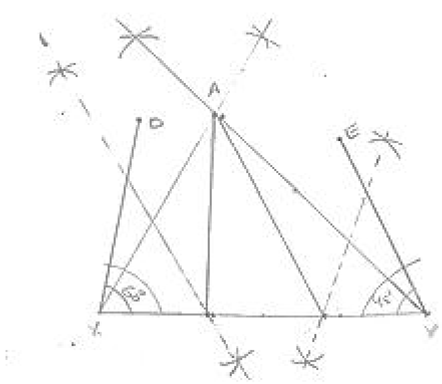
Steps of construction:

1. Draw a line segment BC of $7\,cm$.
2. At B, draw an angle XBC of $60°$
3. With center B and radius $12cm,$ draw an arc which intersects BX at D.
4. Join DC
5. Draw the perpendicular bisector of DC which intersects BD at A.
6. Join AC

$\therefore\ \Delta ABC$ is the required triangle.

5. Construct a triangle whose perimeter is 6.4 cm, and angles at the base are 60° and 45°.

Sol:



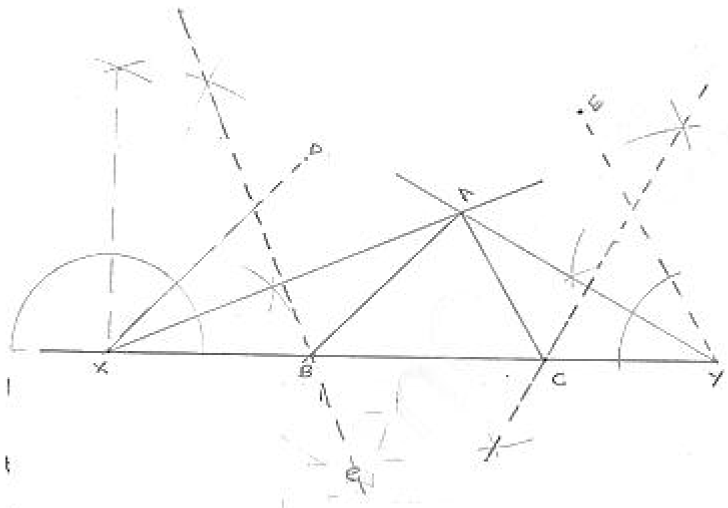
Steps of construction:

1. Draw a line segment XY of $6\cdot4cm$.
2. Draw $\angle DXY = B = 60°\ and\ \angle EYX = \angle C = 45°$
3. Draw the angle bisector of $\angle DXY\ and\ \angle EYX$ which interest each other at A.
4. Draw the perpendicular bisector of AX and AY which interest XY at B and C respectively.
5. Join AB and AC

$\therefore\ \Delta ABC$ is the required triangle.

6. Using ruler and compasses only, construct a ΔABC from the following data:
AB + BC + CA = 12 cm, ∠B = 45° and ∠C = 60°.

Sol:



Steps of construction:

Step 1: Draw a line segment XY of 12cm.

Step 2: Draw $\angle DXY = \angle B = 45^\circ$ *and* $\angle EYX = \angle C = 60^\circ$

Step 3: Draw the angle bisectors of angles of DXY and EYX which intersects each other at A.

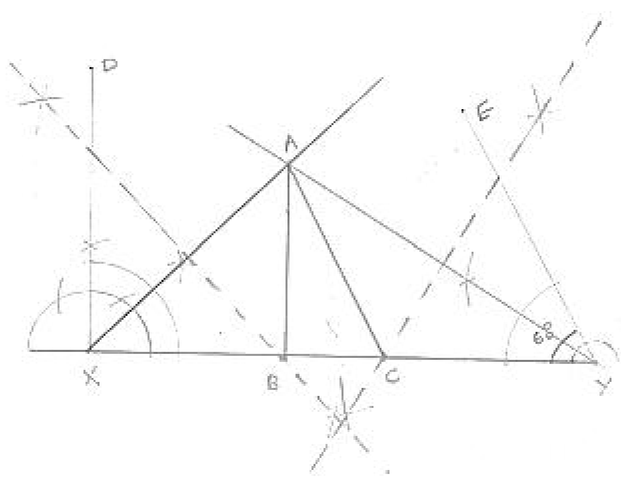
Step 4: Draw the perpendicular of AX and AY which intersect XY at B and C respectively.

Step 5: Join AB and AC

$\therefore \Delta ABC$ is the required triangle

7. Construct a right-angled triangle whose perimeter is equal to 10 cm and one acute angle equal to 60°.

Sol:



Steps of construction:

Step 1: Draw a line segment XY of 10cm.

Step 2: Draw $\angle DXY = \angle B = 90° \ and \ \angle FYX = \angle C = 60°$

Step 3: Draw the angle bisectors of $\angle DXY$ and $\angle EYX$ which intersects each other at A.

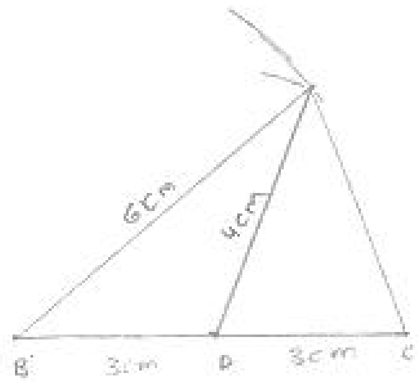
Step 4: Draw the perpendicular of AX and AY which intersect XY at B and C respectively.

Step 5: Join AB and AC

$\therefore \Delta ABC$ is the required triangle

8. Construct a triangle ABC such that BC = 6 cm, AB = 6 cm and median AD = 4 cm.

Sol:



Steps of construction:

Step 1: Draw a line segment BC of 6cm.

Step 2: Take midpoint D of BC.

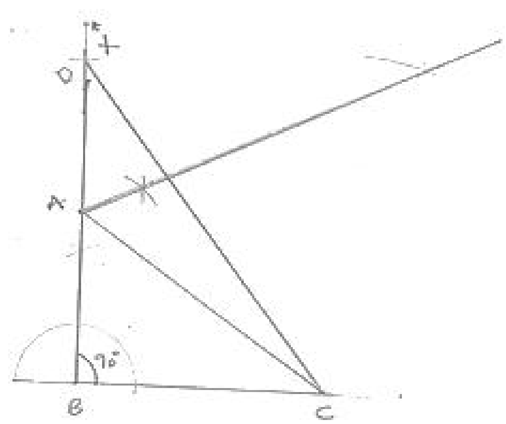
Step 3: with center B and D and radii 6cm and 4cm draw two arcs which intersects each other A

Step 4: Join AB, AD and AC

$\therefore \Delta ABC$ is the required triangle

9. Construct a right triangle ABC whose base BC is 6 cm and the sum of hypotenuse AC and other side AB is 10 cm.

Sol:



Steps of construction:

Step 1: Draw a line segment BC of 6cm.

Step 2: At B draw an angle $le \times BC$ of $90°$.

Step 3: with center B and radius 10cm draw an arc which intersects XB at D.

Step 4: Join DC.

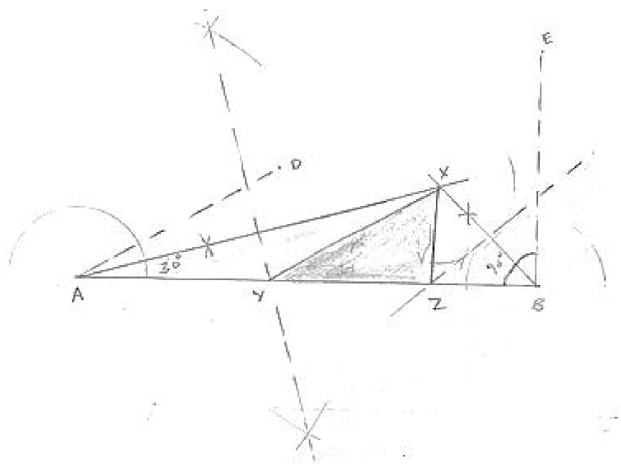
Step 5: Draw the perpendicular bisector of DC which intersects DB at A

Step 6: Join AC

$\therefore \Delta ABC$ is the required triangle

10. Construct a triangle XYZ in which ∠Y = 30°, ∠Z = 90° and XY + YZ + ZX = 11.

Sol:



Steps of construction:

Step 1: Draw a line segment AB of 11cm.

Step 2: Draw $\angle DAB = Y = 30° \ and \ \angle FBA = \angle Z = 90°$

Step 3: Draw the angle bisector of $\angle DAB \ and \ \angle EBA$ which intersect each other at x

Step 4: Draw the perpendicular bisector XA and XB which intersect AB at Y and Z respectively.

Step 5: Join XY and XZ

$\therefore \Delta XYZ$ is the required triangle

Exercise – 18.1

1. Find the lateral surface area and total surface area of a cuboid of length 80 cm, breadth 40 cm and height 20 cm.

Sol:

It is given that

Cuboid length $=80cm=L$

Breath $=40cm=$ b

Height $=20cm=h$

WKT,

Total surface area $=2[lb+bh+hl]$

$=2[(80)(40)+40(20)+20(80)]$

$=2[3200+800+1600]$

$=2[5600]$

$=11,200m^2$

Lateral surface area $=2[l+b]h=2[80+40]20$

$=40(120)$

$=4800cm^2$

2. Find the lateral surface area and total surface area of a cube of edge 10 cm.

Sol:

Cube of edge $a=10cm$

WKT,

Cube lateral surface area $=4a^2$

$=4\times10\times10 \quad [\because a=10]$

$=400cm^2$

Total surface area $a=6a^2$

$=6\times(10)^2$

$=600cm^2$

3. Find the ratio of the total surface area and lateral surface area of a cube.

Sol:

Cube total surface area $=6a^2$

Where, $a=$edge of cube

And, lateral surface area $=LSA=4a^2$

Where $a=$ edge of cube

$\therefore$ Ratio of TSA and LSA $=\frac{6a^2}{4a^2}$ is $\frac{3}{2}$ is $3:2$

4. Mary wants to decorate her Christmas tree. She wants to place the tree on a wooden block covered with coloured paper with picture of Santa Claus on it. She must know the exact quantity of paper to buy for this purpose. If the box has length, breadth and height as 80 cm, 40 cm and 20 cm respectively. How many square sheets of paper of side 40 cm would she require?

Sol:

Given that mary wants to paste the paper on the outer surface of the box; The quantity of the paper required would be equal to the surface area of the box which is of the shape of cuboid. The dimension of the box are

Length $(l) = 80cm$ Breath $(b) = 40cm$ and height (h) = 20cm

The surface area of thee box $= 2[lb + bh + hl]$

$= 2[80(40) + 40(20) + 20(80)]$

$= 2(5600) = 11,200cm^2$

The area of the each sheet of paper $= 40 \times 40cm^2$

$= 1600cm^2$

$\therefore$ Number of sheets required $= \frac{\text{Surface areaa of box}}{\text{area of one sheet of paper}}$

$= \frac{11,200}{1600} = 7$

5. The length, breadth and height of a room are 5 m, 4 m and 3 m respectively. Find the cost of white washing the walls of the room and the ceiling at the rate of Rs. 7.50 m^2.

Sol:

Total area to be washed $= lb + 2(l + b)h$

Where length $(l) = 5m$

Breath $(b) = 4m$

Height $(h) = 3m$

$\therefore$ Total area to be white washed $= (5 \times 4) + 2(5 + 4) \times 3$

$= 20 + 54 = 74m^2$

Now,

Cost of white washing $1m^2$ is Rs. $7 \cdot 50$

$\therefore$ Cost of white washing $74m^2$ is $Rs.(74 \times 7 \cdot 50)$

$= Rs.\ 555$

6. Three equal cubes are placed adjacently in a row. Find the ratio of total surface area of the new cuboid to that of the sum of the surface areas of the three cubes.

Sol:

Length of new cuboid $= 3a$

Breadth of cuboid $= a$

Height of new cuboid $= a$

The total surface area of new cuboid

$\Rightarrow (TSA)_1 = 2[lb + bh + hl]$

$\Rightarrow (TSA)_1 = 2[3a \times a + a \times a + 3a \times a]$

$\Rightarrow (TSA)_1 = 14a^2$

Total surface area of three cubes

$\Rightarrow (TSA)_2 = 3 \times 6a^2 = 18a^2$

$\therefore \frac{(TSA)_1}{(TSA)_2} = \frac{14a^2}{18a^2} = \frac{7}{9}$

$\therefore$ Ratio is $7:9$

7. A 4 cm cube is cut into 1 cm cubes. Calculate the total surface area of all the small cubes.

Sol:

Edge of cube $= 4cm$

Volume of $4cm$ cube $= (4cm)^3 = 64cm^2$

Edge of cube $= 1cm$

Volume of $1cm$ cube $= (1cm)^3 = 1cm^3$

$\therefore$ Total number of small cubes $= \frac{64cm^3}{1cm^3} = 64$

$\therefore$ Total surface area of $64cm$ all cubes

$= 64 \times 6 \times (1cm)^2$

$= 384cm^2$

8. The length of a hall is 18 m and the width 12 m. The sum of the areas of the floor and the flat roof is equal to the sum of the areas of the four walls. Find the height of the hall.

Sol:

Length of the hall $= 18m$

Width of hall $= 12m$

Now given,

Area of the floor and the flat roof = sum of the areas of four walls.

$\Rightarrow 2lb = 2lh + 2bh$

$\Rightarrow lb = lh + bh$

$\Rightarrow h = \frac{lb}{l+b} = \frac{18\times 12}{18+12} = \frac{216}{30}$

$= 7\cdot 2m.$

9. Hameed has built a cubical water tank with lid for his house, with each other edge 1 .5 m long. He gets the outer surface of the tank excluding the base, covered with square tiles of side 25 cm. Find how much he would spend for the tiles, if the cost of tiles is Rs. 360 per dozen.

Sol:

Given that

Hameed is giving 5 outer faces of the tank covered with tiles he would need to know the surface area of the tank, to decide on the number of tiles required.

Edge of the cubic tank $= 1\cdot 5m = 150cm = a$

So, surface area of tank $= 5\times 150\times 150 cm^2$

Area of each square tile $= \frac{\text{surface area of tank}}{\text{area of each title}}$

$= \frac{5\times 150\times 150}{25\times 25} = 180$

Cost of 1 dozen tiles i.e., cost of 12 tiles = Rs. 360

Therefore, cost of 12 balls tiles $= Rs.\ 360$

$\therefore$ cost of one tile $= \frac{360}{12} = Rs.\ 30$

$\therefore$ The cost of 180 tiles $= 180\times Rs.\ 30$

$= Rs.\ 5,400$

10. Each edge of a cube is increased by 50%. Find the percentage increase in the surface area of the cube.

Sol:

Let d be the edge of the cube

$\therefore$ surface area of cube $= 6\times a^2$

i.e, $S_1 = 6a^2$

According to problem when edge increased by 50% then the new edge becomes

$= a + \frac{50}{100}\times a$

$= \frac{3}{2}a$

New surface area becomes $=6\times\left(\frac{3}{2}a\right)^2$

i.e., $S_2=6\times\frac{9}{4}a^2$

$S_2=\frac{27}{2}a^2$

$\therefore$ Increased surface Area $=\frac{27}{2}a^2-6a^2$

$=\frac{15}{2}a^2$

So, increase in surface area $=\frac{\frac{15}{2}a^2}{6a^2}\times 100$

$=\frac{15}{12}\times 100$

$=125\%$

11. The dimensions of a rectangular box are in the ratio of 2 : 3 : 4 and the difference between the cost of covering it with sheet of paper at the rates of Rs. 8 and Rs. 9.50 per m^2 is Rs. 1248. Find the dimensions of the box.

Sol:

Let the ratio be x

$\therefore$ length $=2x$

Breath $=3x$

Height $=4x$

$\therefore$ Total surface area $=2\left[lb+bh+hl\right]$

$=2\left[6x^2+12x^2+8x^2\right]$

$=52x^2m^2$

When cost is at Rs. 9.51 per m^2

$\therefore$ Total cost of $52x^2m^2=Rs.8\times 52x^2$

$=Rs.\ 416x^2$

And when the cost is at $95\,per\ m^2$

$\therefore$ Total cost of $52x^2m^2=Rs.\ 9\cdot 5\times 52x^2$

$=Rs.\ 499x^2$

$\therefore$ Different in cost $=Rs.\ 494x^2-Rs.\ 416x^2$

$\Rightarrow 1248 = 494x^2 - 416x^2$

$\Rightarrow 78x^2 = 1248$

$\Rightarrow x^2 = 16$

$\Rightarrow x = 4$

12. A closed iron tank 12 m long, 9 m wide and 4 m deep is to be made. Determine the cost of iron sheet used at the rate of Rs. 5 per metre sheet, sheet being 2 m wide.

Sol:

Given length $= 12m$, Breadth $= 9m$ and Height $= 4m$.

Total surface area of tank $= 2(lb + bh + hl)$

$= 2[12\times 9 + 9\times 4 + 12\times 4]$

$= 2[108 + 36 + 48]$

$= 384m^2$

Now length of iron sheet $= \dfrac{384}{\text{width of iron sheet}}$

$= \dfrac{384}{2} = 192m.$

Cost of iron sheet = length of sheet $\times$cost rate

$= 192\times 5 = Rs.960.$

13. Ravish wanted to make a temporary shelter for his car by making a box-like structure with tarpaulin that covers all the four sides and the top of the car (with the front face as a flap which can be rolled up). Assuming that the stitching margins are very small, and therefore negligible, how much tarpaulin would be required to make the shelter of height 2.5 m with base dimensions 4 m × 3m?

Sol:

Given that

Shelter length = 4m

Breadth = 3m

Height = $2\cdot 5m$

The tarpaulin will be required for four sides of the shelter

Area of tarpaulin in required $= 2(lb + bh + hl)$

$= \left[2(4)\times 2\cdot 5 + (3\times 2\cdot 5)\right] + 4\times 3\Big]m^2$

$= \left[2(10 + 7\cdot 5) + 12\right]m^2$

$= 47m^2 = 47m^2.$

14. An open box is made of wood 3 cm thick. Its external length, breadth and height are 1.48 m, 1.16 m and 8.3 m. Find the cost of painting the inner surface of Rs 50 per sq. metre.

Sol:

Given

Length $=1\cdot 48m=148cm.$

Breath $=1\cdot 16m=116cm$

Height $=8\cdot 3m=83cm$

Thickness of wood = 3cm

$\therefore$ inner dimensions:

Length $\left(148-2\times 3\right)cm=142cm$

Breadth $\left(116-2\times 3\right)cm=110cm$

Height $=\left(83-3\right)cm=80cm.$

Inner surface area $=2\left(l+b\right)+lb$

$=2\left[\left(142\right)+100\right)80+142\times 110cm^2$

$=2\left(252\right)\left[80\right]+142\times 110cm^2=55,940cm^2$

$=559\ 40m^2$

Hence, cost of painting inner surface area

$=5,5940\times Rs.\ 50$

$=Rs.\ 279\cdot 70$

15. The cost of preparing the walls of a room 12 m long at the rate of Rs. 1.35 per square metre is Rs. 340.20 and the cost of matting the floor at 85 paise per square metre is Rs. 91.80. Find the height of the room.

Sol:

Given that

Length of room = 12m.

Let height of room be ‘h’ m.

Area of 4 walls $=2\left(l+b\right)\times h$

According to question

$\Rightarrow 2\left(l+b\right)\times h\times 1\cdot 35=340\cdot 20$

$\Rightarrow 2\left(12+b\right)\times h\times 1\cdot 35=340\cdot 20$

$\Rightarrow \left(12+b\right)\times h=\dfrac{170\cdot 10}{1\cdot 35}=126 \qquad ....\left(1\right)$

Also area of floor $=l\times b$

$\therefore l\times b\times 0\cdot 85=91\cdot 80$

$\Rightarrow 12\times b\times 0\cdot 85=91\cdot 80$

$\Rightarrow b = 9m$ ......(2)

Substituting $b = 9m$ in equation (1)

$\Rightarrow (12+9)\times h = 126$

$\Rightarrow h = 6m$

16. The dimensions of a room are 12.5 m by 9 m by 7 m. There are 2 doors and 4 windows in the room; each door measures 2.5 m by 1 .2 m and each window 1 .5 m by I m. Find the cost of painting the walls at Rs. 3.50 per square metre.

Sol:

Given length of room $= 12\cdot 5m$

Breadth of room $= 9m$

Height of room $= 7m$

$\therefore$ Total surface area of 4 walls

$= 2(l+b)\times h$

$= 2(12\cdot 5+9)\times 7$

$= 301m^2$

Area of 2 doors $= 2[2\cdot 5\times 1\cdot 2]$

$= 6m^2$

Area to be painted on 4 walls

$= 301-(6+6)$

$= 301-12 = 289m^2$

$\therefore$ cost of painting $= 289\times 3\cdot 50$

$Rs.\ 1011\cdot 5.$

17. The length and breadth of a hall are in the ratio 4: 3 and its height is 5.5 metres. The cost of decorating its walls (including doors and windows) at Rs. 6.60 per square metre is Rs. 5082. Find the length and breadth of the room.

Sol:

Let the length be 4x and breadth be 3x

Height $= 5\cdot 5m$ [given]

Now it is given that cost of decorating 4 walls at the rate of $Rs.\ 6\cdot 601m^2$ is $Rs.\ 5082$

$\Rightarrow$ Area of four walls $\times$rate = total cost of painting

$2(l+b)\times h\times 6.60 = 5082$

$2(4x+3x)\times 5.5\times 6.60 = 5082$

$\Rightarrow 7x = \dfrac{5082}{5\cdot 5\times 2\cdot 6\times 2}$

$\Rightarrow 7x = 10$

$\Rightarrow x = 10$

Length $= 4x = 4 \times 10 = 40m$

Breadth $= 3x = 3 \times 10 = 30m$

18. A wooden bookshelf has external dimensions as follows: Height = 110 cm, Depth = 25 cm, Breadth = 85 cm (See Fig. 18.5). The thickness of the plank is 5 cm everywhere. The external faces are to be polished and the inner faces are to be painted. If the rate of polishing is 20 paise per cm^2 and the rate of painting is 10 paise per cm^2. Find the total expenses required for polishing and painting the surface of the bookshelf.

Sol:

External length of book shelf $= 85cm = l$

Breadth $= 25cm$

Height $= 110cm.$

External surface area of shelf while leaving front face of shelf

$= lh + 2(lb + bh)$

$= \left[85 \times 110 + 21\left(85 \times 25 + 25 \times 110\right)\right] cm^2$

$= 19100 cm^2$

Area of front face $= \left(85 \times 110 - 75 \times 100 + 2\left(75 \times 5\right)\right) cm^2$

$= 1850 + 750 cm^2$

$= 2600 cm^2$

Area to be polished $= 19100 + 2600 cm^2$

$= 21700 cm^2$

Cost of polishing $1cm^2$ area = $Rs.\ 0 \cdot 20$

Cost of polishing $21700cm^2$ area $= Rs.\left[21700 \times 0 \cdot 20\right]$

$= Rs.\ 4340$

Now, length (l), breath (b), height (h) of each row of book shelf is 75cm, 20cm and 30cm $= \left(\dfrac{110 - 20}{3}\right)$ respectively.

Area to be painted in row $= 2\left(l + h\right)b + lh$

$= \left[2\left(75 + 30\right) \times 20 + 75 \times 30\right] cm^2$

$= \left(4200 + 2250\right) cm^2$

$= 6450 cm^2$

Area to be painted in 3 rows $= \left(3 \times 6450\right) cm^2$

$= 19350 cm^2$

Cost of painting $1cm^2$ $area = Rs.\ 0 \cdot 10.$

Cost of painting 19350 $area = Rs.(19350 \times 0 \cdot 10) - Rs.1935$

Total expense required for polishing and painting the surface of the bookshelf

$= Rs.(4340 + 1935) = Rs.\ 6275.$

19. The paint in a certain container is sufficient to paint on area equal to 9.375 m^2. How many bricks of dimension 22.5 cm × 10 cm × 7.5 cm can be painted out of this container?

Sol:

We know that

Total surface area of one brick $= 2(lb + bh + hl)$

$= 2[22 \cdot 5 \times 10 + 10 \times 7 \cdot 5 + 22 \cdot 5 \times 7.5]cm^2$

$= 2[468 \cdot 75)cm^2$

$= 937 \cdot 5cm^2$

Let n number of bricks be painted by the container

Area of brick $= 937 \cdot 50\ cm^2$

Area that can be painted in the container

$= 93755m^2 = 93750\,cm^2$

$93750 = 937 \cdot 5n$

$n = 100$

Thus, 100 bricks can be painted out by the container.

Exercise – 18.2

1. A cuboidal water tank is 6 m long, 5 m wide and 4.5 m deep. How many litres of water can it hold?

Sol:

Given length $= 6cm$

Breath $= 56m$

Height $= 4 \cdot 5m$

Volume of the tank $= l \times b \times h = 6 \times 5(4 \cdot 5) = 135m^3$ It is given that

$1m^3 = 1000$ liters

$\therefore 135m^3 = (135 \times 1000)$ liters

$= 1,35,000$ liters

$\therefore$ The tank can hold $1,35,000$ liters of water

2. A cubical vessel is 10 m long and 8 m wide. How high must it be made to hold 380 cubic metres of a liquid?

Sol:

Given that

Length of vessel (l) = 10m

Width of vessel (b) = 8m

Let height of the cuboidal vessel be 'h'

Volume of vessel $= 380m^3$

$\therefore l \times b \times h = 380$

$10 \times 8 \times h = 380$

$h = 4 \cdot 75$

$\therefore$ height of the vessel should be $4 \cdot 75m$.

3. Find the cost of digging a cuboidal pit 8 m long, 6 m broad and 3 m deep at the rate of Rs. 30 per m^3.

Sol:

Given length of the cuboidal Pit $(l) = 8m$

Width $(b) = 6m$

Depth $(h) = 3m$

Volume of cuboid pit $= l \times b \times h = (8 \times 6 \times 3)m^3$

$= 144m^3$

Cost of digging $1m^3 = Rs.\ 30$

Cost of digging $144m^3 = 144(Rs.\ 30) = Rs. 4320$.

4. If V is the volume of a cuboid of dimensions a, b, c and S is its surface area, then prove that $\frac{1}{V} = \frac{2}{S}\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$

Sol:

Given that

Length $= a$

Breadth $= b$

Height $= c$

Volume $(v) = l \times b \times h$

$= a \times b \times c = abc$

Surface area $= 2(lb + bh + hl)$

$= 2(ab + bc + ac)$

Now, $\frac{2}{5}\left[\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right]=\frac{2}{2\left(ab+bc+ca\right)}\frac{\left[ab+bc+ca\right]}{abc}$

$=\frac{1}{abc}=\frac{1}{v}$

5. The areas of three adjacent faces of a cuboid are x, y and z. If the volume is V, prove that V^2 = xyz.

Sol:

Let a, b, d be the length, breath and height of cuboid then,

$x=ab$

$y=bd,$

$z=da,\ and$

$v=abd \qquad \left[v=l\times b\times h\right]$

$\Rightarrow xyz=ab\times bc\times ca=\left(abc\right)^2$

And $v=abc$

$v^2=\left(abc\right)^2$

$v^2=xyz$

6. If the area of three adjacent faces of a cuboid are 8 cm^2, 18 cm^3 and 25 cm^3. Find the volume of the cuboid.

Sol:

WKT, if x,y,z denote the areas of three adjacent faces of a cuboid

$\Rightarrow x=l\times b, y=b\times h, z=l\times h.$

Volume V is given by

$V=l\times b\times h.$

Now, $xyz=l\times b\times b\times h\times l\times h=V^2$

Here $x=8$

$y=18$

And $z=25$

$\therefore v^2=8\times 18\times 25=3600$

$\Rightarrow v=60cm^3.$

7. The breadth of a room is twice its height, one half of its length and the volume of the room is 512cu. m. Find its dimensions.

Sol:

We have,

$b = 2h \ and \ b = \frac{1}{2}.$

$\Rightarrow \frac{l}{2} = 2h$

$\Rightarrow l = 4h$

$\Rightarrow l = 4h, b = 2h$

Now,

Volume $= 512 \ m^3$

$\Rightarrow 4h \times 2h \times h = 512$

$\Rightarrow h^3 = 64$

$\Rightarrow h = 4$

So, $l = 4 \times h = 16m$

$b = 2 \times h = 8m$

And $h = 4m$

8. A river 3 m deep and 40 m wide is flowing at the rate of 2 km per hour. How much water will fall into the sea in a minute?

Sol:

Radius of water flow $= 2km \ per \ hour = \left(\frac{2000}{60}\right) m/\min$

$= \left(\frac{100}{3}\right) m/\min$

Depth (h) of river = 3m

Width (b) of river = 40m

Volume of water followed in 1 min $= \frac{100}{3} \times 40 \times 3m^3 - 4000m^3$

Thus, 1 minute $4000 \ m^3 = 4000000$ liters of water will fall in sea.

9. Water in a canal 30 cm wide and 12 cm deep, is flowing with a velocity of l00 km per hour. How much area will it irrigate in 30 minutes if 8 cm of standing water is desired?

Sol:

Given that,

Water in the canal forms a cuboid of

width $(h) = 300cm = 3m$

height $= 12 \ cm = 1 \cdot 2m$

length of cuboid is equal to the distance travelled in 30 min with the speed of 100 km per hour

$\therefore$ length of cuboid $= 100 \times \frac{30}{60} km = 50000$ meters

So, volume of water to be used for irrigation $= 50000 \times 3 \times 1 \cdot 2 m^3$

Water accumulated in the field forms a cuboid of base area equal to the area of the field and height equal to $\frac{8}{100}$ *meters*

$\therefore$ Area of field $\times \frac{8}{100} = 50,000 \times 3 \times 1 \cdot 2$

$\Rightarrow$ Area of field $= \frac{50000 \times 3 \times 1 \cdot 2 \times 100}{8}$

$= 2,250000$ meters

10. Three metal cubes with edges 6 cm, 8 cm and 10 cm respectively are melted together and formed into a single cube. Find the volume, surface area and diagonal of the new cube.

Sol:

Let the length of each edge of the new cube be a cm

Then,

$a^3 = \left(6^3 + 8^3 + 10^3\right) cm^3$

$\Rightarrow a^3 = 1728$

$\Rightarrow a = 12$

$\therefore$ Volume of new cube $= a^3 = 1728 cm^3$

Surface area of the new cube $= 6a^2 = 6 \times 12^2 cm^2$

$= 864 cm^2$.

Diagonals of the new cube $= \sqrt{3a} = 12\sqrt{3} cm.$

11. Two cubes, each of volume 512 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

Sol:

Given that

Volume of cube $= 512 cm^3$

$\Rightarrow side^3 = 512$

$\Rightarrow side^3 = 8^3$

$\Rightarrow side = 8 cm$

Dimensions of new cuboid formed

$l = 8 + 8 = 16 cm, b = 8 cm, h = 8 cm$

Surface area $= 2\left(lb + bh + hl\right)$

$= 2\left[16(8)+8(8)+16(8)\right] = 2\left[256+64\right]$

$= 640cm^2$

$\therefore$ Surface area is $640cm^2$.

12. Half cubic meter of gold-sheet is extended by hammering so as to cover an area of 1 hectare. Find the thickness of the gold-sheet.

Sol:

Given that

Volume of gold $= 0\cdot 5m^3$

Area of gold sheet = 1 hectare = 10000 m^2

$\therefore$ Thickness of gold sheet $= \frac{\text{Volume of gold}}{\text{Area of gold sheet}}$

$= \frac{0\cdot 5m^3}{1\,Hectare}$

$= \frac{0\cdot 5m^3}{10000m^2}$

$= \frac{5}{10000}\times 10m$

$= \frac{100}{20000}m$

Thickness of gold sheet $= \frac{1}{200}cm.$

13. A metal cube of edge 12 cm is melted and formed into three smaller cubes. If the edges of the two smaller cubes are 6 cm and 8 cm, find the edge of the third smaller cube.

Sol:

Volume of large cube = $V_1 + V_2 + V_3$

Let the edge of the third cube be x cm

$123^3 = 6^3 + 8^3 + x^3$ [Volume of cube $= side^3$]

$1728 = 216 + 512 + x^3$

$\Rightarrow x^3 = 1728 - 728 = 1000$

$\Rightarrow x = 10cm$

$\therefore$ Side of third side $= 10cm..$

14. The dimensions of a cinema hall are 100 m, 50 m and 18 m. How many persons can sit in the hall, if each person requires 150 m^3 of air?

Sol:

Given that

Volume of cinema hall $= 100 \times 50 \times 18 m^3$

Volume air required by each person $= 150 m^3$

Number of person who can sit in the hall

$= \frac{\text{volume of cinema hall}}{\text{volume of air req each person}}$

$= \frac{100 \times 50 \times 18 m^3}{150 m^3} = 600$ $\quad [\because V = l \times b \times h]$

$\therefore$ number of person who can sit in the hall

$= 600$ members

15. Given that 1 cubic cm of marble weighs 0.25 kg, the weight of marble block 28 cm in width and 5 cm thick is 112 kg. Find the length of the block.

Sol:

Let the length of the block be 1cm

Then, volume $= l \times 28 \times 5\ cm^3$

$\therefore$ weight $= 140 l \times 0 \cdot 25\ kg$

According to the question

$\Rightarrow 112 = 1401 \times 0 \cdot 25$

$\Rightarrow l = \frac{112}{140 \times 0 \cdot 25} = 3 \cdot 2 cm$

16. A box with lid is made of 2 cm thick wood. Its external length, breadth and height are 25 cm, 18 cm and 15 cm respectively. How much cubic cm of a liquid can be placed in it? Also, find the volume of the wood used in it.

Sol:

Given external dimensions of cuboid are

$l = 25 cm, b = 18 cm, h = 15 cm.$

$\therefore$ External volume $= l \times b \times h$

$= 25 \times 18 \times 15 cm^3$

$= 6750 cm^3$.

Internal dimension of cuboid.

l= $25 - 2 \times$ thickness=25 – 4 =21 cm.

h= 15 – 4 = 11 cm.

Internal volume $= l \times b \times h$

$= 21 \times 14 \times 11 cm^3$

$= 3234 cm^3$

$\therefore$ Volume of liquid that can be placed $= 3234 cm^3$

Now, volume of wood = external volume – Internal volume

$= 6750 - 3324$

$= 3516 cm^3$

17. The external dimensions of a closed wooden box are 48 cm, 36 cm, 30 cm. The box is made of 1.5 cm thick wood. How many bricks of size 6 cm x 3 cm x 0.75 cm can be put in this box?

Sol:

Given internal dimensions are

$l = 48 - 2 \times \text{thickness} = 48 - 3 = 45 cm$

$b = 36 - 3 = 33 cm$

$h = 30 - 3 = 27 cm$

$\therefore$ Internal volume $= 45 \times 33 \times 27 cm^3$

Volume of brick $= 5 \times 3 \times 0 \cdot 75 cm^3$

Hence, number of bricks $= \dfrac{\text{Internal volume}}{\text{volume of 1 brick}}$

$= \dfrac{45 \times 33 \times 27}{6 \times 3 \times 0 \cdot 37}$

$= \dfrac{38880}{13 \cdot 5}$

$= 2970$

$\therefore 2970$ bricks can be kept inside the box

18. How many cubic centimeters of iron are there in an open box whose external dimensions are 36 cm, 25 cm and I 6.5 cm, the iron being 1.5 cm thick throughout? If I cubic cm of iron weighs 15g, find the weight of the empty box in kg.

Sol:

Outer dimensions

$l = 36\ cm$

$b = 25\ cm$

$h = 16 \cdot 5 cm$

Inner dimensions

$l = 36 - (2 \times 1 \cdot 5) = 33 cm$

$b = 25 - (3) = 22cm$

$h = 16 \cdot 5 - 1 \cdot 5 = 15cm$

Volume of iron = outer volume – inner volume

$= (36 \times 25 \times 16 \cdot 5 - 33 \times 12 \times 15) cm^3 = 3960 cm^3$

Weight of iron $= 3960 \times 1 \cdot 5 gm = 59400 gm = 59 \cdot 4 kg$

19. A cube of 9 cm edge is immersed completely in a rectangular vessel containing water. If the dimensions of the base are 15 cm and 12 cm, find the rise in water level in the vessel.

Sol:

Volume of cube $= S^3 = 9^3 = 729 cm^3$

Area of base $l \times b = 15 \times 12 = 180 cm^2$

Rise in water level $= \frac{\text{Volume of cube}}{\text{Area of base of rectangular vessel}}$

$= \frac{729}{180} = 4 \cdot 05 cm$

20. A rectangular container, whose base is a square of side 5 cm, stands on a horizontal table, and holds water up to 1 cm from the top. When a cube is placed in the water it is completely submerged, the water rises to the top and 2 cubic cm of water overflows. Calculate the volume of the cube and also the length of its edge.

Sol:

Let the length of each edge of the cube be x cm

Then,

Volume of cube = volume of water inside the tank + volume of water that over flowed

$x^3 = (5 \times 5 \times 1) + 2 = 25 + 2$

$x^3 = 27$

$x = 3cm$

Hence, volume of cube = $27 cm^3$

And edge of cube $= 3cm.$

21. A field is 200 m long and 150 m broad. There is a plot, 50 m long and 40 m broad, near the field. The plot is dug 7 m deep and the earth taken out is spread evenly on the field. By how many meters is the level of the field raised? Give the answer to the second place of decimal.

Sol:

Volume of earth dug out $= 50 \times 40 \times 7 \, m^3$

$= 14000 m^3$

Let the height of the field rises by h meters

$\therefore$ volume of filed (cuboidal) = Volume of earth dugout

$\Rightarrow 200\times150\times h=14000$

$$\Rightarrow h=\frac{1400}{200\times150}=0\cdot47m.$$

22. A field is in the form of a rectangle of length 18 m and width 15 m. A pit, 7.5 m long, 6 m broad and 0.8 m deep, is dug in a corner of the field and the earth taken out is spread over the remaining area of the field. Find out the extent to which the level of the field has been raised.

Sol:

Let the level of the field be risen by h meters volume of the earth taken out from the pit

$=7\cdot5\times6\times0\cdot8m^3$

Area of the field on which the earth taken out is to be spread (X)

$=18\times15-7\cdot5\times6=225m^2$

Now, area of the field $Xh=$ volume of the earth taken out from the pit

$\Rightarrow 225\times h=7\cdot5\times6\times0\cdot8$

$$\Rightarrow h=\frac{36}{225}=0\cdot16m=16cm.$$

23. A rectangular tank is 80 m long and 25 m broad. Water flows into it through a pipe whose cross-section is 25 cm^2, at the rate of 16 km per hour. How much the level of the water rises in the tank in 45 minutes.

Sol:

Let the level of water be risen by *h* cm.

Then,

Volume of water in the tank $=8000\times2500\times hcm^2$

Area of cross – section of the pipe $=25cm^2$.

Water coming out of the pipe forms a cuboid of base area $25cm^2$ and length equal to the distance travelled in 45 minutes with the speed 16km/hour.

i.e., length $=16000\times100\times\frac{45}{60}cm$

$\therefore$ Volume of water coming out of pipe in 45 minutes

$$=25\times16000\times100\left(\frac{45}{60}\right)$$

Now, volume of water in the tank $=$ volume of water coming out of the pipe in 45 minutes

$$\Rightarrow 8000\times2500\times h=16000\times100\times\frac{45}{60}\times25$$

$$\Rightarrow h=\frac{16000\times100\times45\times25}{8000\times2500\times60}cm=1\cdot5cm.$$

24. Water in a rectangular reservoir having base 80 m by 60 m i s 6.5 m deep. In what time can the water be emptied by a pipe ôf which the cross-section is a square of side 20 cm, if the water runs through the pipe at the rate of 15 km/hr.

Sol:

Given that,

Flow of water $=15km/hr$

$=15000m/hr.$

Volume of water coming out of the pipe in one hour

$=\frac{20}{100}\times\frac{20}{100}\times\pm5000=600m^3$

Volume of the tank $=80\times60\times6\cdot5$

$=31200m^3$

$\therefore$ Time taken to empty the tank

$=\frac{\text{Volume of tank}}{\text{volume of water coming out of the pipe in one hour}}$

$=\frac{31200}{600}$

$=52\ hours.$

25. A village having a population of 4000 requires 150 litres of water per head per day. It has a tank measuring 20 m × 15 m × 6 m. For how many days will the water of this tank last?

Sol:

Given that

Length of the cuboidal tank $(l)=20m$

Breath of the cuboidal tank $(b)=15m.$

Height of cuboidal tank (h)=6 m

Height of the tank $=l\times b\times h=(20\times15\times6)m^3$

$=1800m^3$

$=1800000$ liters.

Water consumed by people of village in one day

$=4000\times150\ litres.$

$=600000\ litres.$

Let water of this tank lasts for n days

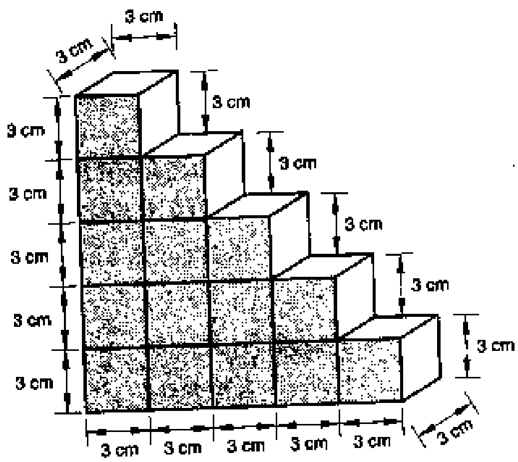
Water consumed by all people of village in n days = capacity of tank

$n\times600000=1800000$

$n=3$

Thus, the water of tank will last for 3 days.

26. A child playing with building blocks, which are of the shape of the cubes, has built a structure as shown in Fig. 18.12 If the edge of each cube is 3 cm, find the volume of the structure built by the child.



Sol:

Volume of each cube = edge $\times$ edge $\times$ edge

$= 3 \times 3 \times 3 cm^3 = 27 cm^3$.

Number of cubes in the surface structure $= 15$

$\therefore$ Volume of the structure $= 27 \times 15 cm^3$

$= 405 cm^3$.

27. A godown measures 40m × 25 m × 10 m. Find the maximum number of wooden crates each measuring 1.5 m × 1.25 m × 0.5 m that can be stored in the godown.

Sol:

Given go down length $(l_1) = 40m$.

Breath $(b_1) = 25m$.

Height $(h_1) = 10m$.

Volume of wooden crate $= l_1 \times b_1 \times h_1 = 40 \times 25 \times 10 m^3$

$= 10000 m^3$

Wood of wooden crate $= l_2 \times b_2 \times h_2$

$= 1 \cdot 5 \times 1 \cdot 25 \times 0 \cdot 25 m^3 = 0 \cdot 9375 m^3$

Let m wooden creates be stored in the go down volume of m wood crates = volume of go down

$0 \cdot 9375 \times n = 10000$

$n = \dfrac{10000}{0 \cdot 9375} = 10,666 \cdot 66,$

Thus, $10,666 \cdot 66$ wooden crates can be stored in go down.

28. A wall of length 10 m was to be built across an open ground. The height of the wall is 4 m and thickness of the wall is 24 cm. If this wall is to be built up with bricks whose dimensions are 24 cm × 12 cm × 8 cm, How many bricks would be required?

Sol:

Given that

The wall with all its bricks makes up the space occupied by it we need to find the volume of the wall, which is nothing but cuboid.

Here, length $= 10m = 1000cm$

Thickness $= 24cm$

Height $= 4m = 400cm$

$\therefore$ The volume of the wall

= length × breadth × height

$= 1000 \times 24 \times 400 cm^3$

Now, each brick is a cuboid with length = 24cm,

Breadth $= 12cm \text{ and height} = 8cm.$

So, volume of each brick = length $\times$breadth $\times$height

$= 24 \times 12 \times 8 cm^3$.

So, number of bricks required $= \frac{\text{Volume of the wall}}{\text{Volume of each brick}}$

$= \frac{1000 \times 24 \times 400}{24 \times 12 \times 8}$

$= 4166 \cdot 6.$

So, the wall requires 167 bricks.

Exercise – 19.1

1. Curved surface area of a right circular cylinder is 4.4 m^2. If the radius of the base of the cylinder is 0.7 m, find its height.

Sol:

Given that

Radius of base of the cylinder $e_r = 0\cdot 7m$

Curved surface area of cylinder $= 4\cdot 4m^2 = 2\pi rh$

Let h be the height of the cylinder

WKT,

$2\pi rh = 4\cdot 4m^2$

$2\times 3\cdot 14\times 0\cdot 7\times h = 4\cdot 4$

$(4\cdot 4)hm - 4\cdot 4m^2$

$h = 1m$

$\therefore$ The height of the cylinder $= 1m.$

2. In a hot water heating system, there is a cylindrical pipe of length 28 m and diameter 5 cm. Find the total radiating surface in the system.

Sol:

Given that

Height of cylinder = length of cylindrical pipe $= 28m.$

Radius (r) of circular end of pipe $= \frac{5}{2}cm = 2\cdot 5cm$

$= 0\cdot 025m.$

Curved surface area of cylindrical pipe $= 2\pi rh$

$= 2\times 3\cdot 14\times 0\cdot 025\times 28 = 4\cdot 4cm$

$\therefore$ The area of radiation surface of the system is $4\cdot 4m^2$ or $44000cm^2$

3. A cylindrical pillar is 50 cm in diameter and 3.5 m in height. Find the cost of painting the curved surface of the pillar at the rate of 12.50 per m^2.

Sol:

Given that

Height of the pillar $= 3\cdot 5m$

Radius of the circular end of the pillar $= \frac{50}{2}cm.$

$= 25cm = 0\cdot 25m$

Curved surface area of pillar $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 0 \cdot 25 \times 3 \cdot 5 m^2$

$= 5 \cdot 5 m^2$

Cost of painting $1m^2$ area – $Rs. 12 \cdot 50$

Cost of painting $5 \cdot 5m^2$ area $= Rs.(5 \cdot 5 \times 12 \cdot 50)$

$= Rs. 68 \cdot 75.$

Thus, the cost of painting the CSA pillar is $Rs. 68, 75$

4. It is required to make a closed cylindrical tank of height 1 m and base diameter 140 cm from a metal sheet. How many square meters of the sheet are required for the same?

Sol:

Height of the cylindrical tank $(h) = 1m.$

Base radius of cylindrical tank $(r) = \frac{140}{2} m = 70cm$

$= 0 \cdot 7m$

Area of sheet required – total surface area of tank $= 2\pi(r + h)$

$= 2 \times 3 \cdot 14 \times 0 \cdot 7(0 \cdot 7 + 1) m^2$

$= 4 \cdot 4 \times 1 \cdot 7 m^2$

$= 7 \cdot 48 m^2$

$\therefore$ So, it will required $7 \cdot 48m^2$ of metal sheet.

5. A solid cylinder has total surface area of 462 cm^2. Its curved surface area is one-third of its total surface area. Find the radius and height of the cylinder.

Sol:

We have

Curved surface area $= \frac{1}{3} \times$ total surface area

$\Rightarrow 2\pi rh = \frac{1}{3}(2\pi rh + 2\pi r^2)$

$\Rightarrow 6\pi rh = 2\pi rh + 2\pi r^2$

$\Rightarrow 4\pi rh = 2\pi r^2$

$\Rightarrow 2h = r$

We know that,

Total surface area $= 462$

$\Rightarrow$ Curved surface Area $= \frac{1}{3} \times 462$

$\Rightarrow 2\pi rh = 154$

$\Rightarrow 2 \times 3 \cdot 14 \times 2h^2 = 154$

$\Rightarrow h^2 = \frac{154 \times 7}{2 \times 22 \times 2}$

$= \frac{49}{4}$

$\Rightarrow h = \frac{7}{2} cm$

$\Rightarrow r = 2h$

$\Rightarrow r = 2 \times \frac{7}{2} cm$

$\Rightarrow r = 7cm.$

6. The total surface area of a hollow cylinder which is open from both sides is 4620 sq. cm, area of base ring is 115.5 sq. cm and height 7 cm. Find the thickness of the cylinder.

Sol:

Let the inner radii of hollow cylinder $\Rightarrow rcm$

Outer radii of hollow cylinder $\Rightarrow Rcm$

Then,

$2\pi rh + 2\pi Rh + 2\pi R^2 - 2\pi r^2 = 4620 \rightarrow (1)$

$\pi R^2 - \pi r^2 = 115.5 \rightarrow (2)$

$\Rightarrow 2\pi h(R + r) + 2(\pi R^2 - \pi r^2) = 4620 \ and \ \pi R^2 - m^2 = 115 \cdot 5$

$\Rightarrow 2\pi h(R + r) + 231 = 4620 \ and \ \pi(R^2 - r^2) = 115 \cdot 5$

$\Rightarrow 2\pi \times 7(r + R) = 4389 \ and \ \pi(R^2 - r^2) = 115 \cdot 5$

$\Rightarrow \pi(R + r) = 313 \cdot 5 \ and \ \pi(R + r)(R - r) = 115 \cdot 5$

$\Rightarrow \frac{\pi(R + r)(R - r)}{\pi(R + r)} = \frac{115 \cdot 5}{313 \cdot 5}$

$\Rightarrow R - r = \frac{7}{19} cm.$

7. Find the ratio between the total surface area of a cylinder to its curved surface area, given that its height and radius are 7.5 cm and 3.5 cm.

Sol:

For cylinder, total surface Area $= 2\pi r(h + r)$

Curved surface area $= 2\pi rh$

$$\frac{\text{Total surface area}}{\text{Curved surface area}} = \frac{2\pi r(h+r)}{2\pi rh} = \frac{h+r}{h}$$

$$\therefore \frac{\text{Total surface area}}{\text{curved surface area}} = \frac{7\cdot 5+3\cdot 5}{7\cdot 5} = \frac{11}{7\cdot 5}$$

$$= \frac{11\times 10}{7\cdot 5} = \frac{22}{15} = 22:15.$$

8. The total surface area of a hollow metal cylinder, open at both ends of external radius 8 cm and height 10 cm is 338 p cm^2. Taking r to be inner radius, obtain an equation in r and use it to obtain the thickness of the metal in the cylinder.

Sol:

Given that,

External radius $(R) = 8cm$

Height $(h) = 10cm$

The total surface area of a hollow metal cylinder $= 338\ IT\ cm^2$

We know that

$2\pi Rh + 2\pi rh + 2\pi R^2 - 2\pi r^2 = 338\pi.$

$\Rightarrow h(R+r)+(R+r)(R-r)=169$

$\Rightarrow 10(8+r)+(8+r)(8-r)=169$

$\Rightarrow 80+10r+64-r^2=169$

$\Rightarrow x^2-10r+25=0$

$\Rightarrow r=5$

$\therefore R-r=8-5cm=3cm$

9. A cylindrical vessel, without lid, has to be tin-coated on its both sides. If the radius of the base is 70 cm and its height is 1.4 m, calculate the cost of tin-coating at the rate of Rs. 3.50 per 1000 cm^2.

Sol:

Given that

$r = 70cm,\ h = 1\cdot 4m = 140cm$

$\therefore$ Area to be tin coated $= 2\left(2\pi rh + \pi r^2\right) = 2\pi r(2h+r)$

$= 2\times\frac{22}{7}\times 70(280+70)$

$= 154000\,cm^2$

Required cost $= \frac{154000\times 3\cdot 50}{1000} = Rs.539.$

10. The inner diameter of a circular well is 3.5 m. It is 10 m deep Find:
(i) inner curved surface area.
(ii) the cost of plastering this curved surface at the rate of Rs. 40 per m^2.
Sol:
Inner radius (r) of circular well $=1\cdot 75m$
Depth (n) of circular well $=10m$
(i) Inner curved surface area $=2\pi rh$
$=2\times\frac{22}{7}\times 1\cdot 75\times 10m^2$
$=(144\times 0\cdot 25\times 10)m^2$
$=110m^2$
(ii) Cost of plastering $1m^2$ area $=Rs.\ 40.$
Cost of plastering $110m^2$ $area = Rs.(110\times 40)$
$=Rs.\ 4400$

11. Find the lateral curved surface area of a cylinderical petrol storage tank that is 4.2 m in diameter and 4.5 m high. How much steel was actually used, if $\frac{1}{12}$ of steel actually used was wasted in making the closed tank?
Sol:
Height (h) cylindrical tank $=4\cdot 5m$
Radius (r) of circular end of cylindrical tank $=\frac{4\cdot 2}{2}m = 2\cdot 1m.$
(i) Lateral or curved surface area of tank $=2\pi rh$
$\Rightarrow 2\times 3\cdot 14\times 2\cdot 1\times 4\cdot 5m^2$
$=59\cdot 4m^2$
(ii) Total surface area of tank $=2\pi r(r+h)$
$=2\left[\frac{22}{7}\right]\times 2\cdot 1(2\cdot 1+4\cdot 5)m^2$
$=87\cdot 12m^2$
Let $A\ m^2$ steel sheet be actually used in making the tank
$\therefore A\left(1-\frac{1}{12}\right)=87\cdot 12m^2$
$\Rightarrow A=\left(\frac{12}{\pi}\times 87\cdot 12\right)m^2$
$\Rightarrow A=95\cdot 04m^2$
Thus, $95\cdot 04m^2$ steel was used in actual while making the tank.

12. The students of a Vidyalaya were asked to participate in a competition for making and decorating pen holders in the shape of a cylinder with a base, using cardboard. Each pen holder was to be of radius 3 cm and height 10.5 cm. The Vidyalaya was to supply the competitors with cardboard. If there were 35 competitors, how much cardboard was required to be bought for the competition?

Sol:

Radius of circular end of cylinder pen holder $= 3cm$

Height of pen holder $= 10\cdot 5cm$

Surface area of 1 pen holder = CSA of penholder + Area of base of SA of 1 penholder =

$2\pi rh + \pi r^2$

$= 2\times 3\cdot 14\times 3\times 10\cdot 5 + 3\cdot 14138$

$= 132\times 1\cdot 5 + \frac{198}{7} cm^2$

$= 198 + \frac{198}{7} cm^2$

$= \frac{1584}{7} cm^2$

Area of car board sheet used by 1 competitor $= \frac{1584}{7} cm^2$

Area of car board sheet used by 35 competitors $= \frac{1584}{7}\times 35 cm^2 = 7920 cm^2$.

13. The diameter of roller 1.5 m long is 84 cm. If it takes 100 revolutions to level a playground, find the cost of levelling this ground at the rate of 50 paise per square metre.

Sol:

Given that,

Diameter of the roller $= 84cm = 0\cdot 84m.$

Length of the roller $= 1\cdot 5m.$

Radius of the roller $= \frac{D}{2} = \frac{0\cdot 84}{2} = 0\cdot 42.$

Area covered by the roller on one revolution = covered surface area of roller

Curved surface of roller $= 2\pi rh = 2\times \frac{22}{7}\times 0\cdot 42\times 1\cdot 5$

$= 0\cdot 12\times 22\times 1\cdot 5 m^2$

Area of the playground $= 100\times$ Area covered by roller in one revolution

$= (100\times 0\cdot 12\times 22\times 1\cdot 5) m^2$

$= 396 m^2$

Now,

Cost of leveling $1m^2 = 50P = \frac{50}{100} \Rightarrow \text{Re} = \frac{1}{2} rs$

Cost of leveling $396m^2 = \frac{1}{2} \times 396 = Rs \cdot 198$

Hence, cost of leveling $396m^2$ is 198

14. Twenty cylindrical pillars of the Parliament House are to be cleaned. If the diameter of each pillar is 0.50 m and height is 4 m. What will be the cost of cleaning them at the rate of Rs. 2.50 per square metre?

Sol:

Diameter of each pillar $= 0 \cdot 5m$

Radius of each pillar $(r) \frac{a}{2} = \frac{0 \cdot 5}{2} = 0 \cdot 25m.$

Height of each pillar $= 4m.$

Curved surface area of each pillar $= 2\pi rh$

$= 2 \times 3 \cdot 14 \times 0 \cdot 25 \times 4m^2$

$= \frac{44}{7} m^2$

Curved surface area of 20 pillars $= 20 \times \frac{44}{7} m^2$

Given, cost of cleaning $= Rs. 2 \cdot 50 \, per$ square meter

$\therefore$ Cost of cleaning 20 pillars $= Rs.\ 2 \cdot 50 \times 20 \times \frac{44}{7}$

$= Rs.\ 314 \cdot 28.$

Exercise – 20.1

1. Find the curved surface area of a cone, if its slant height is 60 cm and the radius of its base is 21 cm.

Sol:

Given that

Radius of its base is 21cm

Slant height $= 60cm$

WKT, Curved surface area of a cone $= \pi rl$

$\therefore$ Curved surface area $= \frac{22}{7} \times 21 \times 60$

$= 3960cm^2$

2. The radius of a cone is 5 cm and vertical height is 12 cm. Find the area of the curved surface.

Sol:

Given,

Radius of cone $= 5cm = r$

Height of a cone $= 12cm = h$

Slant height of the cone $= \sqrt{r^2 + h^2}$

$= \sqrt{5^2 + 12^2} = 13cm$

$\therefore$ Curved surface Area $= \pi rl$

$= \frac{22}{7} \times 5 \times 12$

$= 204 \cdot 28cm^2$

3. The radius of a cone is 7 cm and area of curved surface is 176 cm^2. Find the slant height.

Sol:

Given

Radius of a cone $(r) = 7cm.$

Let $'l'$ be the slant height of a cone

$\therefore$ Curved surface area $= \pi rl.$

$\Rightarrow 176 = \pi \times 7 \times l$

$\Rightarrow l = \frac{176}{7\pi} = \frac{176 \times 7}{7 \times 22} = 8cm.$

4. The height of a cone is 21 cm. Find the area of the base if the slant height is 28 cm.
Sol:
Given that
Slant height 'l' $= 28m.$
Height of cone $(h) = 21cm$

$\therefore$ Radius of cone $(r) = \sqrt{28^2 - 21^2}$ [by Pythagoras theorem]

$= 7\sqrt{7}cm$

$\therefore$ Area of base $= \pi r^2$

$= \frac{22}{7} \times \left(7\sqrt{7}\right)^2$

$= \frac{22}{7} \times 7 \times 7 \times 7 = 1078cm^2.$

5. Find the total surface area of a right circular cone with radius 6 cm and height 8 cm.
Sol:
WKT, Total surface area $= \pi rl + \pi r^2$
Now $l = \sqrt{h^2 + r^2}$ [by Pythagoras theorem]
Here, given
Radius $= 6cm$ and height $= 8cm$
$\Rightarrow$ length $= \sqrt{6^2 + 8^2}$
$= 10cm$
$\therefore$ Total surface area $= \pi rl + \pi r^2$

$= \left(\frac{22}{7} \times 6 \times 10\right) + \left(\frac{22}{7} \times 6 \times 6\right)$

$= \left(\frac{1320}{7}\right) + \frac{792}{7} = 301 \cdot 71cm^2$

6. Find the curved surface area of a cone with base radius 5.25 cm and slant height 10cm.
Sol:
Given that,
Radius of a base of a cone $= 5 \cdot 25cm$
Slant height of cone $= 10cm$
Curved surface area of cone $= \pi rl$

$= \frac{22}{7} \times 5 \cdot 25 \times 10cm^2$

$= \left(22 \times 0 \cdot 75 \times 10\right)cm^2$

$= 165cm^2$

Thus, the curved surface area of a cone is
$165cm^2$

7. Find the total surface area of a cone, if its slant height is 21 m and diameter of its base is 24m.

Sol:

Given that,

Radius of base of cone $=\left(\frac{24}{2}\right)=12m$

Slant height of cone $=21m.$

Total surface area of cone $=\pi r^2+\pi rl$

$=\pi r(r+l)$

$=\frac{22}{7}\times 12\times(12+21)$

$=\frac{22}{7}\times 12\times 33m^2$

$=1244\cdot 57m^2.$

8. The area of the curved surface of a cone is 60 π cm^2. If the slant height of the cone be 8 cm, find the radius of the base?

Sol:

Given that

Curved surface area of cone $=60\pi cm^2$

$\therefore$ Slant height of cone $(l)=8cm.$

$i.e.,\ \pi rl=60\pi$

$\Rightarrow \pi\times r\times 8=60\pi$

$\Rightarrow r=\frac{60}{8}=7\cdot 5$

$\therefore$ Radius of cone $=7\cdot 5cm.$

9. The curved surface area of a cone is 4070 cm^2 and its diameter is 70 cm. What is its slant height? (Use it π = 22/7).

Sol:

Given diameter $=70cm$

$\Rightarrow 2r=70cm$

$\Rightarrow r=35cm$

Now, curved surface area $=4070cm^2$

$\Rightarrow \pi rl=4070$

Where $r=$ radius of the cone

$l=$ slant height of the cone

$\therefore \pi rl=4070$

$\Rightarrow \frac{22}{7} \times 35 \times l = 4070$

$\Rightarrow l = \frac{4070 \times 7}{22 \times 35} = 37cm$

$\therefore$ Slant height of the cone $= 37cm.$

10. The radius and slant height of a cone are In the ratio of 4 : 7. If its curved surface area is 792 cm^2, find its radius. (Use it π = 22/7).

Sol:

Given that,

Curved surface area $= \pi rl = 792.$

Let the radius $(r) = 4x$

Height $(h) = 7x$

Now, CSA $= 792$

$\frac{22}{7} \times 4x \times 7x = 792$

$\Rightarrow 88x^2 = 792$

$\Rightarrow x^2 = \frac{792}{88} = 9.$

$\Rightarrow x = 3.$

$\therefore$ Radius $= 4x = 4 \times 3 = 12cm.$

11. A Joker's cap is in the form of a right circular cone of base radius 7 cm and height 24 cm. Find the area of the sheet required to make 10 such caps.

Sol:

Given that,

Radius of conical cap $(r) = 7cm.$

Height of conical cap $(h) = 24cm.$

Slant height (I) of conical cap $= \sqrt{r^2 + h^2}$

$= \sqrt{(7)^2 + (24)^2} cm$

$= 25cm$

CSA of 1 conical cap $= \pi rl = \frac{22}{7} \times 7 \times 25 cm^2 = 550cm^2$

Curved surface area of such 10 conical caps $= 5500cm^2$

$[\because 550 \times 10]$

Thus, $5500cm^2$ sheet will be req for making of 10 caps.

12. Find the ratio of the curved surface areas of two cones if their diameters of the bases are equal and slant heights are in the ratio 4 : 3.

Sol:

Given that,

Diameter of two cones area equal

$\therefore$ Their radius are equal

Let $r_1 = r_2 = r$

Let ratio be x

$\therefore$ Slant height l_1 of 1^{st} cone $= 4x$

Similarly slant height l_2 of 2^{nd} cone $= 3x$.

$$\therefore \frac{C \cdot S\ A_1}{C \cdot S\ A_2} = \frac{\pi r_1 l_1}{\pi r_2 l_2} = \frac{\pi \times r \times 4x}{\pi \times r \times 3x} = \frac{4}{3}.$$

13. There are two cones. The curved surface area of one is twice that of the other. The slant height of the later is twice that of the former. Find the ratio of their radii.

Sol:

Let curved surface area off 1^{st} cone $= 2x$

CSA of 2^{nd} cone = x

and slant height of 1^{st} cone = h

and slant height of 2^{nd} cone = 2h

$$\therefore \frac{CSA\ of\ 1^{st} cone}{CSA\ of\ 2^{nd}\ cone} = \frac{2x}{x} = \frac{2}{1}.$$

$$\Rightarrow \frac{\pi r_1 l_1}{\pi r_2 l_2} = \frac{2}{1}$$

$$\Rightarrow \frac{r_1 h}{r_2 h} = \frac{2}{1} \Rightarrow \frac{r_1}{r_2} = \frac{4}{1}.$$

i.e., ratio of r_1 *and* r_2 is $(4:1)$

14. The diameters of two cones are equal. If their slant heights are in the ratio 5 : 4, find the ratio of their curved surfaces.

Sol:

Given that,

Diameters of two cones are equal

$\therefore$ Their radius are also equal i.e., $r_1 = r_2$

Let the ratio of slant height be x

$\therefore l_1 = 5x\ and\ l_2 = 4x$

$\therefore$ Ratio of curved surface area $= \dfrac{C_1}{C_2}$

$$\therefore \frac{C_1}{C_2} = \frac{\pi r_1 l_1}{\pi r_2 l_2} = \frac{\pi r_1 Sx}{\pi r_2 (4x)} = \frac{5}{4}$$

$\therefore$ Ratio of curved surface area $= 5:4$

15. Curved surface area of a cone is 308 cm^2 and its slant height is 14 cm. Find the radius of the base and total surface area of the cone.

Sol:

(i) Given that,

Slant height of cone $= 14cm$

Let radius of circular end of cone = r.

Curved surface area of cone $= \pi rh$

$$308cm^2 = \left(\frac{22}{7} \times r \times 14\right)cm \qquad \left[\because CSA = 308cm^2\right)$$

$$\Rightarrow r = \frac{308}{44}cm = 7cm$$

Thus, radius of circular end of cone $= 7cm$

(ii) Given that CSA $= 308cm^2$

WKT, total surface area of cone

= curved surface area of cone + area of base

$= \pi rl + \pi r^2$

$$= \left[308 + \frac{22}{7}(7)^2\right]cm^2$$

$= 308 + 154cm^2$

$= 462cm^2$

Thus, the total SA of the cone is $462cm^2$.

16. The slant height and base diameter of a conical tomb are 25 m and 14 m respectively. Find the cost of white-washing its curved surface at the rate of Rs. 210 per l00 m^2.

Sol:

Given that,

Slant height of conical tomb (l) =25m

Base radius (r) of tomb $= \dfrac{14}{2}m = 7m.$

CSA of conical length tomb $= \pi rl$

$$= \left(\frac{22}{7} \times 7 \times 25\right)m^2$$

$= 550m^2$

Cost of white – washing $100m^2$ area = Rs. 210

Cost of white – washing $550m^2$ area = $Rs.\left(\frac{210\times 550}{100}\right)$

$= Rs.1155.$

Thus the cost of white washing total tomb = Rs. 1155

17. A conical tent is 10 m high and the radius of its base is 24 m. Find the slant height of the tent. If the cost of 1 m^2 canvas is Rs. 70, find the cost of the canvas required to make the tent.

Sol:

(i) Given that

Height of conical tent $(h)=10m$

Radius of conical tent $(r)=24m.$

Let slant height of conical tent be l

$l^2=h^2+r^2=(10m)^2+(24m)^2=(100+576)m^2$

$=676m^2$

$l=26m.$

Thus, the slant height of the conical tent is $26m.$

(ii) Given that

Radius $(r)=24$

Slant height $(l)=26$

CSA of tent $=\pi rl=\frac{22}{7}\times 24\times 26=\frac{13728}{7}m^2$

Cost of $1m^2$ canvas $S=Rs.70.$

Cost of $\frac{13728}{7}m^2$ canvas $=\frac{13728}{7}\times 10$

$=Rs.1,37,280.$

Thus, the cost of canvas required to make the tent is Rs. $137280.$

18. A tent is in the form of a right circular cylinder surmounted by a cone. The diameter of cylinder is 24 m. The height of the cylindrical portion is 11 m while the vertex of the cone is 16 m above the ground. Find the area of the canvas required for the tent.

Sol:

Given that,

Diameter of cylinder $=24m$

$\therefore$ Radius $=\frac{\text{diameter}}{2}=\frac{24cm}{2}=12cm$

Also Radius of cone $=12m.$
Height of cylinder $=11m$
Height of cone $=16-11=5m$
Slant height of cone $=\sqrt{h^2+r^2}$
$=\sqrt{5^2+12^2}=13m$
$\left[\because l=\sqrt{r^2+h^2}\right]$
$\therefore$ area of canvas required for the
tent $=\pi rl+2\pi rh$
$=\frac{22}{7}\left[12\times 13+2\times 12\times 11\right]$
$=490\cdot 285+829\cdot 714$
$=1320m^2.$

19. A circus tent is cylindrical to a height of 3 meters and conical above it. If its diameter is 105 m and the slant height of the conical portion is 53 m, calculate the length of the canvas 5 m wide to make the required tent.
Sol:
Given diameter $=105m$
Radius $=\frac{105}{2}m=52\cdot 5m.$
$\therefore$ Curved surface area of circus tent $=\pi rl+2\pi rh$
$=\frac{22}{7}\times 52\cdot 5\times 53+2\times 52\cdot 5\times 3\times \frac{22}{7}$
$=8745+990$
$=9735m^2$
$\therefore$ Length of the canvas equation for tent $=\frac{\text{Area of canvas}}{\text{width of canvas}}$
$=\frac{9735}{5}=1947m$

20. The circumference of the base of a 10 m height conical tent is 44 metres. Calculate the length of canvas used in making the tent if width of canvas is 2 m. (Use it π= 22/7).
Sol:
WKT, CSA of cone = πrl
Given circumference $=2\pi r$
$\Rightarrow 2\times \frac{22}{7}\times r=44\Rightarrow \frac{r}{7}=1\Rightarrow r=7m$

$\therefore L = \sqrt{r^2 + h^2} = \sqrt{7^2 + 10^2} = \sqrt{149}m$

$\therefore CSA$ of tent $= \pi rl = \frac{22}{7} \times 7 \times \sqrt{149} = 22\sqrt{149}.$

$\therefore$ The length of can vas used in making tent

$= \frac{\text{Area of canvas}}{\text{width of canvas}}$

$= \frac{22\sqrt{149}}{2} = 11\sqrt{149}$

$= 134 \cdot 2m.$

21. What length of tarpaulin 3 m wide will be required to make a conical tent of height 8 m and base radius 6 m? Assume that the extra length of material will be required for stitching margins and wastage in cutting is approximately 20 cm (Use it π = 3.14)

Sol:

Given that,

Height of conical tent $(h) = 8m.$

Radius of base of tent $(r) = 6m.$

Slant height $(l) = \sqrt{r^2 + h^2} = \sqrt{8^2 + 6^2} = \sqrt{100} = 10m$

CSA of conical tent $= \pi rl = (3 \cdot 14 \times 6 \times 10) m^2$

$= 188 \cdot 4m^2$

Let the length of tarpaulin sheet require be L

As 200m will be wasted, So effective length will be $(L - 0 \cdot 2m)$

Breadth of tarpaulin $= 3m$

Area of sheet $=$ CSA of sheet

$(L \times 0 \cdot 2m \times 3) m = 188 \cdot 40m^2$

$\Rightarrow L - 0 \cdot 2m = 62 \cdot 8m$

$\Rightarrow L = 63m$

Thus, the length of the tarpaulin sheet will be $= 163m.$

22. A bus stop is barricated from the remaining part of the road, by using 50 hollow cones made of recycled card-board. Each cone has a base diameter of 40 cm and height 1 m. If the outer side of each of the cones is to be painted and the cost of painting is Rs. 12 per m^2, what will be the cost of painting all these cones. (Use π = 3.14 and $\sqrt{1.04}$ = 1.02)

Sol:

Radius of cone $(r) = \frac{40}{2} = 20m = 0 \cdot 2m.$

Height of cone $= 1m.$

Slant height of cone $(l) = \sqrt{h^2 + r^2}$

$= \sqrt{1^2 + (0 \cdot 2)^2}\ m$

$= \sqrt{1 \cdot 04}m = 1 \cdot 02m$

Curved surface area of each one

$= \pi rl = (3 \cdot 14 \times 0 \cdot 2 \times 1 \cdot 02)m^2$

$= 0 \cdot 64056m^2$

CSA of 50 such cone $= 50 \times 0 \cdot 64056m^2 = 32 \cdot 028m^2$

Cost of painting $1m^2$ area = Rs. 12.

Cost of painting $32 \cdot 028m^2$ area $= Rs.(32 \cdot 028 \times 12)$

$= Rs.\ 384 \cdot 326\ PS.$

Thus, it will cost Rs. 38434 (Approx) in painting the so hollow cones.

23. A cylinder and a cone have equal radii of their bases and equal heights. If their curved surface areas are in the ratio 8:5, show that the radius of each is to the height of each as 3:4.

Sol:

Let us assume radius of cone $= r.$

Also, radius of cylinder $= r.$

Height of cone $= h$

And, height of cylinder $= h.$

Let C_1, be the curved surface area of cone

$\therefore C_1 = \pi r\sqrt{r^2 + h^2}$

Similarly, C_2 be the curved surface area of cone cylinder.

$\therefore C_2 = 2\pi rh$

According to question $\frac{C_2}{C_1} = \frac{8}{5}.$

$\Rightarrow \frac{2\pi rh}{\pi r\sqrt{r^2 + h^2}} = \frac{8}{5}$

$\Rightarrow 10h = 8\sqrt{r^2 + h^2}$

$\Rightarrow 100h^2 = 64r^2 + 64h^2$

$\Rightarrow 36h^2 = 64r^2$

$\frac{h}{r} = \sqrt{\frac{64}{30}}$

$\Rightarrow \left(\frac{h}{r}\right)^2 = \frac{64}{36}$

$\Rightarrow \frac{b}{r} = \sqrt{\frac{64}{30}} = \frac{8}{6} = \frac{4}{3}$

$\therefore \frac{r}{h} = \frac{3}{4}.$

Exercise – 20.2

1. Find the volume of a right circular cone with:

(i) radius 6 cm, height 7 cm.

(ii) radius 3.5 cm, height 12 cm

(iii) height 21 cm and slant height 28 cm.

Sol:

(i) Given that,

Radius of cone $(r) = 6m$

Height of cone $(h) = 7cm$

Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (6)^2 \times 7$

$= 264cm^3$

(ii) Given,

Radius of cone $(r) = 3 \cdot 5cm$

Height of cone $(h) = 12cm$

Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (3 \cdot 5)^2 \times 12$

$= 154cm^3$

(iii) From the relation

$l^2 = r^2 + h^2$, we have

$r = \sqrt{l^2 - h^2} = \sqrt{(28)^2 - (21)^2} = 7\sqrt{7}cm$

So, volume of cone $= \frac{1}{3} \times \pi r^2 \times h$

$= \frac{1}{3} \times \frac{22}{7} \times (21)^2 \times (7\sqrt{7})^2$

$= 7546cm^3$

2. Find the capacity in litres of a conical vessel with:
(i) radius 7 cm, slant height 25 cm (ii) height 12 cm, slant height 13 cm.
Sol:
(i) Radius of cone $(r) = 7cm$

Slant height (l) of cone $= 25cm$

Height (h) of cone $= \sqrt{l^2 - r^2}$

$= \sqrt{(25)^2 - b^2} = \sqrt{25^2 - 7^2} = 24cm.$

Volume of cone $= \frac{1}{3}\pi r^2 h = \left[\frac{1}{3} \times \frac{22}{7} \times (7)^2 \times 24\right] cm^3$

$= 1232\ cm^3.$

(ii) Height (h) of cone = 12cm.

Slant height of cone $(l) = 13cm.$

Radius (r) of cone $= \sqrt{l^2 - r^2} = \sqrt{13^2 - 12^2} cm$

$= 5cm.$

Volume of cone $= \frac{1}{3}\pi r^2 h = \left(\frac{1}{3} \times \frac{22}{7} \times (5)^2 \times 12\right) cm^3$

$= \frac{2200}{7} cm^3$

Capacity of the conical vessel $= \left(\frac{2200}{7000}\right) liters$

$= \frac{11}{35} liters$

3. Two cones have their heights in the ratio 1 : 3 and the radii of their bases in the ratio 3 : 1. Find the ratio of their volumes.
Sol:
Given that, let height $\rightarrow h$ say
Height of 1st cone = h
Height of 2nd cone = 3h
Let the ratio of radii be r
$\therefore$ Radius of 1st cone = 3r
Radius of 2nd cone = r
$\therefore$ ratio of volume $= \frac{V_1}{V_2}$

$$\Rightarrow \frac{V_1}{V_2} = \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{r_1^2 h_1}{r_2^2 h_2}$$

$$= \frac{(3r)^2 \times h}{r^2 \times 3h}$$

$$= \frac{9r^2 h}{3r^2 h}$$

$$= \frac{3}{1}$$

$$\Rightarrow \frac{V_1}{V_2} = \frac{3}{1}.$$

4. The radius and the height of a right circular cone are in the ratio 5 : 12. If its volume is 314 cubic meter, find the slant height and the radius (Use it π = 3.14).

Sol:

Let the ratio be x

$\therefore$ Radius $'r' = 5x$

Height ‘h’ = $12x$

WKT,

$\therefore$ Slant height $= \sqrt{r^2 + h^2} = \sqrt{(5x)^2 + (12x)^2} = 13x$

Now volume $= 314 m^3$ [given data]

$$\Rightarrow \frac{1}{3}\pi r^2 h = 314 m^3$$

$$\Rightarrow \frac{1}{3} \times 3{\cdot}14 \times 25x^2 \times 12x = 314$$

$$\Rightarrow x^3 = \frac{314 \times 3}{3{\cdot}14 \times 25 \times 12}$$

$$\Rightarrow x^3 = 1 \Rightarrow x = 1$$

$\therefore$ Slant height $= 13x = 13m$

Radius $= 5x = 5m.$

5. The radius and height of a right circular cone are in the ratio 5 : 12 and its volume is 2512 cubic cm. Find the slant height and radius of the cone. (Use it π = 3.14).

Sol:

Let the ratio be x

Radius ‘r’ = $5x$

Height ‘h’ = $12x$

$\therefore$ Slant height $'l' = \sqrt{r^2 + h^2} = \sqrt{(5x)^2 + (12x)^2} = 13x.$

Now volume $= 2512 cm^3$

$\Rightarrow \frac{1}{3} \times \pi \times (5x)^2 \times 12x = 2512$

$\Rightarrow \frac{1}{3} \times 3 \cdot 14 \times 25x^2 \times 12x = 2512$

$\Rightarrow x^3 = \frac{2512 \times 3}{314 \times 25 \times 2}$

$\Rightarrow x = 2.$

$\therefore$ Slant height $= 13x = 13 \times 2 = 26 cm$

And, Radius of cone $= 5x = 5 \times 2 = 10 cm.$

6. The ratio of volumes of two cones is 4 : 5 and the ratio of the radii of their bases is 2:3. Find the ratio of their vertical heights.

Sol:

Let ratio of radius be $'r'$

Radius of 1st cone = 2r

Radius of 2nd cone = 3r

Similarly

Let volume ratio be 'v'

Volume of 1st cone $\rightarrow 4v$

Similarly volume of 2nd cone $\rightarrow 5v$

$\therefore \frac{V_1}{V_2} = \frac{4v}{5v} = \frac{4}{5}$

$\Rightarrow \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{4}{5}$

$\Rightarrow \frac{h_1 (2r)^2}{h_2 (3r)^2} = \frac{4}{5}$

$\Rightarrow \frac{h_1}{h_2} \times \frac{4r^2}{9r^2} = \frac{4}{5}$

$\Rightarrow \frac{h_1}{h_2} \times \frac{36}{20} = \frac{18}{20} = \frac{9}{5}$

$\therefore$ Ratio of the inner height is $9:5$

7. A cylinder and a cone have equal radii of their bases and equal heights. Show that their volumes are in the ratio 3:1.

Sol:

Given that,

A cylinder and a cone have equal radii of their equal bases and heights

Let radius of cone = radius of cylinder = r

Let height of cone = height of cylinder = h

Let $V_1 =$ volume of cone

$V_2 =$ volume of cylinder

$$\Rightarrow \frac{V_1}{V_2} = \frac{\frac{1}{3}\pi r^2 h}{\pi r^2 h} = \frac{1}{3}$$

$$\Rightarrow \frac{V_2}{V_1} = \frac{3}{1}$$

Hence their volumes are in the ratio $3:4$.

8. If the radius of the base of a cone is halved, keeping the height same, what is the ratio of the volume of the reduced cone to that of the original cone?

Sol:

Let radius of cone is r and height is h

Volume $V_1 = \frac{1}{3}\pi r^2 h.$

In another case,

Radius of cone = half of radius $= \frac{r}{2}$

Height $= h$

$\therefore$ Volume $= (V_2) = \frac{1}{3}\pi\left(\frac{1}{2}r\right)^2 h$

$$= \frac{1}{3}\pi \times \frac{r^2}{4} \times h$$

$$= \frac{1}{12}\pi r^2 h.$$

$$\therefore \frac{V_1}{V_2} = \frac{\frac{1}{12}\pi r^2 h}{\frac{1}{3}\pi r^2 h} = \frac{3}{12} = \frac{1}{4}.$$

$\therefore$ Ratio will be $(1:4)$.

9. A heap of wheat is in the form of a cone of diameter 9 m and height 3.5 m. Find its volume. How much canvas cloth is required to just cover the heap? (Use π = 3.14).

Sol:

Diameter of heap $d = 9m$

Radius $= \frac{9}{2} m = 4 \cdot 5m.$

Height $(h) = 3 \cdot 5m.$

Volume of heap $= \frac{1}{3} \pi r^2 h$

$= \frac{1}{3}\left[3 \cdot 14 \times (4 \cdot 5)^2 \times 3 \cdot 5\right] m^3$

$= 74 \cdot 18 m^3$

Slant height $l = \sqrt{r^2 + h^2} = \sqrt{(4 \cdot 5)^2 + (3 \cdot 5)^2}$

$= 5 \cdot 70m.$

Area of canvas required = CSA of cone

$= \pi rl$

$= 3 \cdot 14 \times 4 \cdot 5 \times 5 \cdot 7 m^2$

$= 80 \cdot 54 m^2$

10. A heap of wheat is in the form of a cone of diameter 9 m and height 3.5 m. Find its volume. How much canvas cloth is required to just cover the heap? (Use π = 3.14).

Sol:

Given diameter of cone 14cm

$\therefore$ Radius of cone $= 7cm$

Height of cone $= 51cm.$

$\therefore$ Volume of cone $= \frac{1}{3} \times \pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 5 \times 51$

$= 2618 cm^3$

It is given that $1cm^3$ weight 10gm

$\therefore$ $2618cm^3$ weight $(261 \times 10)\, gm$

i.e., $26 \cdot 180kg.$

11. A right angled triangle of which the sides containing he right angle are 6.3 cm and lo cm in length, is made to turn round on the longer side. Find the volume of the solid, thus generated. Also, find its curved surface area.

Sol:

Given, radius of cone $(r) = 6\cdot 3cm$

Height of cone $(h) = 10cm$

$\therefore$ WKT, Slant height $l = \sqrt{(6\cdot 3)^2 + (10)^2}$

$= 11\cdot 819cm \left[l = \sqrt{r^2 + h^2} \right]$

$\therefore$ Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times 3\cdot 14 \times (6\cdot 3)^2 \times 10 = 4158cm^3$

And CSA of cone $= \pi rl$

$= \frac{22}{7} \times 6\cdot 3 \times 11\cdot 819 = 234\cdot 01cm^2$

12. Find the volume of the largest right circular cone that can be fitted in a cube whose edge is 14 cm.

Sol:

For largest circular cone radius of the base of the cone $= \frac{1}{2}$ edge of cube

$= \frac{1}{2} \times 14 = 7cm$

And height of the cone $= 14cm$

Volume of cone $= \frac{1}{3} \times 3\cdot 14 \times 7 \times 7 \times 14$

$= 718\cdot 666 cm^3$.

13. The volume of a right circular cone is 9856 cm^3. If the diameter of the base is 28 cm, find: (i) height of the cone (ii) slant height of the cone (iii) curved surface area of the cone.

Sol:

(i) Radius of cone $= \left(\frac{28}{2}\right) cm = 14cm$

Let height of cone is h

Volume of cone $= 9856cm^3$

$\Rightarrow \frac{1}{3}\pi r^2 h = 9856cm^2$

$\Rightarrow \left[\frac{1}{3} \times 3.14 \times 7 \times 7 \times h\right] cm^2 = 9856\, cm^2$

$h = 48cm.$

Thus the height of the cone is $48cm.$

(ii) Slant height (l) of cone $= \sqrt{r^2 + h^2}$

$= \left(\sqrt{(14)^2 + (48)^2} cm\right.$

$= \sqrt{196 + 2304} = \sqrt{2500} cm$

$= 50cm$

Thus, the slant height of cone is 50cm.

(iii) CSA of cone $= \pi r l = \left(\frac{22}{7} \times 14 \times 50\right) cm^2$

$= 2200 cm^2.$

14. A conical pit of top diameter 3.5 m is 12 m deep. What is its capacity in kilo litres?

Sol:

Radius (r) of pit $= \frac{3 \cdot 5}{2} m = 1 \cdot 75m.$

Depth (h) of pit = 12m.

Volume of pit $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (1 \cdot 75)^2 \times 12$

$= 38 \cdot 5m^3$

$\therefore$ Capacity of the pit $= (38 \cdot 5 \times 1)$ Kilometers

$= 38 \cdot 5$ Kilo liters

15. Monica has a piece of Canvas whose area is 551 m^2. She uses it to have a conical tent made, with a base radius of 7m. Assuming that all the stitching margins and wastage incurred while cutting, amounts to approximately 1 m^2. Find the volume of the tent that can be made with it.

Sol:

Given that,

Area of canvas $= 551m^2$ and area of the canvas lost in wastage is $1m^2$

$\therefore$ area of canvas available for making the tent is $(551 - 1) m^2 = 550m^2$.

SA of tent $= 550m^2$ required $\cdot$ base radius of conical tent $= 7m.$

CSA of tent $= 550m^2$

$\pi rl = 550m^2$

$\Rightarrow \frac{22}{7} \times 7 \times l = 550$

$\Rightarrow l = \frac{550}{22} = 25m$

Now, WKT

$l^2 = r^2 + h^2$

$\Rightarrow (25)^2 - (7)^2 = h^2$

$\Rightarrow h = \sqrt{625 - 49}$

$= \sqrt{576} = 24m$

So, the volume of the conical tent $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times 3 \cdot 14 \times (7 \times 7)(24) m^3 = 1232m^3.$

Exercise – 21.1

1. Find the surface area of a sphere of radius:
(i) 10.5 cm (ii) 5.6 cm (iii) 14 cm
Sol:
(i) Given radius $=10\cdot 5\,cm$

$\boxed{\text{Surface area}=4\pi r^2}$

$=4\times\frac{22}{7}\times(10\cdot 5)^2$

$=1386\,cm^2$

(ii) Given radius $=5\cdot 6cm$

Surface area $=4\pi r^2=4\times\frac{22}{7}\times(5\cdot 6)^2=394\cdot 24cm^2$

(iii) Given radius = 14cm

Surface area $=4\pi r^2=4\times\frac{22}{7}\times(14)^2=2464cm^2$

2. Find the surface area of a sphere of diameter:
(i) 14 cm (ii) 21 cm (iii) 3.5 cm
Sol:
(i) Diameter = 14cm

Radius $=\frac{Diameter}{2}=\frac{14}{2}=7cm$

$\therefore$ Surface area $=4\pi r^2=4\times\frac{22}{7}\times(7)^2=616cm^2$

(ii) Diameter = 21cm

Radius $=\frac{Diameter}{2}=\frac{21}{2}=10\cdot 5\,cm$

$\therefore$ Surface area $=4\pi r^2=4\pi\times(10\cdot 5)^2=4\times\frac{22}{7}\times 10\cdot 5^2=1386cm^2$

(iii) Diameter = $3\cdot 5cm$

Radius $=3\cdot 5cm/2=1\cdot 75cm$

$\therefore$ Surface area $=4\pi r^2=4\times\frac{22}{7}\times\frac{3\cdot 5}{2^2}=38\cdot 5cm^2$

3. Find the total surface area of a hemisphere and a solid hemisphere each of radius 10 cm. (Use π = 3.14)

Sol:

The surface area of the hemisphere $= 2\pi r^2$

$= 2 \times 3 \cdot 14 \times (10)^2$

$= 628 cm^2$

The surface area of solid hemisphere $= 3\pi r^2$

$= 3 \times 3 \cdot 14 \times (10)^2$

$= 942 cm^2$

4. The surface area of a sphere is 5544 cm^2, find its diameter.

Sol:

Surface area of a sphere is $5544 cm^2$

$\Rightarrow 4\pi r^2 = 5544$

$\Rightarrow \frac{4 \times 22}{7} \times r^2 = 5544$

$\Rightarrow r^2 = \frac{5544 \times 7}{88}$

$\Rightarrow r = \sqrt{21cm \times 21cm} = \sqrt{(21)^2} cm$

$\Rightarrow r = 21cm.$

Diameter $= 2$ (radius)

$= 2(21cm)$

$= 42cm.$

5. A hemispherical bowl made of brass has inner diameter 10.5 cm. Find the cost of tin- plating it on the inside at the rate of Rs. 4 per 100 cm^2.

Sol:

Given

Inner diameter of hemisphere bowl $= 10 \cdot 5cm$

Radius $= \frac{10 \cdot 5}{2} cm = 5 \cdot 25cm.$

Surface area of hemispherical bowl $= 2\pi r$

$= 2\left[\frac{22}{7}\right] \times (5 \cdot 25)^2 \; cm^2$

$= 173 \cdot 25 cm^2.$

Cost of tin planning $100 cm^2$ area = Rs. 4

Cost of tin planning $173\cdot 25cm^2$ area = $Rs.\left(\frac{4\times 173\cdot 25}{100}\right)$

$= Rs.\ 6\cdot 93$

Thus, The cost of tin plating the inner side of hemisphere bowl is $Rs. 6\cdot 93$

6. The dome of a building is in the form of a hemisphere. Its radius is 63 dm. Find the cost of painting it at the rate of Rs. 2 per sq. m.

Sol:

Dome Radius $= 63d\ \ m = 6\cdot 3m$

Inner S·A of dome $= 2\pi r^2 = 2\times\frac{22}{7}\times(6\cdot 3)^2 = 249\cdot 48m^2$

Now, cost of $1m^2 = Rs.\ 2.$

$\therefore$ Cost of $249\cdot 48m^2 = Rs.[2\times 249\cdot 48]$

$= Rs.\ 498\cdot 96.$

7. Assuming the earth to be a sphere of radius 6370 km, how many square kilo metres is area of the land, if three-fourth of the earth's surface is covered by water?

Sol:

$\frac{3}{4}^{th}$ of earth surface is covered by water

$\therefore \frac{1}{4}^{th}$ earth surface is covered by land

$\therefore$ Surface area covered by land $= \frac{1}{4}\times 4\pi r^2$

$= \frac{1}{4}\times 4\times\frac{22}{7}\times 6370^2$

$= 127527.4\,km^2$

8. A cylinder of same height and radius is placed on the top of a hemisphere. Find the curved surface area of the shape if the length of the shape be 7 cm.

Sol:

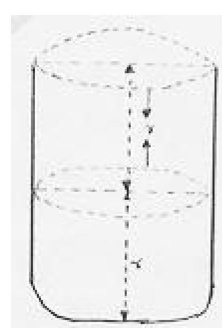
Given length of the shape $= 7cm$

But length $= r + r$

$\Rightarrow 2r = 7cm$

$\Rightarrow r = \frac{7}{2}cm$

$\Rightarrow r = 3\cdot 5cm$



Also; $h = r$

Total S.A of shape $= 2\pi rh + 2\pi r^2 = 2\pi r \times r + 2\pi r^2$

$= 2\pi r^2 + 2\pi r^2$

$= 4\pi r^2$

$= 4 \times \frac{22}{7} \times (3\cdot5)^2$

$= 154 cm^2$

9. A wooden toy is in the form of a cone surmounted on a hemisphere. The diameter of the base of the cone is 16 cm and its height is 15 cm. Find the cost of painting the toy at Rs. 7 per 100 cm^2.

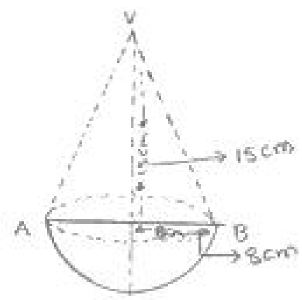
Sol:

Diameter of cone = 16cm.

$\therefore$ Radius of cone = 8cm.

Height of cone = 15cm

Slant height of cone $= \sqrt{8^2 + 15^2}$



$= \sqrt{64 + 225}$

$= \sqrt{289}$

$= 17 cm$

$\therefore$ Total curved surface area of toy

$= \pi rl + 2\pi r^2$

$= \frac{22}{7} \times 8 \times 17 + 2 \times \frac{22}{7} \times 8^2$

$= \frac{5808}{7} cm^2$

Now, cost of $100 cm^2 = Rs. 7$

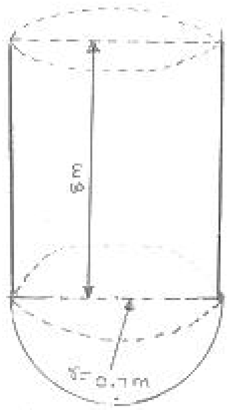
$1 cm^2 = Rs. \frac{7}{100}$

Hence, cost of $\frac{5808}{7} cm^2 = Rs. \left(\frac{5808}{7} \times \frac{7}{100}\right)$

$= Rs. 58\cdot08$

10. A storage tank consists of a circular cylinder with a hemisphere adjoined on either end. If the external diameter of the cylinder be 1.4 m and its length be 8 m, find the cost of painting it on the outside at the rate of Rs. 10 per m^2.

Sol:



Diameter of cylinder $= 1 \cdot 4m$

$\therefore$ Radius of cylinder $= \frac{1 \cdot 4}{2} = 0 \cdot 7m$

Height of cylinder = 8m.

$\therefore S \cdot A$ of tank $= 2\pi rh + 2\pi r^2$

$= 2 \times \frac{22}{7} \times 0 \cdot 7 \times 8 + 2 \times \frac{22}{7} \times (0 \cdot 7)^2$

$= \frac{176}{5} + \frac{77}{25}$

$= \frac{957}{25} = 38 \cdot 28cm^2$

Now, cost of $1m^2 = Rs. 10.$

$\therefore$ Cost of $38 \cdot 28m^2 = Rs.[10 \times 38 \cdot 28]$

$= Rs.382 \cdot 80$

11. The diameter of the moon is approximately one fourth of the diameter of the earth. Find the ratio of their surface areas.

Sol:

Let the diameter of the earth is d then, diameter of moon will be $\frac{d}{4}$

Radius of earth $= \frac{d}{2}$

Radius of moon $= \frac{\frac{d}{2}}{4} = \frac{d}{8}$

$S \cdot A$ of moon $= 4\pi\left(\frac{d}{8}\right)^2$

Surface area of earth $= 4\pi\left(\frac{d}{2}\right)^2$

Required ratio $= \dfrac{4\pi\left(\frac{d}{8}\right)^2}{4\pi\left(\frac{d}{2}\right)^2} = \dfrac{4}{64} = \dfrac{1}{16}$

Thus, the required ratio of the surface areas is $\frac{1}{16}$.

12. A hemi-spherical dome of a building needs to be painted. If the circumference of the base of the dome is 17.6 cm, find the cost of painting it, given the cost of painting is Rs. 5 per l00 cm^2

Sol:

Given that only the rounded surface of the dome to be painted, we would need to find the curved surface area of the hemisphere to know the extent of painting that needs to be done.

Now, circumference of the dome $= 17 \cdot 6m.$

Therefore, $17 \cdot 6 = 2\pi r.$

$2 \times \frac{22}{7} r = 17 \cdot 6m.$

So, the radius of the dome $= 17 \cdot 6 \times \frac{7}{2 \times 22} m = 2 \cdot 8m$

The curved surface area of the dome $= 2\pi r^2$

$= 2 \times \frac{22}{7} \times 2 \cdot 8 \times 2 \cdot 8cm^2$

$= 49 \cdot 28m^2$

Now, cost of painting $100cm^2$ is Rs. 5.

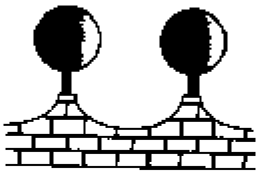
So, cost of painting $1m^2 = Rs.\ 500$

Therefore, cost of painting the whole dome

$= Rs.\ 500 \times 49 \cdot 28$

$= Rs. 24640$

13. The front compound wall of a house is decorated by wooden spheres of diameter 21 cm, placed in small supports as shown in Fig. below. Eight such spheres are used for this purpose, and are to be painted silver. Each support is a cylinder of radius 1 .5 cm and height 7 cm and is to be painted black. Find the cost of paint required if silver paint costs 25 paise per cm^2 and black paint costs 5 paise per cm^2.



Sol:

Wooden sphere radius $=\left(\frac{21}{2}\right)cm = 10\cdot 5cm.$

Surface area of a wooden sphere

$= 4\pi r^2 = 4\left[\frac{22}{7}\right][10\cdot 5]^2\ cm^2 = 1386cm^2$

Radius $\left(r^1\right)$ of cylindrical support $= 1\cdot 5cm$

Height $\left(h^1\right)$ of cylindrical support $= 7cm$

CSA of cylindrical support $= 2\pi r^1 h\left[2\times\frac{22}{7}\times 1\cdot 5\times 7\right]$

$= 66cm^2$

Area of circular end of cylindrical support $= \pi r^2 = \left[\frac{22}{7}(1\cdot 5)^2\right] = 7\cdot 07cm^2$

Area to be painted silver $= \left[8\times\left(1386 - 7\cdot 07\right)\right]cm^2$

$= 8\left(1378\cdot 93\right)cm^2$

$= 11031\cdot 44cm^2$

Cost occurred in painting silver color

$= Rs.\ \left(11031\cdot 44\times 0\cdot 25\right) = Rs.\ 2757\cdot 86$

Area to painted black $= \left(8\times 66\right)cm^2 = 528\ cm^2$

Cost occurred in painting black color $= Rs.\left(528\times 0\cdot 05\right) = Rs.\ 26\cdot 40$

$\therefore$ Total cost occurred in painting $= Rs.\left(2757\cdot 86 + 26\cdot 40\right) = Rs. 2784\cdot 26$

Exercise – 21.2

1. Find the volume of a sphere whose radius is:
(i) 2 cm (ii) 3.5 cm (iii) 10.5 cm
Sol:
(i) Radius $(r) = 2cm$

$\therefore$ Volume $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times (2)^3 = 33 \cdot 52 cm^3$

(ii) Radius $(r) = 3 \cdot 5cm$

$\therefore$ Volume $= (3 \cdot 5)^3 \times \pi \times \frac{4}{3} = \frac{4}{3} \times \frac{22}{7} \times (3 \cdot 5)^3 = 179 \cdot 666 cm^3$

(iii) Radius $(r) = 10 \cdot 5cm$

Volume $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (10 \cdot 5)^3 = 4851 cm^3$

2. Find the volume of a sphere whose diameter is:
(i) 14 cm (ii) 3.5 dm (iii) 2.1 m
Sol:
(i) Diameter $= 14cm$, radius $= \frac{14}{2} = 7cm$

$\Rightarrow$ Volume $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (7)^3 = 1437 \cdot 33 cm^3$

(ii) Diameter $= 3 \cdot 5d\,m$, radius $= \frac{3 \cdot 5}{2} dm = 1 \cdot 75 dm$

$\therefore$ Volume $= \frac{4}{3} \times \frac{22}{7} \times \left(\frac{3 \cdot 5}{2}\right)^3 = 22 \cdot 46 dm^3$

(iii) Diameter $= 2 \cdot 1m \Rightarrow r = \frac{2 \cdot 1}{2} m$

$\therefore$ Volume $= \frac{4}{3} \times \left(\frac{22}{7}\right) \times \left(\frac{2 \cdot 1}{2}\right)^3$

$= 4 \cdot 851 m^3.$

3. A hemispherical tank has inner radius of 2.8 m. Find its capacity in litres.

Sol:

Radius of tank $= 2\cdot 8m$

$\therefore$ Capacity $= \frac{2}{3}\times\frac{22}{7}\times(2\cdot 8)^3$

$= 45\cdot 994m^3$

$\therefore$ Capacity in liters $= 45994$ liters $\left[1m^3 = 1000\right]$

4. A hemispherical bowl is made of steel 0.25 cm thick. The inside radius of the bowl is 5 cm. Find the volume of steel used in making the bowl.

Sol:

Inner radius $= 5cm$

Outer radius $= 5 + 0\cdot 25$

$= 5\cdot 25$

Volume of steel used = outer volume – inner volume

$= \frac{2}{3}\times\pi\times\left(R^3 - r^3\right)$

$= \frac{2}{3}\times\frac{22}{7}\left[5\cdot 25^3 - 5^3\right]$

$= 41\cdot 282cm^3$

5. How mañy bullets can be made out of a cube of lead, whose edge measures 22 cm, each bullet being 2 cm in diameter?

Sol:

Cube edge $= 22cm$

$\therefore$ Volume of cube $= (22)^3$

$= 10648cm^3$

And,

Volume of each bullet $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\times\frac{22}{7}\times\left(\frac{2}{2}\right)^3$

$= \frac{4}{3}\times\frac{22}{7}$

$= \frac{88}{21}cm^3$

$\therefore$ No. of bullets $= \dfrac{\text{Volume of cube}}{\text{Volume of bullet}}$

$= \dfrac{10648}{\frac{88}{21}} = 2541$

6. A shopkeeper has one laddoo of radius 5 cm. With the same material, how many laddoos of radius 2.5 cm can be made.

Sol:

Volume of laddoo having radius $= 5cm$

i.e volume $(V_1) = \dfrac{4}{3}\pi r^3$

$V_1 = \dfrac{4}{3} \times \dfrac{22}{7} \times (5)^3$

$V_1 = \dfrac{11000}{21} cm^3$

Also volume of laddoo having radius $= 2 \cdot 5cm$

i.e., $V_2 = \dfrac{4}{3}\pi r^3$

$V_2 = \dfrac{4}{3} \times \dfrac{22}{7} \times (2 \cdot 5)^3$

$V_2 = \dfrac{1375}{21} cm^3$

$\therefore$ No. of laddoos $= \dfrac{V_1}{V_2} = \dfrac{11000}{1375} = 8.$

7. A spherical ball of lead 3 cm in diameter is melted and recast into three spherical balls. If the diameters of two balls be $\frac{3}{2}$ cm and 2 cm, find the diameter of the third ball.

Sol:

Volume of lead ball $= \dfrac{4}{3}\pi r^3$

$= \dfrac{4}{3} \times \dfrac{22}{7} \times \left(\dfrac{3}{2}\right)^3$

$\therefore$ According to question,

Volume of lead ball $= \dfrac{4}{3} \times \pi \left(\dfrac{3}{4}\right)^3 + \dfrac{4}{3}\pi \left(\dfrac{2}{2}\right)^3 + \dfrac{4}{3}\pi \left(\dfrac{d}{2}\right)^3$

$$\Rightarrow \frac{4}{3}\pi\left(\frac{3}{2}\right)^3 = \frac{4}{3}\pi\left(\frac{3}{4}\right)^3 + \frac{4}{3}\left[\pi\left(\frac{2}{2}\right)^3 + \left(\frac{d}{2}\right)^3\right]$$

$$\Rightarrow \frac{4}{3}\pi\left[\left(\frac{3}{2}\right)^3\right] = \frac{4}{3}\pi\left[\left(\frac{3}{4}\right)^3 + \left(\frac{2}{2}\right)^3 + \left(\frac{d}{2}\right)^3\right]$$

$$\Rightarrow \frac{27}{8} = \frac{27}{64} + \frac{8}{8} + \frac{d^3}{8}$$

$$\Rightarrow \left[\frac{27}{8} - \frac{27}{64} - 1\right]8 = d^3$$

$$\Rightarrow \frac{d^3}{8} = \frac{125}{64}$$

$$\Rightarrow \frac{d}{2} = \frac{5}{4}$$

$$\Rightarrow d = \frac{10}{4}$$

$$\Rightarrow d = 2\cdot 5cm$$

8. A sphere of radius 5 cm is immersed in water filled in a cylinder, the level of water rises $\frac{5}{3}$ cm. Find the radius of the cylinder.

Sol:

Volume of sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3}\pi(5)^3$$

$\therefore$ Volume of water rise in cylinder = Volume of sphere

Let r be the radius of the cylinder

$$\pi r^2 h = \frac{4}{3}\pi r^3$$

$$\Rightarrow r^2 \times \frac{5}{3} = \frac{4}{3}(5)^3$$

$$\Rightarrow r^2 = 20\times 5$$

$$\Rightarrow r^2 = 100$$

$$\Rightarrow r^1 = 10cm$$

9. If the radius of a sphere is doubled, what is the ratio of the volume of the first sphere to that of the second sphere?

Sol:

Let V_1 *and* V_2 be the volumes of first sphere and second sphere respectively

Radius of 1st sphere = r

2nd sphere radius $= 2r$

$$\therefore \frac{Volume\ 1^{st}}{Volume\ 2^{nd}} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi (2r)^3} = \frac{1}{8}.$$

10. A cone and a hemisphere have equal bases and equal volumes. Find the ratio of their heights.

Sol:

Given that

Volume of thee cone = Volume of the hemisphere

$$\Rightarrow \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^3$$

$$\Rightarrow r^2 h = 2r^3$$

$$\Rightarrow h = 2r$$

$$\Rightarrow \frac{h}{r} = \frac{1}{1} \times 2 = \frac{2}{1}$$

$\therefore$ Ratio of the their height is $2:1$

11. A vessel in the form of a hemispherical bowl is full of water. Its contents are emptied in a right circular cylinder. The internal radii of the bowl and the cylinder are 3.5 cm and 7 cm respectively. Find the height to which the water will rise in the cylinder.

Sol:

Given that

Volume of water in the hemisphere bowl = Volume of water in the cylinder

Let n be the height to which water rises in the cylinder.

Inner radii of bowl $= 3\cdot 5cm = r_1$

Inner radii of bowl $= 7cm = r_2$

$$\Rightarrow \frac{2}{3}\pi r_1^3 = \pi r_2^2 h$$

$$\Rightarrow h = \frac{2r_1^3}{3r_2^2} = \frac{2(3\cdot 5)^3}{3(7)^2}$$

$$\Rightarrow h = \frac{7}{12} cm.$$

12. A cylinder whose height is two thirds of its diameter, has the same volume as a sphere of radius 4 cm. Calculate the radius of the base of the cylinder.

Sol:

Given that,

Height of cylinder $= \frac{2}{3}$(diameter)

We know that,

Diameter $= 2$(radius)

$h = \frac{2}{3} \times 2r = \frac{4}{3} r$

Volume of the cylinder = volume of the sphere

$\Rightarrow \pi r^2 \times \frac{4}{3} r = \frac{4}{3} \pi (4)^3$

$\Rightarrow r^3 = 4^3$

$\Rightarrow r = 4cm$

13. A vessel in the form of a hemispherical bowl is full of water. The contents are emptied into a cylinder. The internal radii of the bowl and cylinder are respectively 6 cm and 4 cm. Find the height of water in the cylinder.

Sol:

It is given that,

Volume of water is hemisphere bowl = volume of cylinder

$\Rightarrow \frac{2}{3} \pi (6)^3 = \pi (4)^2 h$

$\Rightarrow h = \frac{2}{3} \times \frac{6 \times 6 \times 6}{4 \times 4}$

$\Rightarrow h = 9cm$

$\therefore$ Height of cylinder $= 9cm.$

14. A cylindrical tub of radius 16 cm contains water to a depth of 30 cm. A spherical iron ball is dropped into the tub and thus level of water is raised by 9 cm. What is the radius of the ball?

Sol:

Let r be the radius of the iron ball

Then, Volume of iron ball = Volume of water raised in the hub

$\Rightarrow \frac{4}{3} \pi r^3 = \pi r^2 h$

$\Rightarrow \frac{4}{3} r^3 = (16)^2 \times 9$

$\Rightarrow r^3 = \frac{27 \times 16 \times 16}{4}$

$\Rightarrow r^3 = 1728$

$\Rightarrow r = 12\, cm$

Therefore, radius of the ball = 12cm.

15. A cylinder of radius 12 cm contains water to a depth of 20 cm. A spherical iron ball is dropped into the cylinder and thus the level of water is raised by 6.75 cm. Find the radius of the ball. (Use π = 22/7).

Sol:

Given that,

Radius of cylinder $=12cm=r_1$

Raised in height $=6\cdot 75cm=h$

$\Rightarrow$ Volume of water raised = Volume of the sphere

$$\Rightarrow \pi r_1^2 h=\frac{4}{3}\pi r_2^3$$

$$\Rightarrow 12\times 12\times 6\cdot 75=\frac{4}{3}r_2^3$$

$$\Rightarrow \frac{12\times 12\times 6\cdot 75\times 3}{4}=r_2^3$$

$$\Rightarrow r_2^3=729$$

$$\Rightarrow \boxed{r_2=9cm}$$

Radius of sphere is 9cm.

16. The diameter of a coper sphere is 18 cm. The sphere is melted and is drawn into a long wire of uniform circular cross-section. If the length of the wire is 108 m, find its diameter.

Sol:

Given that diameter of a coper sphere = 18cm.

Radius of the sphere = 9cm

Length of the wire = 108m

= 10,800cm

Volume of cylinder = volume of sphere

$$\Rightarrow \pi r_1^2 h=\frac{4}{3}\pi r_2^3$$

$$\Rightarrow r_1^2\times 10800=\frac{4}{3}\times 9\times 9\times 9\Rightarrow r_1^2=0\cdot 09$$

$\therefore$ Diameter = $2\times 0\cdot 3=0\cdot 6cm$

17. A cylindrical jar of radius 6 cm contains oil. Iron spheres each of radius 1 .5 cm are immersed in the oil. How many spheres are necessary to raise the level of the oil by two centimetres?

Sol:

Given that,

Radius of cylinder jar $=6cm=r_1$

Level to be rised = 2cm=h

Radius of each iron sphere $=1\cdot 5cm = r_2$

Number of sphere $=\dfrac{\text{Volume of cylinder}}{\text{Volume of sphere}}$

$$=\frac{\pi r_1^2 h}{4\pi r_2^3}$$

$$=\frac{r_1^2 h}{r_2^3 \times \frac{4}{3}} = \frac{6\times 6\times 2}{\frac{4}{3}\times 1\cdot 5\times 1\cdot 5\times 1\cdot 5}$$

Number of sphere = 16.

18. A measuring jar of internal diameter 10 cm is partially filled with water. Four equal spherical balls of diameter 2 cm each are dropped in it and they sink down in water completely. What will be the change in the level of water in the jar?

Sol:

Given that,

Diameter of jar = 10cm

Radius of jar = 5cm

Let the level of water raised by 'h'

Diameter of spherical ball = 2cm

Radius of the ball =1cm

Volume of jar = 4(Volume of spherical)

$$\Rightarrow \pi r_1^2 h = 4\left(\frac{4}{3}\pi r_2^3\right)$$

$$\Rightarrow r_1^2 h = 4\times \frac{4}{3} r_2^3$$

$$\Rightarrow r_1^2 h = 4\times \frac{4}{3}\times 1\times 1\times 1$$

$$\Rightarrow h = \frac{4\times 4\times 1}{3\times 5\times 5}$$

$$\Rightarrow h = \frac{16}{75} cm.$$

$$\therefore \text{Height of water in jar} = \frac{16}{75} cm.$$

19. The diameter of a sphere is 6 cm. It is melted and drawn into a wire of diameter 0.2 cm. Find the length of the wire.

Sol:

Given that,

Diameter of sphere = 6cm

Radius of sphere $= \frac{d}{2} = \frac{6}{2} cm = 3cm = r_1$

Diameter of the wire = $0 \cdot 2cm$

Radius of the wire $= 0 \cdot 1cm = r_2$

Volume of sphere = Volume of wire

$\Rightarrow \frac{4}{3}\pi r_1^3 = \pi r_2^2 h$

$\Rightarrow \frac{4}{3} \times 3 \times 3 \times 3 = 0 \cdot 1 \times 0 \cdot 1 \times h$

$\Rightarrow \frac{4 \times 3 \times 3}{0 \cdot 1 \times 0 \cdot 1} = h$

$\Rightarrow h = 3600$

$\Rightarrow h = 36m.$

$\therefore$ Length of wire = 36m.

20. The radius of the internal and external surfaces of a hollow spherical shell are 3 cm and 5 cm respectively. If it is melted and recast into a solid cylinder of height $2\frac{2}{3}$ cm. Find the diameter of the cylinder.

Sol:

Given that,

Internal radius of the sphere $= 3cm = r_1$

External radius of the sphere $= 5cm = r_2$

Height of cylinder $= 2\frac{2}{3} cm = \frac{8}{3} cm = h$

Volume of spherical shell = Volume of the cylinder

$\Rightarrow \frac{4}{3}\pi\left(r_2^3 - r_1^3\right) = \pi r_3^2 h$

$\Rightarrow \frac{4}{3}\left(5^3 - 3^3\right) = \frac{8}{3} r_3^2$

$\Rightarrow \frac{4 \times 98 \times 3}{3 \times 8} = r_3^2$

$\Rightarrow r_3^2 = \sqrt{49}$

$\Rightarrow r_3 = 7cm$

$\therefore$ Diameter of the cylinder = 2 (radius) = 14cm

21. A hemisphere of lead of radius 7 cm is cast into a right circular cone of height 49 cm. Find the radius of the base.

Sol:

Given radius of hemisphere = 7cm = r_1

Height of cone $h = 49cm$

Volume of hemisphere = Volume of cone

$$\Rightarrow \frac{2}{3}\pi r_1^3 = \frac{1}{3}\pi r_2^2 h$$

$$\Rightarrow \frac{2}{3}\times 7^3 = \frac{1}{3} r_2^2 \times 49$$

$$\Rightarrow \frac{2\times 7\times 7\times 7\times 3}{3\times 49} = r_2^2$$

$$\Rightarrow r_2^2 = 3\cdot 74cm$$

$\therefore$ Radius of the base $= 3\cdot 74cm.$

22. A hollow sphere of internal and external radii 2 cm and 4 cm respectively is melted into a cone of base radius 4 cm. Find the height and slant height of the cone.

Sol:

Given that

Hollow sphere external radii $= 4cm = r_2$

Internal radii $(r_1) = 2cm$

Cone base radius $(R) = 4cm$

Height = ?

Volume of cone = Volume of sphere

$$\Rightarrow \frac{1}{3}\pi r^2 H = \frac{4}{3}\pi\left(R_2^3 - R_1^3\right)$$

$$\Rightarrow 4^2 H = 4\left(4^3 - 2^3\right)$$

$$\Rightarrow H = H = \frac{4\times 56}{16} = 14cm$$

Slant height $= \sqrt{R^2 + H^2} = \sqrt{4^2 + 14^2}$

$$\Rightarrow l = \sqrt{16+196} = \sqrt{212}$$

$$= 14\cdot 56cm.$$

23. A metallic sphere of radius 10.5 cm is melted and thus recast into small cones, each of radius 3.5 cm and height 3 cm. Find how many cones are obtained.

Sol:

Given that

Metallic sphere of radius $=10\cdot 5cm$

Cone radius $=3\cdot 5cm$

Height of radius $=3cm$

Let the number of cones obtained be x

$V_s = x \times V$ cone

$\Rightarrow \frac{4}{3}\pi r^3 = x \times \frac{1}{3}\pi r^2 h$

$\Rightarrow \frac{4\times 10\cdot 5\times 10\cdot 5\times 10\cdot 5}{3\cdot 5\times 3\cdot 5\times 3} = x$

$\Rightarrow x = 126$

$\therefore$ Number of cones $=126$

24. A cone and a hemisphere have equal bases and equal volumes. Find the ratio of their heights.

Sol:

Given that

A cone and a hemisphere have equal bases and volumes

$V_{cone} = V_{hemisphere}$

$\Rightarrow \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^3$

$\Rightarrow r^2 h = 2r^3$

$\Rightarrow h = 2r$

$\Rightarrow h : r - 2r : r - 2 : 1$

25. A cone, a hemisphere and a cylinder stand on equal bases and have the same height. Show that their volumes are in the ratio 1 : 2 : 3.

Sol:

Given that,

A cone, hemisphere and a cylinder stand one equal bases and have the same weight

We know that

$V_{cone} : V_{hemisphere} : V_{cylinder}$

$\Rightarrow \frac{1}{3}\pi r^2 h : \frac{2}{3}\pi r^3 : \pi r^2 h$

Multiplying by 3

$\Rightarrow \pi r^2 h : 2\pi r^3 : 3\pi r^2 h$ or

$\pi r^3 : 2\pi r^3 : 3\pi r^3 \left[\therefore r = h \because r^2 h = r^3\right]$

Or $1:2:3$

26. A cylindrical tub of radius 12 cm contains water to a depth of 20 cm. A spherical form ball is dropped into the tub and thus the level of water is raised by 6.75 cm. What is the radius of the ball?

Sol:

A cylindrical tub of radius $=12cm$

Depth $=20cm.$

Let $r\,cm$ be the radius of the ball

Then, volume of ball = volume of water raised

$$=\frac{4}{3}\pi r^3=\pi r^2 h$$

$$=\frac{4}{3}\pi r^3=\pi\times(12)^2\times 6\cdot 75$$

$$\Rightarrow r^3=\frac{144\times 6\cdot 75\times 3}{4}$$

$$\Rightarrow r^3=729$$

$$\Rightarrow r=9cm$$

Thus, radius of the ball $=9cm.$

27. The largest sphere is carved out of a cube of side 10.5 cm. Find the volume of the sphere.

Sol:

Given that,

The largest sphere is carved out of a cube of side $=10\cdot 5cm$

Volume of the sphere = ?

We have,

Diameter of the largest sphere $=10\cdot 5cm$

$2r=10\cdot 5$

$$\Rightarrow r=5\cdot 25cm$$

Volume of sphere $=\frac{4}{3}\times\frac{22}{7}\times(5\cdot 25)^3=\frac{4}{3}\times\frac{22}{7}\times 5\cdot 25\times 5\cdot 25\times 5\cdot 25$

$$\Rightarrow \text{Volume }=\frac{11\times 441}{8}cm^3=606\cdot 37cm^3.$$

28. A sphere, a cylinder and a cone have the same diameter. The height of the cylinder and also the cone are equal to the diameter of the sphere. Find the ratio of their volumes.

Sol:

Let r be the common radius thus,

$h=$ height of the cone = height of the cylinder = 2r

Let

$V_1 = \text{Volume of sphere} = \frac{4}{3}\pi r^3$

$V_2 = \text{Volume of cylinder } = \pi r^2 \times 2r = 2\pi r^3$

$V_3 = \text{Volume of the cone} = \frac{1}{3}\pi r^2 \times 2r = \frac{2}{3}\pi r^3$

Now,

$V_1 : V_2 : V_3 = \frac{4}{3}\pi r^3 : 2\pi r^3 : \frac{2}{3}\pi r^3$

$= 4:6:2$

$= 2:3:1$

29. A cube of side 4 cm contains a sphere touching its side. Find the volume of the gap in between.

Sol:

It is given that

Cube side = 4cm

Volume of cube $= (4cm)^3 = 64cm^3$

Diameter of the sphere = Length of the side of the cube = 4cm

$\therefore$ Radius of sphere = 2cm

Volume of the sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (2)^3 = 33\cdot 52cm^3$

$\therefore$ Volume of gap $=$ Volume of gap – Volume of sphere

$= 64cm^2 - 33\cdot 52cm^3 = 30\cdot 48cm^3.$

30. A hemispherical tank is made up of an iron sheet 1 cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank.

Sol:

Given that,

Inner radius (r_1) of hemispherical tank $= 1m = r_1$

Thickness of hemispherical tank $= 1cm = 0\cdot 01m$

Outer radius (r_2) of the hemispherical $= (1 + 0\cdot 01m) = 1\cdot 01m = r_2$

Volume of iron used to make the tank $= \frac{2}{3}\pi\left(r_2^3 - r_1^3\right)$

$= \frac{2}{3} \times \frac{22}{7}\left[(1\cdot 01)^3 - 1^3\right]$

$= \frac{44}{21}[1\cdot 030301 - 1]m^3$

$= 0\cdot 06348m^3$ (Approximately)

31. A capsule of medicine is in the shape of a sphere of diameter 3.5 mm. How much medicine (in mm^3) is needed to fill this capsule?

Sol:

Given that,

Diameter of capsule $= 3\cdot 5mm$

Radius $= \frac{3\cdot 5}{2} = 1\cdot 75mm$

Volume of spherical capsule $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\times\frac{22}{7}\times(1\cdot 75)^3\ mm^3$

$= 22\cdot 458mm^3$

$\therefore 22\cdot 46mm^3$ of medicine is required.

32. The diameter of the moon is approximately one fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?

Sol:

Given that,

The diameter of the moon is approximately one fourth of the diameter of the earth.

Let diameter of earth bed. So radius $= \frac{d}{2}$

Then, diameter of moon $= \frac{d}{4}$, radius $= \frac{\frac{d}{2}}{4} = \frac{d}{8}$

Volume of moon $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{d}{8}\right)^3 = \frac{4}{3}\times\frac{1}{512}\pi d^3$

Volume of earth $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 = \frac{1}{8}\times\frac{4}{3}\pi d^3$

$$\frac{\text{Volume of moon}}{\text{Volume of earth}} = \frac{\frac{1}{512}\times\frac{4}{3}\pi d^3}{\frac{1}{8}\times\frac{4}{3}\pi d^3} = \frac{1}{64}.$$

Thus, the volume of moon is $\frac{1}{64}$ of volume of earth.

Exercise – 22.1

1. What do you understand by the word "statistics" in
(i) Singular form
(ii) Plural form?
Sol:
The word "Statistics" is used in both its singular as well as its plural senses.
(i) **In singular sense:** Statistics may be defined as the science of collection, presentation, analysis and interpretation of numerical data
(ii) **In plural sense:** Statistics means numerical facts or observations collected with definite purpose
(iii) **For example:** Income and expenditure of persons in a particular locality, number males and females in a particular town are statistics.

2. Describe some fundamental characteristics of statistics.
Sol:
Fundamental characteristics of statistics:
1. A single observation does not form statistics are a sum total of observation
2. Statics are expressed quantitatively not qualitatively
3. Statistics are collected with definite purpose
4. Statistics in an experiment are comparable and can be classified various groups.

3. What are (i) primary data? (ii) secondary data? Which of the two – the primary or the secondary data – is more reliable and why?
Sol:
The word data means information statistical data and of two types
(i) Primary data, (ii) Secondary data
(i) **Primary data:** when an investigator collects data himself with a definite plan or design in his (her) mind is called primary data
(ii) **Secondary data:** data which are not originally collected rather obtained from published or unpublished sources are known as secondary data.
Secondary data are collected by an individual or an institution for some purpose and are used by someone else in another context
Primary data are reliable and relevant because they are original in character and are collected by some individuals or by research bodies.

4. Why do we group data?
Sol:
The data obtained in original form are called raw data. Raw data does not give any useful information and is rather confusing to mind. Data is grouped so that it becomes

understandable can be interpreted. According to various characteristics groups are formed by us. After grouping the data, we are in position to make calculations of certain values which will help us in describing and analyzing the data.

5. Explain the meaning of the following terms:

(i) Variable
(ii) Class-integral
(iii) Class-size
(iv) Class-mark
(v) Frequency
(vi) Class limits
(vii) True class limits

Sol:

(i) Variable: Any character that can vary from one individual to another is called variable or variable
(ii) Class interval: In the data each group into which raw data is considered is called a class-interval.
(iii) Class-size: The different between the true upper limit and lower limit is called the class size of that class.
(iv) Class mark: The middle value of the class is called as the class mark.
$$\text{Class mark} = \frac{\text{Upper Limit} + \text{Lower Limit}}{2}$$
(v) Frequency: The number of observations corresponding to class is called its frequency.
(vi) Class limits: Each class is bounded by two figures, called the class limits. The figures on the left side of the classes are called lower limits while figures on the right side are called upper limits.
(vii) True class limits: if classes are inclusive eg $15-19, 20-24, 25-29, 30-34$...........
Then, true lower limit of class = upper limit of class $-0\cdot 5$
And true upper limit of class = upper limit of class $+0\cdot 5$
Example: true limits of the class 15-19 are $14\cdot 5$ and $19\cdot 5$
But if because classes are exclusive use like $10-20, 20-30, 30-40$..........
Here class limits and true class limits are the same.

6. The ages of ten students of a group are given below. The ages have been recorded in years and months:

8 – 6, 9 – 0, 8 – 4, 9 – 3, 7 – 8, 8 – 11,8 – 7, 9 – 2, 7 – 10, 8 – 8

(i) What is the lowest age?
(ii) What is the highest age?
(iii) Determine the range?

Sol:

The ages of ten students of a group are given below

$8-6, 9-0, 8-4, 9-3, 7-8, 8-11, 8-7, 9-2, 7-10, 8-8.$

(i) Lowest age is 7 years, 8 months

(ii) Highest age is 9 years, 3 months.

(iii) Range = highest age – lowest age

= 9 year, 3 months, 7 year, 8 months

= 1 year, 7 months

7. The monthly pocket money of six friends is given below: Rs. 45, Rs. 30, Rs. 40, Rs. 50, Rs. 25, Rs. 45.

(i) What is the highest pocket money?

(ii) What is the lowest pocket money?

(iii) What is the range?

(iv) Arrange the amounts of pocket money in ascending order.

Sol:

The monthly pocket money of six friends is given below:

$Rs.\ 45,\ Rs.\ 30,\ Rs.40,$ Rs. 50, $Rs.\ 25,\ Rs.\ 45.$

(i) Highest pocket money $= Rs.\ 50$

(ii) Lowest pocket money $= Rs.25$

(iii) Range $= 50-25$

= 25.

(iv) The cumulative amounts of pocket money in ascending order is

$Rs.\ 25, Rs.\ 30, Rs.\ 40, Rs.\ 45, Rs.\ 45, Rs.\ 50.$

8. Write the class-size in each of the following:

(i) 0 – 4, 5 – 9, 10 – 14

(ii) 10 – 19, 20 – 29, 30 – 39

(iii) 100 – 120, 120 – 140, 160 – 180

(iv) 0 – 0.25, 0.25 – 0.50, 0.50 – 0.75

(v) 5 – 5.01, 5.01 – 5.02, 5.02 – 5.03

Sol:

(i) $0-4,\ 5-9, 10-14.$

True class limits are $0\cdot5-4\cdot5, 4\cdot5-9\cdot5, 9\cdot5-14\cdot5.$

$\therefore$ Class size $= 14\cdot5-9\cdot5 = 5$

(ii) $10-19,\ 20-29,\ 30-39.$

True class limits $\rightarrow 10\cdot5-19\cdot5, 19\cdot5-29\cdot5, 29\cdot5-39\cdot5$

Class size $= 39\cdot5-29\cdot5$

$= 10$

(iii) $100-120, 120-140, 160-180.$

Here the class limits and true class limits are the same

$\therefore$ Class size $=120-100$

$=20$

(iv) $0-0\cdot25, 0\cdot25-0\cdot50, 0\cdot50-0\cdot75$

Here the class limits and true class limits are the same

$\therefore$ Class size $=0\cdot25-0=0\cdot25$

(v) $5-5\cdot01, 5\cdot01-5\cdot02, 5\cdot02-5\cdot03.$

Here the class limits ad true class limits are the same

$\therefore$ Class size $=5\cdot01-5\cdot0=0\cdot01.$

9. The final marks in mathematics of 30 students are as follows:

53, 61, 48, 60, 78, 68, 55, 100,67,90

75,88,77,37,84,58,60,48,62,56

44, 58, 52, 64, 98, 59, 70, 39, 50, 60

(i) Arrange these marks in the ascending order, 30 to 39 one group, 40 to 49 second group etc.

Now answer the following:

(ii) What is the highest score?

(iii) What is the lowest score?

(iv) What is the range?

(v) If 40 is the pass mark how many have failed?

(vi) How many have scored 75 or more?

(vii) Which observations between 50 and 60 have not actually appeared?

(viii) How many have scored less than 50?

Sol:

The final marks in mathematics of 30 students are as follows:

53, 61, 48, 60, 78, 68, 55, 100, 67, 90, 75, 88, 77, 37, 84, 58, 60, 48, 62, 56, 44, 58, 52, 64, 98, 59, 70, 39, 50, 60.

(i)	Group	Observations
	I(30–39)	37, 39
	II(40–49)	44, 48, 48
	III(50–59)	50, 52, 53, 55, 56, 58, 58, 59
	IV(60–69)	60, 60, 60, 61, 62, 64, 67, 68
	V(70–79)	70, 55, 77, 78
	VI(80–89)	84, 88
	VII(90–99)	90, 98
	VIII(100–109)	100

(ii) Highest score = 100

(iii) Lowest score = 37

(iv) Range = 100 – 37 = 63
(v) If 40 is the pass mark, 2 students have failed
(vi) 8 students have scored 75 or more
(vii) Observation 51, 54, 57 between 50 and 60 have not actually appeared
(viii) 5 students have scored less than 50

10. The weights of new born babies (in kg) in a hospital on a particular day are as follows:
2.3, 2.2, 2.1, 2.7, 2.6, 3.0, 2.5, 2.9, 2.8, 3.1, 2.5, 2.8, 2.7, 2.9, 2.4
(i) Rearrange the weights in descending order.
(ii) Determine the highest weight.
(iii) Determine the lowest weight.
(iv) Determine the range.
(v) How many babies were born on that day?
(vi) How many babies weigh below 2.5 kg?
(vii) How many babies weigh more than 2.8 kg?
(viii) How many babies weigh 2.8 kg?
Sol:
The weights of new born babies (in kg) area as follows
$2\cdot3, 2\cdot2, 2\cdot1, 2\cdot7, 2\cdot6, 2\cdot5, 3\cdot0, 2\cdot8, 2\cdot9, 3\cdot1, 2\cdot5, 2\cdot8, 2\cdot7, 2\cdot9, 2\cdot4.$
(i) The weights in descending order
$3\cdot1, 3\cdot0, 2\cdot9, 2\cdot9, 2\cdot8, 2\cdot8, 2\cdot7, 2\cdot7, 2\cdot6, 2\cdot5, 2\cdot5, 2\cdot4, 2\cdot3, 2\cdot2, 2\cdot1.$
(ii) The highest weight $= 3\cdot1kg$
(iii) The lowest weight $= 2\cdot1kg$
(iv) Range $= 3\cdot1 - 2\cdot1 = 1\cdot0kg$
(v) 15 babies were born on that particular day
(vi) 4 babies weight below $2\cdot5kg$
(vii) Weight more than $2\cdot8kg$ are 4-babies.
(viii) Weight $2\cdot8kg \to 2$ babies

11. The number of runs scored by a cricket. player in 25 innings are as follows:
26, 35, 94, 48, 82, 105, 53, 0, 39, 42, 71, 0, 64, 1.5, 34, 67, 0, 42, 124, 84, 54, 48, 139, 64, 47.
(i) Rearrange these runs in ascending order.
(ii) Determine the player, is highest score.
(iii) How many times did the player not score a run?
(iv) How many centuries did he score?
(v) How many times did he score more than 50 runs?
Sol:
The numbers of runs scored by a player in 25 innings are

26, 35, 94, 48, 82, 105, 53, 0, 39, 42, 71, 0, 64, 15, 34, 15, 34, 67, 0, 42, 124, 84, 54, 48, 139, 64, 47

(i) Runs in ascending order are 0, 0, 0, 15, 26, 34, 35, 39, 42, 42, 47, 48, 48, 53, 54, 64, 64, 67, 71, 82, 84, 94, 105, 124, 139.

(ii) The highest number = 139

(iii) The player did not score any run 3 times

(iv) He scored 3 centuries

(v) He scored more than 50 run 12 times.

12. The class size of a distribution is 25 and the first class-interval is 200-224. There are seven class-intervals.

(i) Write the class-intervals.

(ii) Write the class-marks of each interval.

Sol:

Given,

Class size = 25

First class interval $=200-224$

(i) Seven class intervals are:

$200-240, 225-249, 250-274, 275-299, 300-324, 325-349, 350-374.$

(ii) Class mark of $200-224=\frac{200+224}{2}=\frac{424}{2}=212$

Class mark of $225-249=\frac{225+249}{2}=\frac{474}{2}=237$

Class mark of $250-274=\frac{250+274}{2}=\frac{524}{2}=262$

Class mark of $300-324=\frac{300+324}{2}=\frac{624}{2}=312$

Class mark of $325-349=\frac{325+349}{2}=\frac{674}{2}=337$

Class mark of $350-374=\frac{350+374}{2}=\frac{724}{2}=362.$

13. Write the class size and class limits in each of the following:

(i) 104, 114, 124, 134, 144, 154, and 164

(ii) 47, 52, 57, 62, 67, 72, 77, 82, 87, 92, 97 and 102

(iii) 12.5, 17.5, 22.5, 27.5, 32.5, 37.5, 42.5, 47.5

Sol:

(i) 104, 114, 124, 134, 144, 154 and 102

Class size = 114 – 104 = 10

Class mark	Lower class limit	Upper class limit	Class limit
104	$104-\frac{10}{2}=99$	$104+\frac{10}{2}=109$	99-109
114	$114-\frac{10}{2}=109$	$114+\frac{10}{2}=119$	109-119
124	119	129	119-129
134	129	139	129-139
144	139	149	139-149
154	149	159	149-159
164	159	169	159-169

(ii) 47, 52, 57, 62, 67, 72, 79, 82, 87, 92, 97, 1102

Class size $=52-47=5$

Class mark	Lower class limit	Upper class limit	Class limit
47	$47\cdot\frac{5}{2}=44\cdot5$	$47+\frac{5}{2}=49\cdot5$	$44\cdot5-49\cdot5$
52	$49\cdot5$	$54\cdot5$	$49.5-54\cdot5$
57	$54\cdot5$	$59\cdot5$	$54\cdot5-59\cdot5$
62	$59\cdot5$	$64\cdot5$	$59\cdot5-64\cdot5$
67	$64\cdot5$	$69\cdot5$	$64\cdot5-69\cdot5$
72	$69\cdot5$	$74\cdot5$	$69\cdot5-74\cdot5$
77	$74\cdot5$	$79\cdot5$	$74\cdot5-79\cdot5$
82	$79\cdot5$	$84\cdot5$	$79\cdot5-84\cdot5$
87	$84\cdot5$	$89\cdot5$	$84\cdot5-89\cdot5$
92	$89\cdot5$	$94\cdot5$	$89\cdot5-94\cdot5$
97	$94\cdot5$	$99\cdot5$	$94\cdot5-99\cdot5$
102	$99\cdot5$	$104\cdot5$	$99\cdot5-104\cdot5$

(iii) Class size $=17\cdot5-12\cdot5=5$

Class mark	Lower class limit	Upper class limit	Class limit
$12\cdot5$	$12\cdot5-\frac{5}{2}=10$	$12\cdot5+\frac{5}{2}=15$	$10-15$
$17\cdot5$	$17\cdot5-2\cdot5=15$	$17\cdot5+2\cdot5=20$	$15-20$
$22\cdot5$	$22\cdot5-2\cdot5=20$	$22\cdot5+2\cdot5=25$	$20-25$
$27\cdot5$	$27\cdot5-2\cdot5=25$	$27\cdot5+2\cdot5=30$	$25-30$

$32\cdot5$	$32\cdot5-2\cdot5=30$	$32\cdot5+2\cdot5=35$	$30-35$
$37\cdot5$	$37\cdot5-2\cdot5=35$	$37\cdot5+2\cdot5=40$	$35-40$
$42\cdot5$	$42\cdot5-2\cdot5=40$	$42\cdot5+2\cdot5=40$	$40-45$
$47\cdot5$	$47\cdot5-2\cdot5=45$	$47\cdot5+2\cdot5=45$	$45-50$

14. Following data gives the number of children in 40 families:
1,2,6,5,1,5, 1,3,2,6,2,3,4,2,0,0,4,4,3,2,2,0,0,1,2,2,4,3, 2,1,0,5,1,2,4,3,4,1,6,2,2.
Represent it in the form of a frequency distribution.
Sol:

Number of children	Tally marks	Number of families
0	|||||	5
1	||||| ||	7
2	||||| ||||| ||	12
3	|||||	5
4	||||| |	6
5	|||	3
6	|||	3

15. The marks scored by 40 students of class IX in mathematics are given below:
81, 55, 68, 79, 85, 43, 29, 68, 54, 73, 47, 35, 72, 64, 95, 44, 50, 77, 64, 35, 79, 52, 45, 54, 70, 83, 62, 64, 72, 92, 84, 76, 63, 43, 54, 38, 73, 68, 52, 54.
Prepare a frequency distribution with class size of 10 marks.
Sol:

Marks	Tally marks	Frequency
20-30	|	1
30-40	|||	3
40-50	|||||	5
50-60	||||| |||	8
60-70	||||| |||	8
70-80	||||| ||||	9
80-90	||||	4
90-100	||	2
		Total = 40

16. The heights (in cm) of 30 students of class IX are given below:
155, 158, 154, 158, 160, 148, 149, 150, 153, 159, 161, 148, 157, 153, 157, 162, 159, 151, 154, 156, 152, 156, 160, 152, 147, 155, 163, 155, 157, 153.
Prepare a frequency distribution table with 160-164 as one of the class intervals.

Sol:

Height (in cm)	Tally marks	Frequency
145-149	||||	4
150-154	||||| ||||	9
155-159	||||| ||||| ||	12
160-164	|||||	5
		Total = 30

17. The monthly wages of 30 workers in a factory are given below:
83.0, 835, 890, 810, 835, 836, 869, 845, 898, 890, 820, 860, 832, 833, 855, 845, 804, 808, 812, 840, 885, 835, 836, 878, 840, 868, 890, 806, 840, 890.
Represent the data in the form of a frequency distribution with class size 10.
Sol:

Height (in cm)	Tally marks	Frequency
800-810	|||	3
810-820	||	2
820-830	|	1
830-840	|||| ||||	8
840-850	|||||	5
850-860	|	1
860-870	|||	3
870-880	|	1
880-890	|	1
890-900	|||||	5
		Total = 30

18. The daily maximum temperatures (in degree celsius) recorded in a certain city during the month of November are as follows:
25.8, 24.5, 25.6, 20.7, 21.8, 20.5, 20.6, 20.9, 22.3, 22.7, 23.1, 22.8, 22.9, 21.7, 21.3, 20.5, 20.9, 23.1, 22.4, 21.5, 22.7, 22.8, 22.0, 23.9, 24.7, 22.8, 23.8, 24.6, 23.9, 21.1
Represent them as a frequency distribution table with class size 1°C.
Sol:

Maximum temperature (in degree celcius)	Tally marks	Frequency
$20\cdot0-21\cdot0$	||||| |	6
$21\cdot0-22\cdot0$	|||||	5
$22\cdot0-23\cdot0$	||||| ||||	9
$23\cdot0-24\cdot0$	|||||	5
$24\cdot0-25\cdot0$	|||	3

$25\cdot0-26\cdot0$	||	2
		Total = 30

19. Construct a frequency table with equal class intervals from the following data on the monthly wages (in rupees) of 28 laborers working in a factory, taking one of the class intervals as 210-230 (230 not included):

220, 268, 258, 242, 210, 268, 272, 242, 311, 290, 300, 320, 319, 304, 302, 318, 306, 292, 254, 278, 210, 240, 280, 316, 306, 215, 256, 236.

Sol:

Monthly wages (in rupees)	Tally marks	Frequency
210-230	||||	4
230-250	||||	4
250-270	|||||	5
270-290	|||	3
290-310	||||| ||	7
310-330	|||||	5
		Total = 28

20. The daily minimum temperatures in degrees Ce1siu& recorded in a certain Arctic region are as follows:

−12.5, −10.8, −18.6, −8.4, −10.8, −4.2, −4.8, −6.7, −13.2, −11.8, −2.3, 1.2, 2.6, 0, 2.4, 0, 3.2, 2.7, 3.4, 0, − 2.4, − 2.4, 0, 3.2, 2.7, 3.4, 0, − 2.4, − 5.8, -8.9, 14.6, 12.3, 11.5, 7.8, 2.9.

Represent them as frequency distribution table taking − 19.9 to − 15 as the first class interval.

Sol:

Since first class interval is frequency distribution with lower limit included and upper limit excluded is

Temperature	Tally marks	Frequency
$-19\cdot9$ *to* -15	||	2
-15 *to* $-10\cdot1$	||| ||||	7
$-10\cdot1$ *to* $-5\cdot2$	|||||	5
$-5\cdot2$ *to* $-0\cdot3$	||||	4
$-0\cdot3$ *to* $-4\cdot6$	||||| ||||| ||||| ||	17
Total		35

21. The blood groups of 30 students of class VIII are recorded as follows:
A, B, O, O, AB, O, A, O, B, A, O, B, A, O, O, A, AB, O, A, A, O, O, AB, B, A, O, B, A, B, O
Represent this data in the form of a frequency distribution table. Find out which is the most Common and which is the rarest blood group among these students.

Sol: Here 9 students have blood groups $A, 6\ as\ B, 3$ as AB and as O.

So the table representing the data is as follows

Blood group	Number of students
A	9
B	8
AB	3
O	12
Total	30

As 12 students have the blood group O and 3 have their blood group is AB. Clearly, the most common blood group among these students is 0 and the rarest blood group among these students is 0 and the rarest blood group among these students is AB

22. Three coins were tossed 30 times. Each time the number of head occurring was noted down as follows:

0	1	2	2	1	2	3	1	3	0
1	3	1	1	2	2	0	1	2	1
3	0	0	1	1	2	3	2	2	0

Sol:

By observing the data given above following frequency distribution table can be constructed

Number of heads	Number of times (frequency)
0	6
1	10
2	9
3	5
Total	30

23. Thirty children were asked about the number of hours they watched T.V. programmers in the previous week. The results were found as follows:

1	6	2	3	5	12	5	8	4	8
10	3	4	12	2	8	15	1	17	6
3	2	8	5	9	6	8	7	14	12

(i) Make a grouped frequency distribution table for this data, taking class width 5 and one of the class intervals as 5 – 10.

(ii)How many children watched television for 15 or more hours a week?

Sol:

(i) Class intervals will be $0-5, 5-10, 10-15, ...$

The grouped frequency distribution table is as follows

Hours	Number of children
$0-5$	10
$5-10$	13
$10-15$	5
$15-20$	2
Total	30

(ii) The number of children, who watched TV for 15 or more hours a week is 2 (i.e., number of children in class interval $15-20$).

Exercise – 22.2

1. Define cumulative frequency distribution.

Sol:

Cumulative frequency distribution: A table which displays the manner in which cumulative frequencies are distributed over various classes is called a cumulative frequencies (or) cumulative frequencies distribution table.

2. Explain the difference between a frequency distribution and a cumulative frequency distribution.

Sol:

Frequencies table or frequency distribution is a method to represent raw data in the form from which one can easily understand the information contained in a raw data , where as a table which plays the manner in which cumulative frequencies are distributed over various classes is called a cumulative frequency distribution.

3. The marks scored by 55 students in a test are given below:

Marks	0-5	5-10	10-15	15-20	20-25	25-30	30-35
No. of students	2	6	13	17	11	4	2

Prepare a cumulative frequency table:

Sol:

Marks	No. of students	Marks	Cumulative frequency
0-5	2	Less than 5	2
5-10	6	Less than 10	8
10-15	13	Less than 15	21

15-20	17	Less than 20	38
20-25	11	Less than 25	49
25-30	4	Less than 30	53
30-35	2	Less than 35	55
	N = 55		

4. Following are the ages of360 patients getting medical treatment in a hospital on a day:

Age (in years):	10-20	20-30	30-40	40-50	50-60	60-70
No. of Patients:	90	50	60	80	50	30

Construct a cumulative frequency distribution.

Sol:

Age (in years)	No. of students	Age (in years)	Cumulative frequency
10-20	90	Less than 20	90
20-30	50	Less than 30	140
30-40	60	Less than 40	200
40-50	80	Less than 50	280
50-60	50	Less than 60	330
60-70	30	Less than 70	360
	N = 360		

5. The water bills (in rupees) of 32 houses in a certain street for the period 1.1.98 to. 31.3.98 are given below:

56, 43, 32, 38, 56, 24, 68, 85, 52, 47, 35, 58, 63, 74, 27, 84, 69, 35, 44, 75, 55, 30, 54, 65, 45, 67, 95, 72, 43, 65, 35, 59.

Tabulate the data and resent the data as a cumulative frequency table using 70-79 as one of the class intervals.

Sol:

The minimum bill is Rs 24.

Maximum bill is Rs 95.

Range = maximum bill – minimum bill

$=95-24$

$=71$

Given class in terval is $70-79$, So class size

$=79-70=9$

$\therefore$ Number of classes $=\dfrac{\text{Range}}{\text{class size}}=\dfrac{71}{9}$

$=7\cdot 88$

$\therefore$ Number of classes = 8

The cumulative frequency distribution table is as

Bills	No. of houses (Frequency)	Cumulative frequency
16-25	1	1
25-34	3	4
34-43	5	9
43-52	4	13
52-61	7	20
61-70	6	26
70-79	3	29
79-88	2	31
88-97	1	32

6. The number of books in different shelves of a library are as follows:

30, 32, 28, 24, 20, 25, 38, 37, 40, 45, 16, 20
19, 24, 27, 30, 32, 34, 35, 42, 27, 28, 19, 34,
38, 39, 42, 29, 24, 27, 22, 29, 31, 19, 27, 25
28, 23, 24, 32, 34, 18, 27, 25, 37, 31, 24, 23,
43, 32, 28, 31, 24, 23, 26, 36, 32, 29, 28, 21.

Prepare a cumulative frequency distribution table using 45-49 as the last class interval.

Sol:

Minimum number of book shelves is 16 and maximum number of book shelves is 45

Range = maximum B. S – Minimum B. S

= 45 – 16

= 29

Given class interval = 45 – 49. So class size

$= 49 - 45 = 4.$

$\therefore$ Number of classes $= \dfrac{\text{Range}}{\text{Class size}} = \dfrac{29}{4} = 7 \cdot 25$

$\Rightarrow$ Number of classes = 8.

The cumulative frequencies distribution is as

No. of Books	No. of shelves (Frequency)	Cumulative frequency
13-17	1	1
17-21	6	7
21-25	11	18
25-29	15	33
29-33	12	45
33-37	5	50

37-41	6	56
41-45	3	59
45-49	1	60

7. Given below are the cumulative frequencies showing the weights of 685 students of a school. Prepare a frequency distribution table.

Weight (in kg)	No. of students
Below 25	0
Below 30	24
Below 35	78
Below 40	183
Below 45	294
Below 50	408
Below 55	543
Below 60	621
Below 65	674
Below 70	685

Sol:

Weight (in kg)	No. of students	Class interval	Frequency
Below 30	24	25-30	24-0=24
Below 35	78	30-35	78-24=54
Below 40	183	35-40	183-78=105
Below 45	294	40-45	294-183=111
Below 50	408	45-50	408-294=114
Below 55	543	50-55	543-408=135
Below 60	621	55-60	621-543=78
Below 65	674	60-65	674-621=53
Below 70	685	65-70	685-674=11

8. The following cumulative frequency distribution table shows the daily electricity consumption (in kW) of 40 factories in an industrial state:

Consumption (in kW)	No. of Factories
Below 240	1
Below 270	4
Below 300	8
Below 360	24
Below 390	38
Below 420	40

(i) Represent this as a frequency distribution table.

(ii) Prepare a cumulative frequency table.

Sol:

(i)

Consumption (in kg)	No. of factories	Class interval	Frequency
Below 240	1	0-240	1
Below 270	4	240-270	4-1=3
Below 300	8	270-300	8-4=4
Below 330	24	300-330	24-8=16
Below 360	33	330-360	33-24=9
Below 390	38	360-390	38-33=5
Below 420	40	390-420	40-38=2

(ii)

Class interval	Frequency	Consumption (in kw)	No. of factories
0-240	1	More than 0	40
240-270	3	More than 240	40-1=39
270-300	4	More than 270	39-3=36
300-330	16	More than 300	36-4=32
330-360	9	More than 330	32-16=16
360-390	5	More than 360	16-9=7
390-420	2	More than 390	7-5=2
		More than 420	2-2=0
	N = 40		

9. Given below is a cumulative frequency distribution table showing the ages of people living in a locality:

Ace in years	No. of persons
Above 108	0
Above 96	1
Above 84	3
Above 72	5
Above 60	20
Above 48	158
Above 36	427
Above 24	809
Above 12	1026
Above 0	1124

Prepare a frequency distribution table

Sol:

Age (in years)	No. of persons	Class interval	Frequency
Above 0	1124	0-12	1124-1026=98
Above 12	1026	12-24	217
Above 24	809	24-36	382
Above 36	427	36-48	269
Above 48	158	48-60	138
Above 60	20	60-72	15
Above 72	5	72-84	5-3=2
Above 84	3	84-96	3-1=2
Above 96	1	96-108	1-0=1

Exercise – 23.1

1. Read the bar graph shown in Fig. 23.8 and answer the following questions:

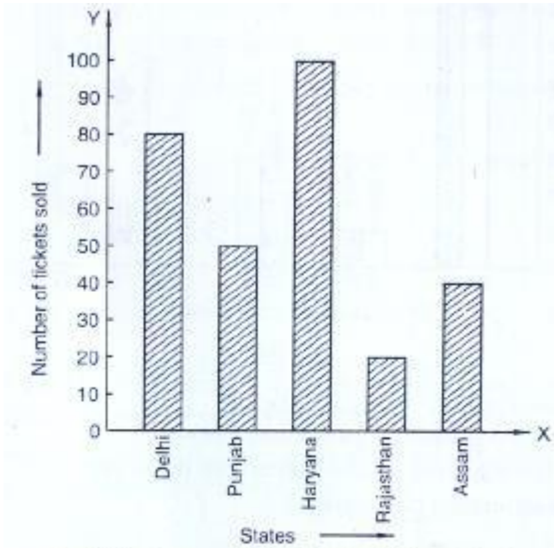


Fig. 23.8 : Bar graph of the tickets of different state lotteries sold by an agent on a day.

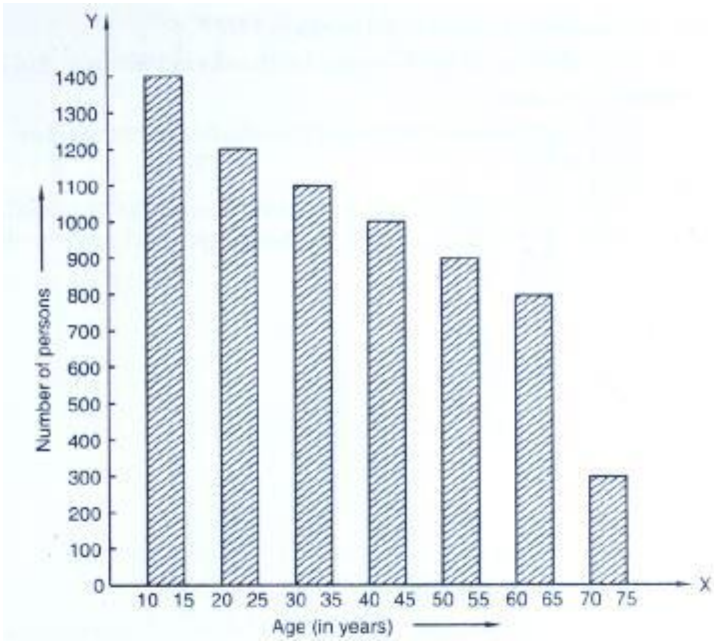
(i) What is the information given by the bar graph?
(ii) How many tickets of Assam State Lottery were sold by the agent?
(iii) Of which state, were the maximum number of tickets sold?
(iv) State whether true or false.
The maximum number of tickets sold is three times the minimum number of tickets sold.
(v) Of which state were the minimum number of tickets sold?

Sol:

(i) The given bar graph represents the number of the tickets of different state lotteries sold by an agent on a day
(ii) Number of tickets of assam state lottery were sold by agent is 40
(iii) Haryana sold the maximum number of tickets
(iv) Minimum number of tickets sold = 20
Maximum number of tickets sold = 100
$\therefore 100 = 5 \times 20$
So, the given statement is false
(v) Rajasthan sold the minimum number of tickets.

2. Study the bar graph representing the number of persons in various age groups in a town shown in Fig. below. Observe the bar graph and answer the following questions:
(i) What is the percentage of the youngest age-group persons over those in the oldest age group?
(ii) What is the total population of the town?

(iii) What is the number of persons in the age group 60 - 65?
(iv) How many persons are more in the age-group 10 - 15 than in the age group 30 - 35?
(v) What is the age-group of exactly 1200 persons living in the town?
(vi) What is the total number of persons living in the town in the age-group 50 - 55?
(vii) What is the total number of persons living in the town in the age-groups 10 - 15 and 60 - 65?



(viii) Whether the population in general increases, decreases or remains constant with the increase in the age-group.

Sol:

(i) The percentage of the youngest age group persons over those in the oldest age group

$$= \frac{1400}{300} \times 100$$

$$= 466\frac{2}{3}.$$

(ii) Total population of the town

$= 1400 + 1200 + 1100 + 1000 + 900 + 800 + 300$

$= 6700.$

(iii) The number of persons in the age group of $60 - 65$ *is* 800.

(iv) The number of persons are more in the age group 10-15 than in the age group $= 30 - 35 = 1400 - 1100 = 300.$

(v) The age group in which exactly 1200 persons living in the town is $20 - 25$

(vi) The total number of person living in the town is 20-25

(vii) The total number of persons living in the town in the age group 10 to 15 and 60-65 $= 1400 + 800$

$= 2,200$

(viii) The population decreases with the increase in the group.

3. Read the bar graph shown in Fig. 23.10 and answer the following questions

(i) What is the information given by the bar graph?

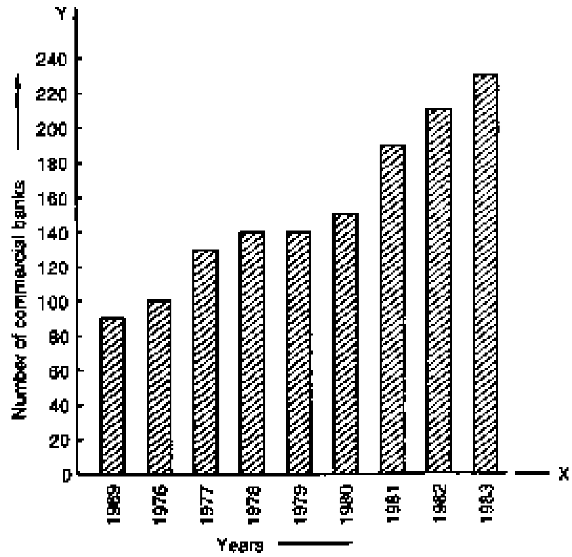


Fig. 23.10 : Bar graph of the number of commercial banks in India during some years.

(ii) What was the number of commercial banks in 1977?

(iii) What is the ratio of the number of commercial banks in 1969 to that in 1980?

(iv) State whether true or false:

The number of commercial banks in 1983 is less than double the number of commercial banks in 1969.

Sol:

(i) Given bar graph represents the number of commercial banks in India during some years.

(ii) The number of commercial banks in 1977 was 130.

(iii) The ratio of the number of commercial banks in 1969 to that in 1980 $= \frac{90}{150}$

$= 3 : 5$

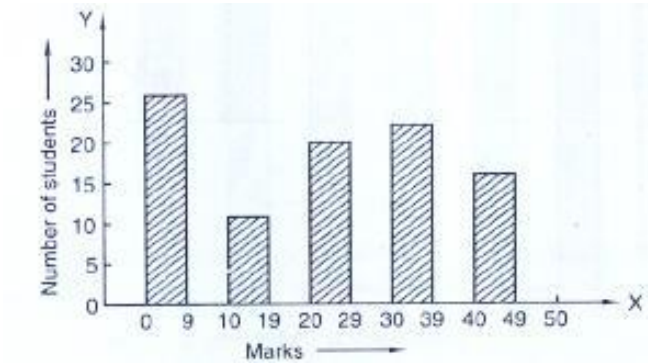
(iv) The number of commercial banks in 1983 = 230

The number of commercial banks in 1980 = 150

Clearly, the number of commercial banks in 1983 is not less than double the number of commercial banks in 1969.

So, the given statement is false.

4. Given below (Fig. below) is the bar graph indicating the marks obtained out of 50 in mathematics paper by 100 students. Read the bar graph and answer the following questions:



(i) (i) It is decided to distribute work books on mathematics to the students obtaining less than 20 marks, giving one workbook to each of such students. If a work book costs Rs 5, what sum is required to buy the work books?

(ii) Every student belonging to the highest mark group is entitled to get a prize of Rs. 10. How much amount of money is required for distributing the prize money?

(iii) Every student belonging to the lowest mark—group has to solve 5 problems per day. How many problems, in all, will be solved by the students of this group per day?

(iv) State whether true or false.

a. 17% students have obtained marks ranging from 40 to 49.

b. 59 students have obtained marks ranging from 10 to 29.

(v) What is the number of students getting less than 20 marks?

(vi) What is the number of students getting more than 29 marks?

(vii) What is the number of students getting marks between 9 and 40?

(viii) What is the number of students belonging to the highest mark group?

(ix) What is the number of students obtaining more than 19 marks?

Sol:

(i) Total number of students obtaining less than 20 marks $= 27 + 12 = 39$.

The cost off one – work book = Rs 5.

$\therefore$ The cost of 30 work books $= 39 \times 5$

= Rs 195.

(ii) The number of students belonging to the highest mark group = 17

The cost of a prize = 10.

$\therefore$ The cost of 17 prizes $= 10 \times 17$

$= Rs\ 170$

(iii) The number of students belonging to the lowest mark group $= 27$

The number of problems solved by 1 student = 5

$\therefore$ The total number of problems solved by 27 students $= 5 \times 27$

$= 135.$

(iv) (a) Total number of students = 100

The number of students in range 40-49=17.

% of students obtaining marks ranging

$40-49=\frac{17}{100}\times100$

$=17\%$

So, the given statement is true.

(b) The number of students in range $10-29=12=120=32.$

Percentage of students obtaining marks ranging $10-29=\frac{32}{100}\times100=32\%$

So, the given statement is false (False).

(v) No. of students getting more than 20 marks = 39

(vi) Total no. of students getting more than 29 marks = 41

(vii) The number of students getting marks between 9 and $40=12+20+24=56.$

(viii) The number of students belonging to the highest mark group = 17.

(ix) The number of students obtaining more than 19 marks $=100-27-12=61$

5. Read the following bar graph (Fig. 23.12) and answer the following questions:

(i) What is the information given by the bar graph?

(ii) State each of the following whether true or false.

a. The number of government companies in 1957 is that of 1982 is 1 :9.

b. The number of government companies have decreased over the year 1957 to 1983.

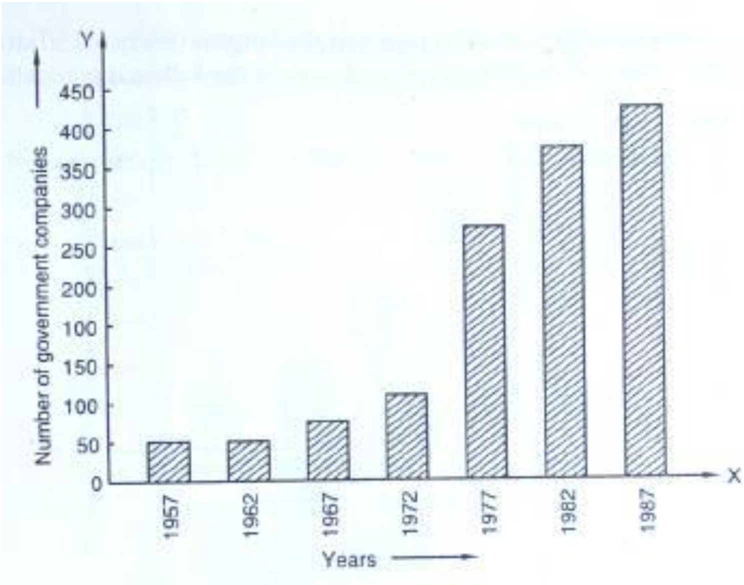


Fig. 23.12 Bar graph of the number of government companies in India during some years

Sol:

(i) The given Bar graph represents the number of government companies in india during some years.

(ii) (a) no. of government companies in 1957=50
Number of government companies in 1982=375

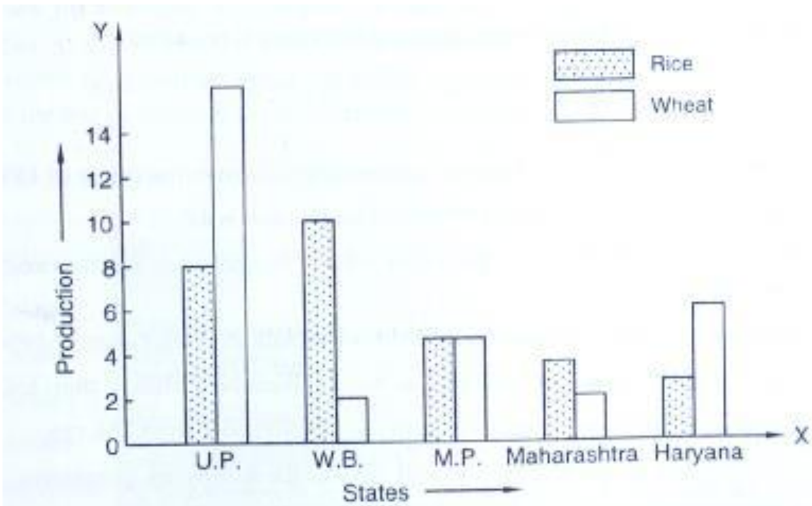
$\therefore$ The number of government companies in 1957 is that of 1982 $=\frac{50}{375}$

$=\frac{2}{15}\neq\frac{1}{9}$

So, the given statement is false

(b) The height of the bars increases over the years hence, the statement is false.

6. Read the following bar graph and answer the following questions:

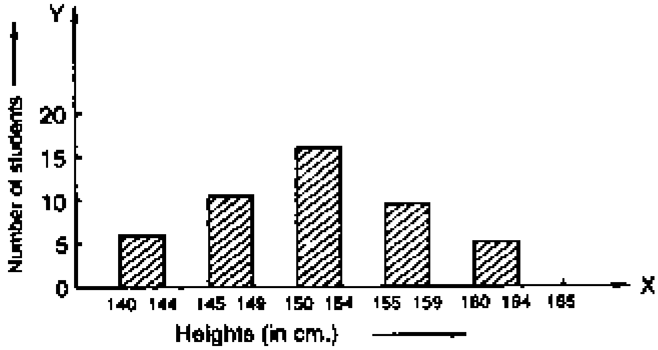


(i) What information is given by the bar graph?
(ii) Which state is the largest producer of rice?
(iii) Which state is the largest producer of wheat?
(iv) Which state has total production of rice and wheat as its maximum?
(v) Which state has the total production of wheat and rice minimum?

Sol:

(i) The given information is about rice and wheat production in various states of India.
(ii) $W \cdot B$ is the largest producer of rice
(iii) $U \cdot P$ is the largest producer of wheat
(iv) The total production of rice and wheat is maximum in $U \cdot P$
(v) The total production of rice and wheat is minimum in Maharashtra

7. The following bar graph (Fig. 23. 1 4) represents the heights (in cm) of 50 students of Class XI of a particular school. Study the graph and answer the following questions:



(i) What percentage of the total number of students have their heights more than 149cm?

(ii) How many students in the class are in the range of maximum height of the class?

(iii) The school wants to provide a particular type of tonic to each student below the height of 150 cm to improve his height. If the cost of the tonic for each student comes out to be Rs. 55, how much amount of money is required?

(iv) How many students are in the range of shortest height of the class?

(v) State whether true or false:

a. There are 9 students in the class whose heights are in the range of 155 - 159 cm.
b. Maximum height (in cm) of a student in the class is 17.
c. There are 29 students in the class whose heights are in the range of 145- 154 cm.
d. Minimum height (in cm) of a student is the class is in the range of 140 – 144 cms.
e. The number of students in the class having their heights less than 150 cm is 12.
f. There are 14 students each of whom has height more than 154. cm.

Sol:

(i) Total number of students have their heights more than $149cm = 16 + 10 + 5 = 31$.
The percentage of the total number of students has their heights more than 149 cm
$= \frac{31}{50} \times 100 = 31 \times 2 = 62\%$

(ii) The number of students in the range of maximum height of the class is 5.

(iii) Total number of students below height of 150 cm $= 7 + 12 = 19$.
The cost of the tonic for each student = Rs. 55
The cost of the tonic for 19 student $= 19 \times 55$
$= Rs\ 1045$

(iv) The number of students are in the range of shortest height of the class = 7.

(v) (a) True
(b) False
(c) Total number of students in the range of $145 - 154 = 12 + 17 = 29$
So the given statement is true
(d) True
(e) The number of students whose height more than $154cm = 9 + 5 = 14$.
(f) So, the given statement is true.

8. Read the following bar graph(Fig. 23.15)and answer the following questions:

(i) What information is given by the bar graph?

(ii) What was the production of a student in the year 1980 - 81?

(iii) What is the minimum and maximum productions of cement and corresponding years?

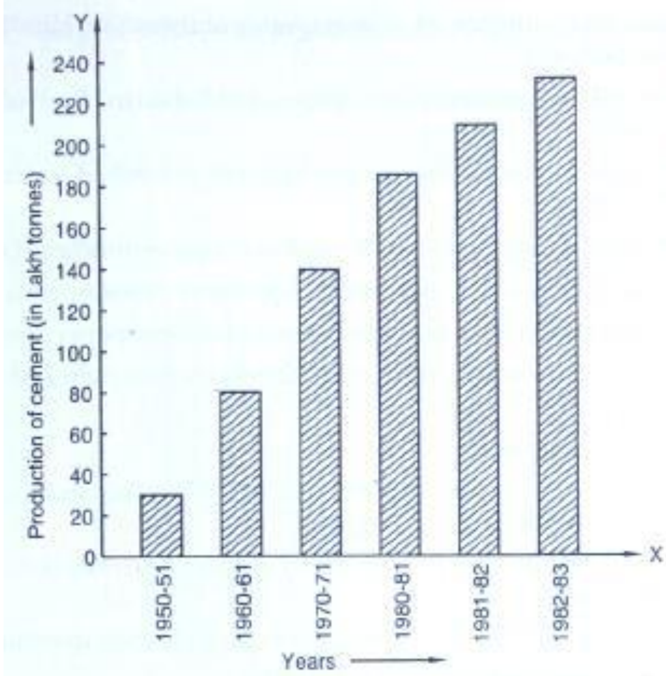


Fig. 23.15 Bar graph of the industrial production of cement in different years in India

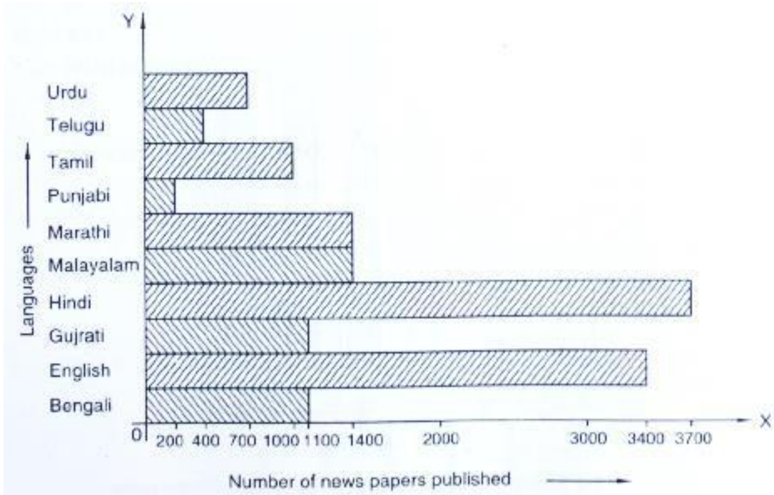
Sol:

(i) It gives information regarding industrial production of cement in different years in India.

(ii) The production of cement in the year 1980 – 1981 = 186 lakh tones.

(iii) The minimum production is 30 lakh tones in 1950 – 1951 and maximum production 232 lakh tones in 1982 – 1983

9. The bar graph shown in Fig 23.16 represents the circulation of newspapers in 10 languages. Study the bar graph and answer the following questions:



(i) What is the total number of newspapers published in Hindi, English, Urdu, Punjabi and Bengali?

(ii) What percent is the number of news papers published in Hindi of the total number of newspapers?

(iii) Find the excess of the number of newspapers published in English over those published in Urdu.

(iv) Name two pairs of languages which publish the same number of newspapers.

(v) State the language in which the smallest number of newspapers are published.

(vi) State the language in which the largest number of newspapers are published.

(vii) State the language in which the number of newspapers published is between 2500 and 3500.

(viii) State whether true or false:

a. The number of newspapers published in Malayalam and Marathi together is less than those published in English.

b. The number of newspapers published in Telugu is more than those published in Tamil.

Sol:

(i) Total number of newspapers published in Hindi, English, Urdu, Punjabi and Bengali

$=3700+3400+700+200+1100$

$=9100.$

(ii) The number of newspapers published in Hindi $=3700$

The total number of newspapers is published $=700+400+1000+200+1400+1400+700+1100+3400+1100=14,400$

The percentage of Hindi news papers $\frac{3700}{14400}\times 100 = 25\cdot 69 = 25\cdot 71$

(iii) The total number of newspaper are published = 700 + 400 + 1000 + 200 +1500+1400+3700+1100+3400+1100=14400

The percentage of hindi news paper $\frac{3700}{14400}\times 100 = 25.69 = 25.71\%$

(iv) Bengali, Gujrati, and Marathi, Malayalam are the two pairs of languages which publish the same number of newspaper.

(v) Punjabi is the Languages in which the smallest number of newspaper were published.

(vi) Hindi is the language in which the largest numbers of newspaper were published.

(vii) English is the language in which the number of newspaper were published in between 2500 and 3500

(viii) (a) Total number of newspaper were published in Malayalam and Marathi $= 1400 + 1400 = 2800$.

Number of newspaper were published in English = 3400

$\therefore$ The number of newspapers published in Malayalam and Marathi together is less than those published in English so the given statement is true.

(b) Number of news. Papers published in Telugu = 400

Number of newspaper published in Tamil = 1000

$\therefore$ The number of newspapers published in Telugu is more less than those published in Tamil.

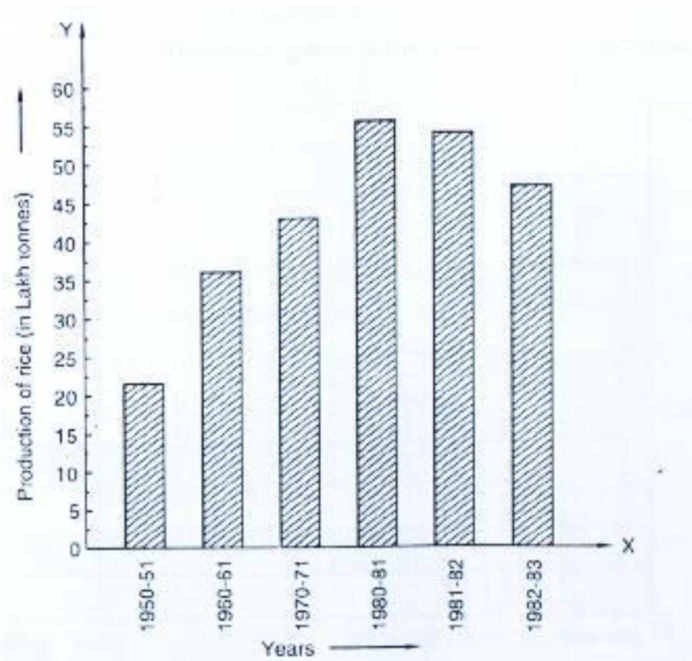
So, the given statement is false

10. Read the bar graph given in Fig. 23.17 and answer the following questions:

(i) What information is given by the bar graph?

(ii) What was the crop-production of rice in 1970 - 71?

(iii) What is the difference between the maximum and minimum production of rice?



23.17 Bar graph of the production of rice crop in India in different years

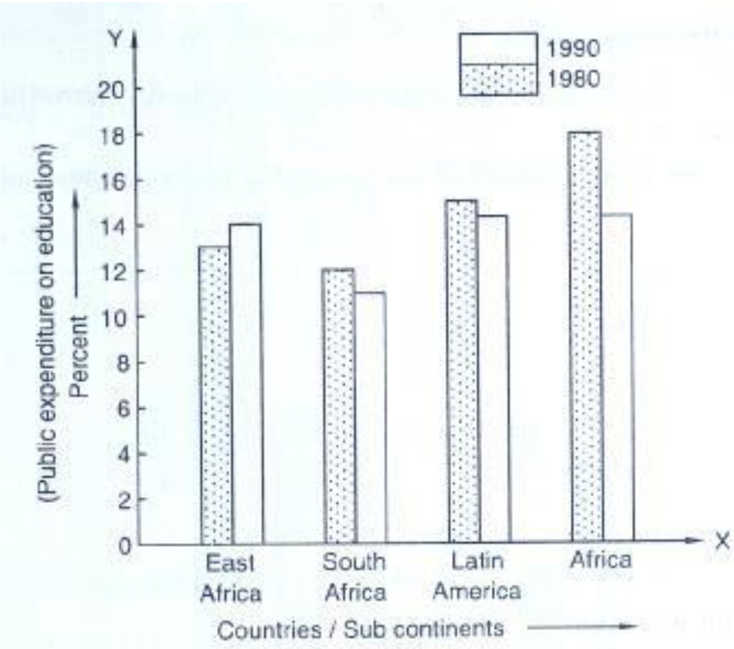
Sol:

(i) It gives information regarding the production of rice crop in India in different years

(ii) The crop production of rice in $1970-71=42\cdot5$ lakh tonnes.

(iii) The difference between the maximum and minimum production of rice = 55 – 22 = 33 lakh tonnes

11. Read the bar graph given in Fig. below and answer the following questions:



(i) What information does it give?

(ii) In which part the expenditure on education is maximum in 1980?

(iii) In which part the expenditure has gone up from 1980 to 1990?

(iv) In which part the gap between 1980 and 1990 is maximum?

Sol:

(i) It gives the information about the public expenditure one education by various state subcontinents

(ii) In Africa the expenditure education is maximum in 1980

(iii) In east Africa. The expenditure has gone by from 1980 to 1990

(iv) In Africa the gap between 1980 and 1990 is maximum.

12. Read the bar graph given in Fig. 23.19 and answer the following questions:

(i) What information is given by the bar graph?

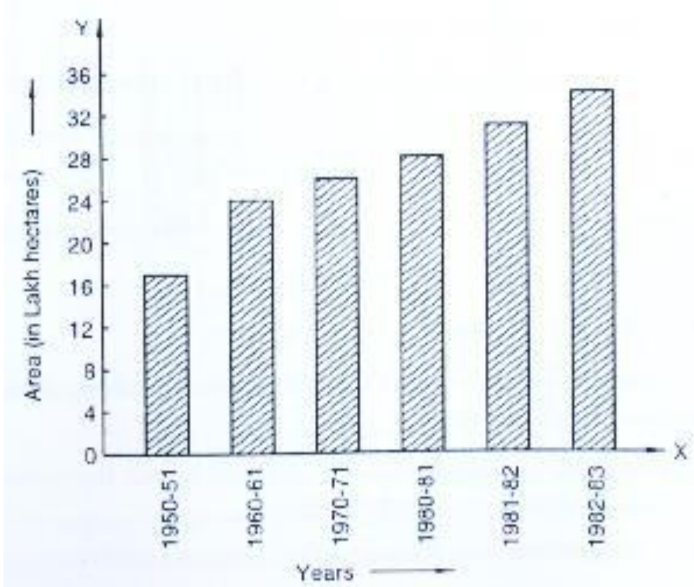


Fig. 23.19 : Bar graph of the area under the sugarcane crop during different years in India

(ii) In which years the areas under the sugarcane crop were the maximum and the minimum?

(iii) State whether true or false:
The area under the sugarcane crop in the year 1982 - 83 is three times that of the year 1950 - 51

Sol:

(i) It gives the information about the areas under sugarcane crop during different years in India

(ii) The areas under the sugarcane crop were the maximum and the minimum in $1982-1983$ and $1950-51$ respectively

(iii) The area under sugarcane crop in the year $1982-1983=34$ lakh hectors.
The area under sugarcane crop in the year $1950-51=17$ lakh hectors
Clearly, the area under the sugarcane crop in the year $1982-83$ is not three tomes that of the year $1950-51$
So, the given statement is false

13. Read the bar graph given in Fig. 23.20 and answer the fol1owing questions:

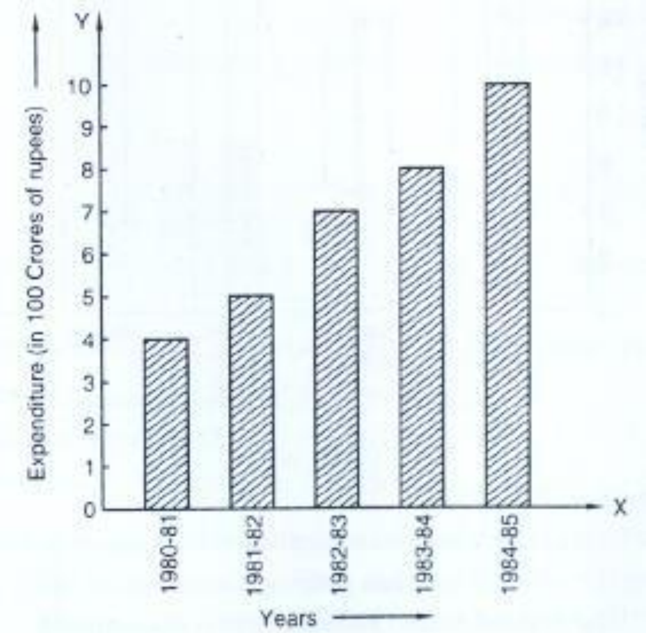


Fig. 23.20 Bar graph of the expenditure on health and family planning during the Sixth Five Year Plan in India

(i) What information is given by the bar graph?

(ii) What was the expenditure on health and family planning in the year 1982-83?

(iii) In which year is the increase in expenditure maximum over the expenditure in previous year? What is the maximum increase?

Sol:

(i) It gives the information about the expenditure on health and family planning during sixth five plan in India

(ii) The expenditure on health and many family planning in the year $1982-83 = Rs\ 700$ cores

(iii) $1984-85$ is the year in which the increase in expenditure maximum over the expenditure in previous year.

The maximum increase $= 1000 - 780$

$= 220$ crores.

14. Read the bar graph given in Fig. 23.21 and answer the following questions:

(i) What is the information given by the bar graph?

(ii) What is the number of families having 6 members?

(iii) How many members per family are there in the maximum number of families? Also tell the number of such families.

(iv) What are the number of members per family for which the number of families are equal? Also, tell the number of such families?

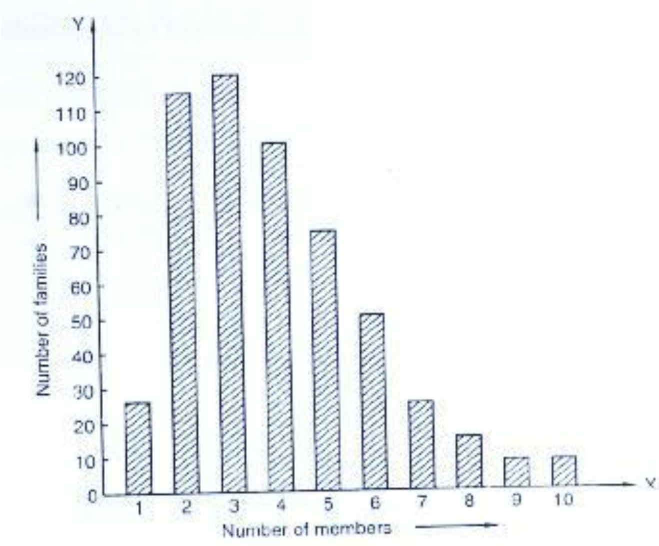


Fig. 23.21 : Bar graph of the number of families with different number of members in a locality

Sol:

(i) It gives the information about the number of families with different number of members in a locality.

(ii) The number of families having 6 members = 50

(iii) 3 members per family are there in the maximum number of families
The number of families which have 3 members = 120.

(iv) 9 and 10 are the number of members per family for which the number of families are equal
The number of such families is 5.

15. Read the bar graph given in Fig. 23.22 and answer the following questions:

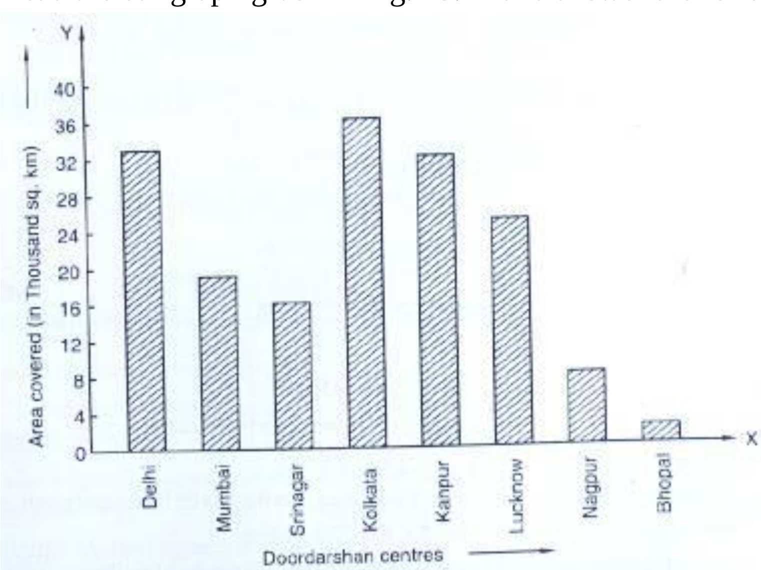


Fig. 23.22 Bar graph showing the coverage of some Doordarshan Centres of India

(i) What information is given by the bar graph?
(ii) Which Doordarshan centre covers maximum area? Also tell the covered area.
(iii) What is the difference between the areas covered by the centres at delhi and Bombay?
(iv) Which Doordarshan centres are in U.P State? What are the areas covered by them?

Sol:

(i) It gives the information above the overage of some Door Darshan centers of India
(ii) Kolkata Door Darshan center convers maximum area.
The area covered by Kolkata door darshan
Centre $= 36000\, sq - km$
(iii) The difference between the areas covered by the centers of Delhi and Bombay
$= 33,000 - 19,000$
$= 14,000\ sq - km$
(iv) Kanpur and Luck now door darshan center are in U·P state
The area covered by Kanpur door darshan center $= 32,000\ sq.km$
The area covered by Lucknow door darshan center $= 25,000\ sq.km$

Exercise – 23.2

1. Explain the reading and interpretation of bar graphs.

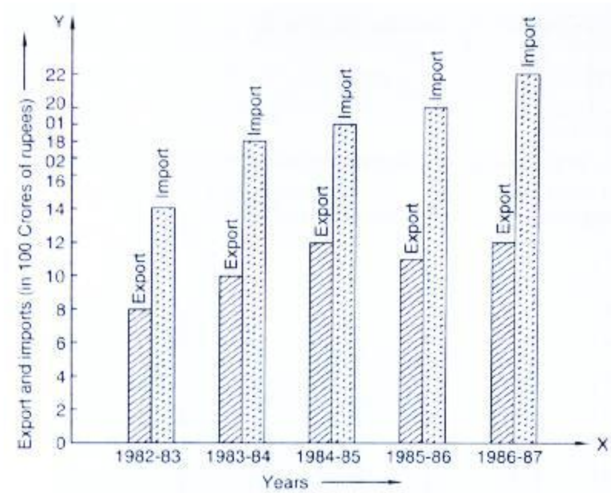
Sol:

First step in reading a bar graph is to know what it represents or what is the information given by it, for this we read the captions. Which are generally written just below the horizontal line $(x - axis)$ and adjacent to vertical line $(y - axis)$

After knowing that what a bar graph represents, we read the scale so that we can know the precise value in the given data.

After reading a bar graph one must be able to draw certain condusions from it. Drawing some conditions from a given bar graph means interpretation of the bar grph.

2. Read the following bar graph and answer the following questions:
(i) What information is given by the bar graph?
(ii) In which year the export is minimum?
(iii) In which year the import is maximum?
(iv) In which year the difference of the values of export and import is maximum?



Sol:

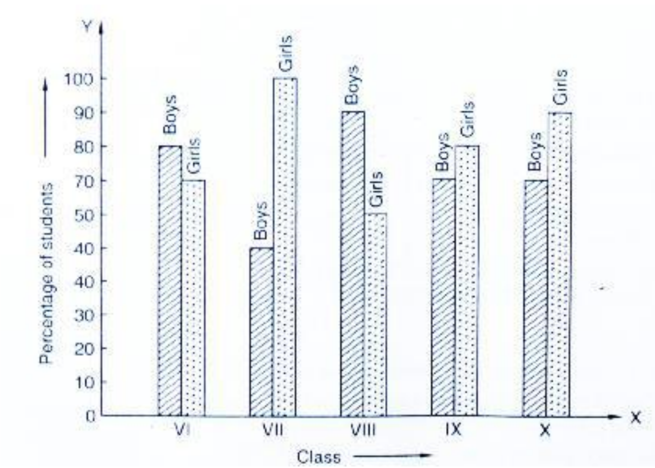
(i) It gives the information regarding import and export from $1982-83$ to $1986-1987$

(ii) The export is minimum in the years $1982-83$

(iii) The imports is maximum in $1986-87$.

(iv) The different of value of export and import is maximum in 1986-87

3. The following bar graph shows the results of an annual examination in a secondary school. Read the bar graph (Fig. 23.28) and choose the correct alternative in each of the following:



(i) The pair of classes in which the results of boys and girls are inversely proportional are:
(a) VI, VIII (b) VI, IX (c) VIII, IX (d) VIII, X

(ii) The class having the lowest failure rate of girls is
(a) VII (b) X (c) IX (d) VIII

(iii) The class having the lowest pass rate of students is
(a) VI (b) VII (c) VIII (d) IX

Sol:

(i) (b) VI, IX

(ii) (a) VII

(iii) (b) VII

Exercise – 24.1

1. If the heights of 5 persons are 140 cm, 150 cm, 152 cm, 158 cm and 161 cm respectively, find the mean height.

Sol:

It is given that,

The heights of 5 persons are $-140cm, 150cm, 152cm, 158cm\ and\ 161cm.$

$$\therefore \text{Mean height } = \frac{Sum\ of\ \ heights}{Total\ \ No.of\ \ \ persons}$$

$$= \frac{140+150+152+158+161}{5}$$

$$= \frac{761}{5}$$

$$= 152\cdot 2.$$

2. Find the mean of 994, 996, 998, 1002 and 1000.

Sol:

Given numbers are $-994, 996, 998, 1000\ and\ 1002.$

$$\therefore Mean = \frac{Sum\ of\ Numbers}{Total\ \ Numbers}$$

$$= \frac{994+996+998+1000+1002}{5}$$

$$= \frac{4990}{5}$$

$$= 998.$$

3. Find the mean of first five natural numbers.

Sol:

Given that,

The first five natural numbers are $1, 2, 3, 4, 5$

$$\therefore Mean = \frac{Sum\ of\ Numbers}{Total\ \ Numbers}$$

$$= \frac{1+2+3+4+5}{5}$$

$$= \frac{15}{5}$$

$$\boxed{Mean = 3}$$

4. Find the mean of all factors of 10.

Sol:

All factors of 10 are $-1, 2, 5, 10$

$\therefore Mean = \frac{Sum\ of\ \ factors}{Total\ \ factors}$

$= \frac{1+2+5+10}{4}$

$= \frac{18}{4}$

$= \frac{9}{2}$

$= 4 \cdot 5$

$\boxed{\therefore Mean = 4 \cdot 5}$

5. Find the mean of first 10 even natural numbers.

Sol:

Given that,

The first 10 natural numbers be $-2, 4, 6, 8, 10, 12, 14, 16, 18, 20$

$\therefore Mean = \frac{Sum\ of\ all\ Numbers}{Total\ \ Numbers}$

$= \frac{2+4+6+8+10+12+14+16+18+20}{10} = \frac{110}{10}$

$= \frac{110}{10} = 11$

$\boxed{Mean = 11}$

6. Find the mean of x, x + 2, x + 4, x +6, x + 8.

Sol:

Numbers be $x, x+2, x+4, x+6\ and\ x+8$

$\therefore Mean = \frac{Sum\ of\ \ Numbers}{Total\ \ Numbers}$

$= \frac{x+x+2+x+4+x+6+x+8}{5}$

$= \frac{5x+20}{5}$

$= \frac{5(x+4)}{5}$

$= x+4$

7. Find the mean of first five multiples of 3.

Sol:

First five multiple of 3: $3, 6, 9, 12, 15$

$\therefore Mean = \frac{Sum\ of\ Numbers}{Total\ Numbers}$

$= \frac{3+6+9+12+15}{5}$

$= \frac{45}{5} = 9.$

8. Following are the weights (in kg) of 10 new born babies in a hospital on a particular day:

3.4, 3.6, 4.2, 4.5, 3.9, 4.1, 3.8, 4.5, 4.4, 3.6. Find the mean $\bar{X}$.

Sol:

The weight (in kg) of 10 new born babies

$= 3\cdot 4, 3\cdot 6, 4\cdot 2, 4\cdot 5, 3\cdot 9, 4\cdot 1, 3\cdot 8, 4\cdot 5, 4\cdot 4, 3\cdot 6$

$\therefore Mean(\bar{x}) = \frac{Sum\ of\ weights}{Total\ babies}$

$\frac{= 3\cdot 4+3\cdot 6+4\cdot 2+4\cdot 5+3\cdot 9+4\cdot 1+3\cdot 8+4\cdot 5+4\cdot 4+3\cdot 6}{10}$

$= \frac{40}{10} 4kg.$

9. The percentage of marks obtained by students of a class in mathematics are : 64, 36, 47, 23, 0, 19, 81, 93, 72, 35, 3, 1. Find their mean.

Sol:

The percentage marks obtained by students are

$= 64, 36, 47, 23, 0, 19, 81, 93, 72, 35, 3, 1.$

$\therefore$ Mean marks $= \frac{64+36+47+23+0+19+81+93+72+35+3+1}{12}$

$= \frac{474}{12} = 39\cdot 5$

$\therefore$ Mean marks $= 39\cdot 5$

10. The numbers of children in 10 families of a locality are:

2, 4, 3, 4, 2, 0, 3, 5, 1, 1, 5. Find the mean number of children per family.

Sol:

The number of children in 10 families is

$\Rightarrow 2, 4, 3, 4, 2, 3, 5, 1, 1, 5.$

$\therefore$ Mean number of children per family

$= \frac{Total\ no.\ of\ children}{Total\ \ families}$

$= \frac{2+4+3+4+2+3+5+1+1+5}{10}$

$= \frac{30}{10}$

$= 3.$

11. If M is the mean of x_1 , x_2 , x_3 , x_4 , x_5 and x_6, prove that
$(x_1 - M) + (x_2 - M) + (x_3 - M) + (x_4 - M) + (x_5 - M) + (x_6 - M) = 0.$

Sol:

Let m be the mean of x_1, x_2, x_3, x_4, x_5 *and* x_6

Then $M = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6}{6}$

$\Rightarrow x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 6M$

To prove: $(x_1 - M) + (x_2 - M) + (x_3 - M) + (x_4 - M) + (x_5 - M) + (x_6 - M)$

$= (x_1 + x_2 + x_3 + x_4 + x_5 + x_6) - (M + M + M + M + M + M)$

$= 6M - 6M$

$= 0$

$= RHS$

12. Durations of sunshine (in hours) in Amritsar for first 10 days of August 1997 as reported by the Meteorological Department are given below:
9.6, 5.2, 3.5, 1.5, 1.6, 2.4, 2.6, 8.4, 10.3, 10.9

(i) Find the mean $\bar{X}$ (ii) Verify that $= \sum_{i=1}^{10} (x_i - \bar{x}) = 0$

Sol:

Duration of sunshine (in hours) for 10 days are

$= 9\cdot6, 5\cdot2, 3\cdot5, 1\cdot5, 1\cdot6, 2\cdot4, 2\cdot6, 8\cdot4, 10\cdot3, 10\cdot9$

(i) $Mean\ \bar{x} = \frac{Sum\ of\ all\ numbers}{Total\ \ numbers}$

$= \frac{9\cdot6+5\cdot2+3\cdot5+1\cdot5+1\cdot6+2\cdot4+2\cdot6+8\cdot4+10\cdot3+10\cdot9}{10}$

$= \frac{56}{10} = 5\cdot6$

(ii) LHS $=\sum_{i=1}^{10}\left(x_i-\overline{x}\right)$

$=\left(x_1-\overline{x}\right)+\left(x_2-\overline{x}\right)+\left(x_3-\overline{x}\right)+.............+\left(x_{10}-\overline{x}\right)$

$=(9\cdot 6-5\cdot 6)+(5\cdot 2-5\cdot 6)+(3\cdot 5-5\cdot 6)+(1\cdot 5-5\cdot 6)+(1\cdot 6-5\cdot 6)+(2\cdot 4-5\cdot 6)$

$=(4)+(-0\cdot 4)+(-2\cdot 1)-4\cdot 1-4-3\cdot 2-3+2\cdot 8+4\cdot 7+5\cdot 3$

$=16\cdot 8-16\cdot 8$

$=0.$

13. Explain, by taking a suitable example, how the arithmetic mean alters by (i) adding a constant k to each term, (ii) subtracting a constant k from each them, (iii) multiplying each term by a constant k and (iv) dividing each term by a non-zero constant k.

Sol:

Let us say numbers are be $3,4,5$

$\therefore Mean=\frac{Sum\ of\ \ number}{Total\ \ number}$

$=\frac{3+4+5}{3}$

$=\frac{12}{3}$

$=4$

(i) Adding constant term $k=2$ in each term

New numbers are $=5,6,7.$

$\therefore$ New mean $=\frac{5+6+7}{3}$

$=\frac{18}{3}=6=4+2$

$\therefore$ New mean will be 2 more than the original mean.

(ii) Subtracting constant term $k=2$ in each term New number are $=1,2,3.$

$\therefore$ New mean $=\frac{1+2+3}{3}=\frac{6}{3}=2=4-2.$

$\therefore$ New mean will be 2 less than the original mean

(iii) Multiplying by constant term $k=2$ in each term

New numbers are = 6, 8, 10

New mean = $\frac{6+8+10}{3}$

$= \frac{24}{3}$

$= 8$

$= 4 \times 2$

$\therefore$ New mean will be 2 times of the original mean.

(iv) Divide by constant term $k = 2$ in each term

New number are $= 1 \cdot 5, 2, 2 \cdot 5$

$\therefore$ New mean $= \frac{1 \cdot 5 + 2 + 2 \cdot 5}{3}$

$= \frac{6}{3} = 2 = \frac{4}{2}$

$\therefore$ New mean will be half of the original mean.

14. The mean of marks scored by 100 students was found to be 40. Later on it was discovered that a score of 53 was misread as 83. Find the correct mean.

Sol:

Mean marks of 100 students = 40

$\Rightarrow$ Sum of marks of 100 students $= 100 \times 40 = 4000$

Correct value $= 53.$

Incorrect value $= 83.$

Correct sum $= 4000 - 83 + 53$

$= 3970$

$\therefore$ Correct mean $= \frac{3970}{100}$

$= 39 \cdot 7.$

15. The traffic police recorded the speed (in kmlhr) of 10 motorists as 47, 53, 49, 60, 39, 42, 55, 57, 52, 48. Later on an error in recording instrument was found. Find the correct overage speed of the motorists if the instrument recorded 5 km/hr less in each case.

Sol:

The speed of 10 motorists are $47, 53, 49, 60, 39, 42, 55, 57, 52, 48$

Later on it was discovered that the instrument recorded 5km/hr less than in each case

Corrected values are 52, 58, 54, 65, 44, 47, 60, 62, 57, 53

$\therefore$ Correct mean $= \frac{52 + 58 + 54 + 65 + 44 + 47 + 60 + 62 + 57 + 53}{10}$

$= \frac{552}{10}$

$= 55 \cdot 2 \;\; km / hr$

16. The mean of five numbers is 27. If one number is excluded, their mean is 25. Find the excluded number.

Sol:

The mean of the numbers 27

The, sum of five numbers $= 5 \times 27$

$= 135.$

If one number is excluded, then the new mean is 25

$\therefore$ Sum of numbers $= 4 \times 25 = 100$

$\therefore$ Excluded number $= 135 - 100$

$= 35$

17. The mean weight per student in a group of 7 students is 55 kg. The individual weights of 6 of them (in kg) are 52, 54, 55, 53, 56 and 54. Find the weight of the seventh student.

Sol:

The mean weight per student in a group of 7 students is 55kg.

Weight of 6 students (in kg) $= 52, 54, 55, 53, 56$ and 54.

Let weight of 7^{th} student = x kg

$\therefore$ Mean $= \frac{Sum\ of\ all\ weights}{Total\ students}$

$\Rightarrow 55 = \frac{52+54+55+53+56+54+x}{7}$

$\Rightarrow 385 = 324 + x$

$\Rightarrow x = 385 - 324$

$\Rightarrow x = 61\ kg$

$\therefore$ Weight of 7^{th} student = 61kg

18. The mean weight of 8 numbers is 15. If each number is multiplied by 2, what will be the new mean?

Sol:

We have,

The mean weight of 8 numbers is 15

Then, The sum of 8 numbers $= 8 \times 15 = 120.$

If each number is multiplied by 2

Then, new mean $= 120 \times 2$

$= 240$

$\therefore$ New mean $= \frac{240}{8} = 30.$

19. The mean of 5 numbers is 18. If one number is excluded, their mean is 16. Find the excluded number.

Sol:

The mean of 5 numbers is 18

Then, the sum of 5 numbers $=5\times18$

$=90$

If the one number is excluded

Then, the mean of 4 numbers = 16.

$\therefore$ Sum of 4 numbers $=4\times16$

$=64$

Excluded number $=90-64$

$=26.$

20. The mean of 200 items was 50. Later on, it was discovered that the two items were misread as 92 and 8 instead of 192 and 88. Find the correct mean.

Sol:

The mean of 200 items = 50

Then the sum of 200 items $=200\times50$

$=10,000$

Correct values $=192$ *and* 88

Incorrect values $=92=8$

$\therefore$ Correct sum $=10000-92-8+192+88$

$=10180$

$\therefore$ Correct mean $=\frac{10180}{200}=50\cdot9$

$=\frac{101\cdot8}{2}=50\cdot9.$

21. Find the values of n and $\bar{X}$ in each of the following cases:

(i) $\sum_{i=1}^{n}(x_i-12)=-10$ $\sum_{i=1}^{n}(x_i-3)=62$

(ii) $\sum_{i=1}^{n}(x_i-10)=30$ $\sum_{i=1}^{n}(x_i-6)=150.$

Sol:

(i) Given $\sum_{i=1}^{n}(x_n-12)=-10$

$\Rightarrow(x_1-12)+(x_2-12)+.......+(x_n-12)=-10$

$\Rightarrow(x_1+x_2+x_3+x_4+x_5+...........+x_n)-(12+12+12+......+12)=-10$

$\Rightarrow \sum x - 12n = -10 \qquad ....(1)$

And $\sum_{i=1}^{n}(x_i - 3) = 62$

$\Rightarrow (x_1 - 3) + (x_2 - x_3) + (x_3 - 3) + ... + (x_n - 3) = 62.$

$\Rightarrow (x_1 + x_2 + ... + x_n) - (3+3+3+3+......+37) = 62$

$\Rightarrow \Sigma x - 3n = 62 \qquad ....(2)$

By subtracting equation (1) from equation (2)

We get

$\Sigma x - 3n - \Sigma x + 12n = 62 + 10$

$\Rightarrow 9n = 72$

$\Rightarrow n = \frac{72}{9} = 8.$

Put value of n in equation (1)

$\Sigma x - 12 \times 8 = -10$

$\Rightarrow \Sigma x - 96 = -10$

$\Rightarrow \Sigma x = -10 + 96 = 86$

$\therefore \bar{x} = \frac{\Sigma x}{x} = \frac{86}{8} = 10 \cdot 75$

(ii) Given $\sum_{i=1}^{n}(x_i - 10) = 30$

$\Rightarrow (x_1 - 10) + (x_2 - 10) + ... + (x_n - 10) = 30$

$\Rightarrow (x_1 + x_2 + x_3 + ... + x_n) - (10+10+10+......+10) = 30$

$\Rightarrow \sum x - 10n = 30 \qquad ....(1)$

And $\sum_{i=1}^{n}(x_i - 6) = 150.$

$\Rightarrow (x_1 - 6) + (x_2 - 6) + ... + (x_n - 6) = 150.$

$\Rightarrow (x_1 + x_2 + x_3 + ... + x_n) - (6+6+6+......+6) = 150$

$\Rightarrow \Sigma x - 6n = 150 \qquad ....(2)$

By subtracting equation (1) from equation (2)

$\Sigma x - 6n - \Sigma x + 10n = 150 - 30$

$\Rightarrow \Sigma x - \Sigma x + 4n = 120$

$\Rightarrow n = \frac{120}{4}$

$\Rightarrow n = 30$

Put value of n in equation (1)

$\Sigma x - 10 \times 30 = 30$
$\Rightarrow \Sigma x - 300 = 30$
$\Rightarrow \Sigma x = 30 + 300 = 330$
$\therefore \overline{x} = \frac{\Sigma x}{n} = \frac{330}{30} = 11.$

22. The sums of the deviations of a set of n values $x_1, x_2, \dots . x_{11}$ measured from 15 and -3 are $-$ 90 and 54 respectively. Find the valùe of n and mean.

Sol:

(i) Given $\sum_{i=1}^{n}(x_i + 5) = -90$

$\Rightarrow (x_1 - 15) + (x_2 - 15) + ... + (x_n - 15) = -90$
$\Rightarrow (x_1 + x_2 + ... + x_n) - (15 + 15 + ... + 15) = -90$
$\Rightarrow \sum x - 15n = -90$ ....(1)

And $\sum_{i=1}^{n}(x_i + 3) = 54$

$\Rightarrow (x_1 - 3) + (x_2 - 3) + ... + (x_n + 3) = 54.$
$\Rightarrow (x_1 + x_2 + x_3 + ... + x_n) + (3 + 3 + 3 + ... + 37) = 54$
$\Rightarrow \Sigma x + 3n = 54$ ....(2)

By subtracting equation (1) from equation (2)

$\Sigma x - 30 - \Sigma x + 15n = 54 + 90$
$\Rightarrow 18n = 144$
$\Rightarrow n = \frac{144}{18} = 8.$

Put value of n in equation (1)

$\Sigma x - 15 \times 8 = -90$
$\Rightarrow \Sigma x - 120 = -90$
$\Rightarrow \Sigma x = -90 + 120 = 30$
$\therefore Mean = \frac{\Sigma x}{n} = \frac{30}{8} = \frac{15}{4}.$

23. Find the sum of the deviations of the variate values 3, 4, 6, 7, 8, 14 from their mean.

Sol:

Values are $3, 4, 6, 7, 8, 14.$

$\therefore Mean = \frac{Sum\ of\ \ numbers}{Total\ \ number}$

$= \frac{3+4+6+7+8+14}{6}$

$= \frac{42}{6}$

$= 7.$

$\therefore$ Sum of deviation of values from their mean

$\Rightarrow (3-7)+(4-7)+(6-7)+(7-7)+(8-7)+(14-7)$

$\Rightarrow (-4)+(-3)+(-1)+(0)+(1)+(7)$

$\Rightarrow -8+8$

$= 0.$

24. If $\bar{X}$ is the mean of the ten natural numbers x_1, x_2, x_3 , ..., x_{10} , show that, $(x_1 - \bar{X}) + (x_2 - \bar{X}) + ... + (x_{10} - \bar{X}) = 0$

Sol:

We have, $\bar{x} = \frac{x_1 + x_2 + ... + x_{10}}{10}$

$\Rightarrow x_1 + x_2 + ... + x_{10} = 10\bar{x} \quad ......(i)$

Now, $(x_1 - \bar{x}) + (x_2 - \bar{x}) + ... + (x_{10} - \bar{x})$

$= (x_1 + x_2 + ... + x_{10}) - (\bar{x} + \bar{x} + ...$ up to 10 terms)

$\Rightarrow 10\bar{x} - 10\bar{x}$ [By equation (i)]

$= 0$

$\therefore (x_1 - \bar{x}) + (x_2 - \bar{x}) + ... + (x_{10} - \bar{x}) = 0$ Hence proved.

Exercise – 24.2

1. Calculate the mean for the following distribution:

x:	5	6	7	8	9
f:	4	8	14	11	3

Sol:

x	f	Fx
5	4	20
6	8	48
7	14	98
8	11	88
9	3	27
	N = 40	$\Sigma fx = 281.$

$\therefore$ Mean $\bar{x} = \frac{\Sigma fx}{N}$

$= \frac{281}{40}$

$= 7 \cdot 025$

2. Find the mean of the following data:

x:	19	21	23	25	27	29	31
f:	13	15	16	18	16	15	13

Sol:

x	f	fx
19	13	247
21	15	315
23	16	368
25	18	450
27	16	432
29	15	435
31	13	403
	N = 106	$\Sigma fx = 2650$

$\therefore$ Mean $\bar{x} = \frac{\Sigma fx}{N} = \frac{2650}{106} = 25.$

3. The mean of the following data is 20.6. Find the value of p.

x:	10	15	p	25	35
f:	3	10	25	7	5

Sol:

x	*f*	*Fx*
10	3	30
15	10	150
P	25	25p
25	7	175
35	5	175
	N = 50	$\Sigma fx = 25P + 530$

It is given that

$\text{Mean} = 20 \cdot 6$

$\Rightarrow \frac{\Sigma fx}{N} = 20 \cdot 6$

$\Rightarrow \frac{25p + 530}{50} = 20 \cdot 6$

$\Rightarrow 25p + 530 = 20 \cdot 6(50) = 1030$

$\Rightarrow 25p = 1030 - 530$

$\Rightarrow 25p = 500$

$\Rightarrow p = \frac{500}{25} = 20$

$\Rightarrow p = 20$

$\boxed{\therefore P = 20}$

4. If the mean of the following data is 15, find p.

x:	5	10	15	20	25
f:	6	p	6	10	5

Sol:

x	*f*	*Fx*
5	6	30
10	P	10p
15	6	90
20	10	200
25	5	125
	N = P + 27	$\Sigma fx = 10P + 445$

Given mean = 15

$\Rightarrow \frac{\Sigma xf}{N} = 15$

$\Rightarrow \frac{10p+445}{p+27} = 15$

$\Rightarrow 10p+445 = 15p+405$

$\Rightarrow 15p-10p = 445-405$

$\Rightarrow 5p = 40$

$\Rightarrow p = \frac{40}{5}$

$\therefore p = 8.$

5. Find the value of p for the following distribution whose mean is 16.6

x:	8	12	15	p	20	25	30
f:	12	16	20	24	16	8	4

Sol:

x	f	fx
8	12	96
12	16	192
15	20	300
P	24	24p
20	16	320
25	8	200
30	4	120
	N = 100	$\Sigma fx = 24P + 1228$

Given mean $= 16 \cdot 6$

$\Rightarrow \frac{\Sigma fx}{N} = 16 \cdot 6$

$\Rightarrow \frac{24p+1228}{100} = 16 \cdot 6$

$\Rightarrow 24p = 1660 - 1228$

$\Rightarrow 24p = 432$

$\Rightarrow p = \frac{432}{24} = 18$

6. Find the missing value of p for the following distribution whose mean is 12.58.

x:	5	8	10	12	p	20	25
f:	2	5	8	22	7	4	2

Sol:

x	*f*	*fx*
5	2	10
8	5	40
10	8	80
12	22	264
P	7	7p
20	4	80
25	2	50
	N = 50	$\Sigma fx = 7P + 524.$

Given mean $= 12 \cdot 58$

$$\Rightarrow \frac{\Sigma fx}{N} = 12 \cdot 58$$

$$\Rightarrow \frac{7p + 524}{50} = 12 \cdot 58$$

$$\Rightarrow 7p + 524 = 629$$

$$\Rightarrow 7p = 629 - 524$$

$$\Rightarrow 7p = 105$$

$$\Rightarrow p = \frac{105}{7} = 15$$

7. Find the missing frequency (p) for the following distribution whose mean is 7.68.

x:	3	5	7	9	11	13
f:	6	8	15	p	8	4

Sol:

x	*f*	*Fx*
3	6	18
5	8	40
7	15	105
9	P	9p
11	8	88
13	4	52
	N = P + 41	$\Sigma fx = 9P + 303.$

Given mean $= 7 \cdot 68$

$\Rightarrow \frac{\Sigma fx}{N} = 7 \cdot 68$

$\Rightarrow \frac{9p + 303}{p + 41} = 7 \cdot 68$

$\Rightarrow 9p + 303 = 7 \cdot 68p + 314 \cdot 88$

$\Rightarrow 9p - 7 \cdot 68p = 314 \cdot 88 - 303$

$\Rightarrow 1 \cdot 32p = 11 \cdot 88$

$\Rightarrow p = \frac{11 \cdot 88}{1 \cdot 32}$

$\Rightarrow p = 9.$

8. Find the mean of the following distribution:

x:	10	12	20	25	35
f:	3	10	15	7	5

Sol:

x	*f*	*Fx*
10	3	30
12	10	120
20	15	300
25	7	175
35	5	175
	N = 40	$\Sigma fx = 800$

$\therefore Mean(\bar{x}) = \frac{\Sigma fx}{N}$

$= \frac{800}{40} = 20$

$\bar{x} = 20.$

9. Candidates of four schools appear in a mathematics test. The data were as follows:

Schools	No. of candidates	Average score
I	60	75
II	48	80
III	N A	55
IV	40	50

If the average score of the candidates of all the four schools is 66, find the number of candidates that appeared from school III.

Sol:

Let no. of candidates appeared from school $III = x.$

School	No. of candidates	Average score
I	60	75
II	48	80
III	*x*	55
IV	40	50

Given, average score of all school = 66.

$$\Rightarrow \frac{N_1\overline{x}_1 + N_2\overline{x}_2 + N_3\overline{x}_3 + N_4\overline{x}_4}{N_1 + N_2 + N_3 + N_4} = 66$$

$$\Rightarrow \frac{60+75+48+80+x\times55+40\times50}{60+48+x+40} = 66$$

$$\Rightarrow \frac{4500+3840+55x+2000}{148+x} = 66$$

$$\Rightarrow \frac{10340+55x}{148+x} = 66$$

$$\Rightarrow 10340+55x = 66x+9768$$

$$\Rightarrow 10340-9768 = 66x-55x$$

$$\Rightarrow 11x = 572$$

$$\Rightarrow x = \frac{572}{11} = 52.$$

$\therefore$ No. of candidates appeared from school (3) – 52.

10. Five coins were simultaneously tossed 1000 times and at each toss the number of heads were observed. The number of tosses during which 0, 1, 2, 3, 4 and 5 heads were obtained are shown in the table below. Find the mean number of heads per toss.

No. of heads per toss	No. of tosses
0	38
1	144
2	342
3	287
4	164
5	25
Total	1000

Sol:

No. of heads per toss (*x*)	No. of tosses (f)	*fx*
0	38	0
1	144	144
2	342	684
3	287	861
4	164	656

5	25	125
	N = 100	$\Sigma fx = 2470$

$\therefore$ Mean number of heads per toss $= \frac{\Sigma fx}{N}$

$= \frac{2470}{1000}$

$= 2 \cdot 47.$

11. Find the missing frequencies in the following frequency distribution if its known that the mean of the distribution is 50.

x:	10	30	50	70	90	
f:	17	f_1	32	f_2	19	Total 120

Sol:

x	f	fx
10	17	170
30	f_1	30 f_1
50	32	1600
70	f_2	70 f_2
90	19	1710
	N = 120	$\Sigma fx = 3480 + 30 f_1 + 70 f_2$

It is give that

Mean = 50

$\Rightarrow \frac{\Sigma fx}{N} = 50$

$\Rightarrow \frac{3480 + 30 f_1 + 70 f_2}{N} = 50$

$\Rightarrow 3480 + 30 f_1 + 70 f_2 = 50(120)$

$\Rightarrow 30 f_1 + 70 f_2 = 6000 - 3480$

$\Rightarrow 10(3 f_1 + 7 f_2) = 10(252)$

$\Rightarrow 3 f_1 + 7 f_2 = 252$ .....(1) [$\because$ Divide by 10]

And $N = 120$

$\Rightarrow 17 + f_1 + 32 + f_2 + 19 = 120$

$\Rightarrow 68 + f_1 + f_2 = 120$

$\Rightarrow f_1 + f_2 = 120 - 68$

$\Rightarrow f_1 + f_2 = 52$

Multiply with '3' on both sides

$\Rightarrow 3 f_1 + 3 f_2 = 156$ .....(2)

Subtracting equation (2) from equation (1)

$3f_1 + 7f_2 - 3f_1 - 3f_2 = 252 - 156$

$\Rightarrow 4f_2 = 96$

$\Rightarrow f_2 = \frac{96}{4}$

$\Rightarrow f_2 = 24$

Put value of f_2 in equation (1)

$\Rightarrow 3f_1 + 7 \times 24 = 250$

$\Rightarrow 3f_1 = 252 - 168 - 84$

$\Rightarrow f_1 = \frac{84}{3} = 28.$

Exercise – 24.3

Find the median of the following data (1-8)

1. 83, 37, 70, 29, 45, 63, 41, 70, 34, 54

Sol:

Given numbers are

83, 37, 70, 29, 45, 63, 41, 70, 34, 54

Arrange the numbers is ascending order

29, 34, 37, 41, 45, 54, 63, 70, 70, 83

n = 10 (even)

$$\therefore \text{Median } = \frac{\frac{n}{2}^{th} \ value + \left(\frac{n}{2}+1\right)^{th} \ value}{2}$$

$$= \frac{\frac{10}{2}^{th} \ value + \left(\frac{10}{2}+1\right)^{th} \ value}{2}$$

$$= \frac{5^{th} \ value + 6^{th} value}{2}$$

$$= \frac{45+54}{2} = \frac{99}{2} = 49 \cdot 5$$

2. 133, 73, 89, 108, 94, 1O4, 94, 85, 100, 120

Sol:

Given numbers are 133, 73, 89, 108, 94, 104, 94, 85, 100, 120

Arrange in ascending order

73, 85, 89, 94, 94, 100, 104, 105, 120, 133

$n = 10$ (even)

$$\therefore Median = \frac{\frac{n}{2}^{th} value + \left(\frac{n}{2}+1\right)^{th} value}{2}$$

$$= \frac{\frac{10}{2}^{th} value + \left(\frac{10}{2}+1\right)^{th} value}{2}$$

$$= \frac{5^{th} value + 6^{th} value}{2}$$

$$= \frac{90+104}{2} = 97$$

3. 31 , 38, 27, 28, 36, 25, 35, 40

Sol:

Given numbers are 31, 38, 27, 28, 36, 35, 40

Arranging in increasing order

25, 27, 28, 31, 35, 36, 38, 40

$n = 8$ (even)

$$\therefore Median = \frac{\frac{n}{2}^{th} value + \left(\frac{n}{2}+1\right)^{th} value}{2}$$

$$= \frac{\frac{8}{2}^{th} value + \left(\frac{8}{2}+1\right)^{th} value}{2}$$

$$= \frac{4^{th} value + 5^{th} value}{2} = \frac{31+35}{2}$$

$$= \frac{66}{2} = 33$$

4. 15, 6, 16, 8, 22, 21, 9, 18, 25

Sol:

Given numbers are 15, 6, 16, 8, 22, 21, 9, 18, 25

Arrange in increasing order

6, 8, 9, 15, 16, 18, 21, 22, 25

$n = 9$ (odd)

$$\therefore Median = \left(\frac{n+1}{2}\right)^{th} value = \left(\frac{9+1}{2}\right)^{th} value$$

$= 5^{th}\, value$

$= 16$

5. 41, 43, 127, 99, 71, 92, 71, 58, 57

Sol:

Given numbers are 41, 43, 127, 99, 71, 92, 71, 58, 57

Arrange in increasing order

41, 43, 57, 58, 71, 71, 92, 99, 127

$n = 9$ (odd)

$\therefore Median = \left(\frac{n+1}{2}\right)^{th} value$

$= \left(\frac{9+1}{2}\right)^{th} value$

$= 5^{th}\, value$

$= 71$

6. 25, 34, 31, 23, 22, 26, 35, 29, 20, 32

Sol:

Given number are 25, 34, 31, 23, 22, 26, 35, 29, 20, 32

Arranging in increasing order

20, 22, 23, 25, 26, 29, 31, 32, 34, 35

$n = 10$ (even)

$$\therefore Median = \frac{\frac{n}{2}^{th} value + \left(\frac{n}{2}+1\right)^{th} value}{2}$$

$$= \frac{\frac{10}{2}^{th} value + \left(\frac{10}{2}+1\right)^{th} value}{2}$$

$$= \frac{5^{th} value + 6^{th} value}{2}$$

$$= \frac{26+29}{2} = \frac{55}{2}.$$

7. 12, 17, 3, 14, 5, 8, 7, 15

Sol:

Given numbers are 12, 17, 3, 14, 5, 8, 7, 15

Arranging in increasing order 3, 5, 7, 8, 12, 14, 15, 17

$n = 8$ (even)

$$\therefore Median = \frac{\frac{n}{2}^{th} value + \left(\frac{n}{2}+1\right)^{th} value}{2}$$

$$= \frac{\frac{8}{2}^{th} value + \left(\frac{8}{2}+1\right)^{th} value}{2}$$

$$= \frac{4^{th} value + 5^{th} value}{2}$$

$$= \frac{8+12}{2} = \frac{20}{2}$$

$\therefore$ Median = 10

8. 92, 35, 67, 85, 72, 81, 56, 51, 42, 69

Sol:

Given number are

92, 35, 67, 85, 72, 81, 56, 51, 42, 69

Arranging in increasing order

35, 42, 51, 56, 67, 69, 72, 81, 85, 92

$n = 10$ (even)

$$\therefore Median = \frac{\frac{n}{2}^{th} value + \left(\frac{n}{2}+1\right)^{th} value}{2}$$

$$= \frac{5^{th} value + 6^{th} value}{2}$$

$$= \frac{67+69}{2} = 68.$$

9. Numbers 50, 42, 35, 2x + 10, 2x − 8, 12, 11, 8 are written in descending order and their median is 25, find x.

Sol:

Given number of observation, $n = 8$

$$\text{Median } = \frac{\left(\frac{n}{2}\right)^{th} observation + \left(\frac{n}{2}+1\right)^{th} observation}{2}$$

$$= \frac{2x+10+2x-8}{2}$$

$= 2x+1$

Given median $= 25$

$\therefore 2x+1=25$

$\Rightarrow 2x=24$

$\Rightarrow x=12$

10. Find the median of the following observations : 46, 64, 87, 41, 58, 77, 35, 90, 55, 92, 33. If 92 is replaced by 99 and 41 by 43 in the above data, find the new median?

Sol:

Given numbers are

46, 64, 87, 41, 58, 77, 35, 90, 55, 92, 33

Arrange in increasing order

33, 35, 41, 46, 55, 58, 64, 77, 87, 90, 92

$n=1$ (odd)

$\therefore Median=\left(\frac{n+1}{2}\right)^{th} value$

$=\left(\frac{11+1}{2}\right)^{th} value$

$=6^{th} value=58$

If 92 is replaced by 99 and 41 by 43

Then, the new values are

33, 35, 43, 46, 55, 58, 64, 77, 87, 90, 99

$\therefore n=11$ (odd)

New median $=\left(\frac{n+1}{2}\right)^{th} value$

$=\left(\frac{11+1}{2}\right)^{th} value$

$=6^{th} value$

$=58.$

11. Find the median of the following data : 41, 43, 127, 99, 61, 92, 71, 58, 57 If 58 is replaced by 85, what will be the new median.

Sol:

Given numbers are

41, 43, 127, 99, 61, 92, 71, 58 and 57

Arrange in ascending order

41, 43, 57, 58, 61, 71, 92, 99, 127

$n=9$ (odd)

$\therefore \text{Median } = \left(\frac{n+1}{2}\right)^{th} value$

$= \left(\frac{9+1}{2}\right)^{th} value$

$= 5^{th} value$

$= 61$

If 58 is replaced by 85

Then new values be in order are

41, 43, 57, 61, 71, 85, 92, 99, 27

$n = 9$ (odd)

$\therefore \text{Median } = \left(\frac{n+1}{2}\right)^{th} value$

$= \left(\frac{9+1}{2}\right)^{th} value$

$= 5^{th} value$

$= 71$

12. The weights (in kg) of 15 students are: 31, 35, 27, 29, 32, 43, 37, 41, 34, 28, 36, 44, 45, 42, 30. Find the median. If the weight 44 kg is replaced by 46 kg and 27 kg by 25 kg, find the new median.

Sol:

Given numbers are

31, 35, 27, 29, 32, 43, 37, 41, 34, 28, 36, 44, 45, 42, 30

Arranging increasing order

27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 41, 42, 43, 44, 45

$n = 15$(odd)

$\therefore \text{Median } = \left(\frac{n+1}{2}\right)^{th} value$

$= \left(\frac{15+1}{2}\right)^{th} value$

$= 8^{th} value$

$= 35kg$

If the weight 44kg is replaced by 46 kg and 27 kg is replaced by 25 kg

Then, new values in order be

25, 28, 29, 30, 31, 32, 34, 35, 36, 37, 41, 42, 43, 45, 46

$n = 15$(odd)

$\therefore$ Median $= \left(\frac{n+1}{2}\right)^{th}$ *value*

$= \left(\frac{15+1}{2}\right)^{th}$ *value*

$= 8^{th}$ *value*

$= 35kg$

13. The following observations have been arranged in ascending order. If the median of the data is 63, find the value of x: 29, 32, 48, 50, x, x + 2, 72, 78, 84, 95

Sol:

Total number of observation in the given data is 10 (even number). So median of this data will be mean of $\frac{10}{2}$ i.e., 5^{th} *and* $\frac{10}{2}+1$ i.e., 6^{th} observations.

So, median of data $= \frac{5^{th} observation + 6^{th} observation}{2}$

$\Rightarrow 63 = \frac{x + x + 2}{2}$

$\Rightarrow 63 = \frac{2x + 2}{2}$

$\Rightarrow 63 = x + 1$

$\Rightarrow x = 62$

Exercise – 24.4

1. Find out the mode of the following marks obtained by 15 students in a class:
Marks : 4, 6, 5, 7, 9, 8, 10, 4, 7, 6, 5, 9, 8, 7, 7.

Sol:

Marks	4	5	6	7	8	9	10
No. of students	2	2	2	4	2	2	1

Since, the maximum frequency corresponds to the value 7 then mode = 7 marks.

2. Find the mode from the following data:
125, 175, 225, 125, 225, 175, 325, 125, 375, 225, 125

Sol:

Values	125	175	225	325	375
Frequency	4	2	3	1	1

Since, maximum frequency 4 corresponds value 125 then mode = 125

3. Find the mode for the following series :
7.5, 7.3, 7.2, 7.2, 7.4, 7.7, 7.7,7.5, 7.3, 7.2, 7.6, 7.2
Sol:

Values	$7\cdot2$	$7\cdot3$	$7\cdot4$	$7\cdot5$	$7\cdot6$	$7\cdot7$
Frequency	4	2	1	2	1	2

Since, maximum frequency 4 corresponds to value $7\cdot2$ then mode $=7\cdot2$

4. Find the mode of the following data in each case:
(i) 14, 25, 14, 28, 18, 17, 18, 14, 23, 22, 14, 18
(ii) 7, 9, 12, 13, 7, 12, 15, 7, 12, 7, 25, 18, 7
Sol:
(i) Arranging the data in an ascending order
14, 14, 14, 14, 17, 18, 18, 18, 22, 23, 25, 28
Here observation 14 is having the highest frequency
i.e., 4 in given data, so mode of given data is 14.
(ii)

Values	7	9	12	13	15	18	25
Frequency	5	1	3	1	1	1	1

Since, maximum frequency 5 corresponds to value 7 then the mode = 7

5. The demand of different shirt sizes, as obtained by a survey, is given below:

Size:	38	39	40	41	42	43	44	Total
No of persons (wearing it)	26	39	20	15	13	7	5	125

Find the modal shirt sizes, as observed from the survey.
Sol:

Size	38	39	40	41	42	43	44	Total
No. of persons	26	39	20	15	13	7	5	125

Since, maximum frequency 39 corresponds to value – 39 then mode size = 39.

Exercise – 25.1

1. A coin is tossed 1000 times with the following frequencies:
Head: 455, Tail: 545
Compute the probability for each event.
Sol:
It is given that the coin is tossed 1000 times. The total number of trials is 1000.
Let us denote the event of getting head and of getting tail by E and F respectively. Then number of trails in which the E happens = 455

$$\boxed{So,\ probabilites\ of\ E = \frac{Number\ of\ Event\ head}{Total\ no.of\ trails}}$$

$$i.e.,\ p(E) = \frac{455}{1000} = 0\cdot 455$$

Similarly, the probability of the event getting a

$$\text{Tail } = \frac{Number\ of\ Tails}{Total\ number\ of\ Trials}$$

$$i.e.,\ p(E) = \frac{545}{1000} = 0\cdot 545$$

Note: we note that $P(A) + P(B) = 0\cdot 48 + 0\cdot 52$

Therefore, A and B are the only two opposite outcomes.

2. Two coins are tossed simultaneously 500 times with the following frequencies of different outcomes:
Two heads : 95 times
One tail : 290 times
No head : 115 times
Find the probability of occurrence of each of these events.
Sol:

WKT, $$\boxed{\text{probabilty}(\text{E}) = \frac{\text{Number of trails which the event happens}}{\text{Total number of Trials}}}$$

$$\text{P(getting two heads)} = \frac{95}{500} = 0\cdot 19$$

$$\text{P(getting one tail)} = \frac{290}{500} = 0\cdot 58$$

$$\text{P(getting no head)} = \frac{115}{500} = 0\cdot 23$$

3. Three coins are tossed simultaneously 100 times with the following frequencies of different outcomes:

Outcome: No head One head Two heads Three heads
Frequency: 14 38 36 12

If the three coins are simultaneously tossed again, compute the probability of:

(i) 2 heads coming up. (ii) 3 heads coming up.
(iii) at least one head coming up. (iv) getting more heads than tails.
(v) getting more tails than heads.

Sol:

Out come	No head	One head	Two heads	Three heads
Frequency	14	38	36	12

(i) Probability of 2 heads coming up $= \dfrac{Favorable\ outcome}{Total\ outcome}$

$$= \frac{36}{100} = 0\cdot 36$$

(ii) Probability of 3 heads coming up $= \dfrac{Favorable\ outcome}{Total\ outcome}$

$$= \frac{12}{100}$$
$$= 0\cdot 12$$

(iii) Probability of at least one head coming up $= \dfrac{Favorable\ outcome}{Total\ outcome}$

$$= \frac{38+36+12}{100}$$
$$= \frac{86}{100} = 0\cdot 86$$

(iv) Probability of getting more than heads and tails $= \dfrac{Favorable\ outcome}{Total\ outcome}$

$$= \frac{36+12}{100} = \frac{48}{100} = 0\cdot 48$$

(v) Probability of getting more tails than heads

$$= \frac{14+38}{100} = \frac{52}{100} = 0\cdot 52.$$

4. 1500 families with 2 children were selected randomly and the following data were recorded:

Number of girls in a family: 0 1 2
Number of families: 211 814 475

If a family is chosen at random, compute the probability that it has:

(i) No girl (ii) 1 girl (iii) 2 girls

(iv) at most one girl (v) more girls than boys

Sol:

It is given that

Total number of families = 475 + 814 + 211 = 1500.

(i) No. of families having no girl = 211

Required probability $= \dfrac{No.of\ \ families\ having\ no\ girl}{Total\ number\ of\ \ families}$

$= \dfrac{211}{1500} = 0\cdot1406$

(ii) Number of families having 1 girl = 814

Required probability $= \dfrac{No.of\ \ families\ having\ one\ girl}{Total\ number\ of\ \ families}$

$= \dfrac{814}{1500} = \dfrac{407}{750} = 0\cdot5426$

(iii) Number of families having 2 girl = 475

Required probability $= \dfrac{No.of\ \ families\ having\ 2\ girl}{Total\ number\ of\ \ families}$

$= \dfrac{475}{1500} = 0\cdot3166.$

(iv) Number of families having at the most one girl = 211 + 814 = 1025.

Required probability $= \dfrac{\text{Number of families having atmost one girl}}{Total\ number\ of\ \ families}$

$= \dfrac{1025}{1500} = 0\cdot6833$

(v) Probability of families having more girls than boys

$= \dfrac{Number\ of\ \ families\ having\ more\ girls\ than\ Boys}{Total\ number\ of\ \ families}$

$= \dfrac{475}{1500} = 0\cdot31$

5. In a cricket match, a batsman hits a boundary 6 times out of 30 balls he plays. Find the probability that on a ball played:

(i) he hits boundary (ii) he does not hit a boundary.

Sol:

Number of times batsman hits a boundary = 6

Total number of balls played = 30.

$\therefore$ Number of times that batsman does not hit a boundary $= 30 - 6 = 24.$

(i) Probability (he hits a boundary) $= \dfrac{No \cdot of\ \ times\ he\ hits\, boundary}{Total\ \ No \cdot of\ balls\ \ played}$

$= \dfrac{6}{30} = \dfrac{1}{5}$

(ii) P (he does not hits a boundary) $= \dfrac{No \cdot of\ \ times\ he\ hits\, boundary}{Total\ \ No \cdot of\ balls\ \ played}$

$= \dfrac{24}{30} = \dfrac{4}{5}$

6. The percentage of marks obtained by a student in monthly unit tests are given below:

Unit test:	I	II	III	IV	V
Percentage of marks obtained:	69	71	73	68	76

Find the probability that the student gets: (i) more than 70% marks (ii) less than 70% marks (iii) a distinction.

Sol:

(i) Let E be the event of getting more than 70% marks
The number of times E happens is 3

$\therefore P\left(A\right) = \dfrac{3}{5} = 0 \cdot 5$

(ii) Let F be the event of getting less than 70% marks
The number of times B happens is 2

$\therefore P\left(B\right) = \dfrac{2}{5} = 0 \cdot 4$

(iii) Let G be the event of getting a distinction
The number of G happens is 1.

$\therefore P\left(C\right) = \dfrac{1}{5} = 0 \cdot 2.$

7. To know the opinion of the students about Mathematics, a survey of 200 students was conducted. The data is recorded in the following table:

Opinion:	Like	Dislike
Number of students:	135	65

Find the probability that a student chosen at random (i) likes Mathematics (ii) does not like it.

Sol:

Opinion	Like	Dislike
No. of students	135	65

(i) Probability that a student likes mathematics

$$= \frac{Favorable\ outcome}{Total\ outcome} = \frac{135}{200}$$

$$= \frac{135}{200} = 0 \cdot 675$$

(ii) Probability that a student does not like mathematics

$$= \frac{Favorable\ outcome}{Total\ outcome} = \frac{65}{200} = 0 \cdot 325$$

8. The blood groups of 30 students of class IX are recorded as follows:
A, B, O, O, AB, O, A, O, B, A, O, B, A, O, O,
A, AB, O, A, A, O, O, AB, B, A, O, B, A, B, O,
A student is selected at random from the class from blood donation. Find the probability that the blood group of the student chosen is:
(i) A (ii) B (iii) AB (iv) O

Sol:

Blood group	A	B	0	AB	Total
No. of students	9	6	12	3	30

(i) Probability of a student of blood group $A = \frac{Favorable\ outcome}{Total\ outcome}$

$$= \frac{9}{30} = 0 \cdot 3$$

(ii) Probability of a student of blood group $B = \frac{Favorable\ outcome}{Total\ outcome}$

$$= \frac{6}{30} = 0 \cdot 2.$$

(iii) The probability of a student of blood group

$$AB = \frac{Favorable\ outcome}{Total\ outcome} = \frac{3}{30}$$

$$= 0 \cdot 1$$

(iv) The probability of a student of blood group

$$0 = \frac{Favorable\ outcome}{Total\ outcome} = \frac{12}{30} = 0 \cdot 4.$$

9. Eleven bags of wheat flour, each marked 5 kg, actually contained the following weights of flour (in kg):
4.97, 5.05, 5.08, 5.03, 5.00, 5.06, 5.08, 4.98, 5.04, 5.07, 5.00
Find the probability that any of these bags chosen at random contains more than 5 kg of flour.

Sol:

It is given that

Eleven bags of wheat flour, each marked 5kg, actually contained the following weights

$4\cdot 97, 5\cdot 05, 5\cdot 08, 5\cdot 03, 5\cdot 00, 5\cdot 06, 5\cdot 08, 4\cdot 98, 5\cdot 04, 5\cdot 07, 5\cdot 00$

Probability (Bag having more than 5kg of flour)

$$= \frac{No\cdot of\ bags\ having\ more\ than\ 5kg}{Total\ no\cdot oof\ bags}$$

$$= \frac{7}{11}.$$

10. Following table shows the birth month of 40 students of class IX.

Jan	Feb	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec
3	4	2	2	5	1	2	5	3	4	4	4

Find the probability that a student was born in August.

Sol:

The birth month of 40 students of class 1X

Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
3	4	2	2	5	1	2	6	3	4	4	4

Probability (Students was born in August) $= \frac{Favorable\ outcome}{Total\ outcome}$

$$= \frac{6}{40} = \frac{3}{20}.$$

11. Given below is the frequency distribution table regarding the concentration of sulphur dioxide in the air in parts per million of a certain city for 30 days.

Conc. of $S0_2$	0.00-0.04	0.04-0.08	0.08-0.12	0.12-0.16	0.16-0.20	0.20-0.24
No. of days	4	8	9	2	4	3

Find the probability of concentration of sulphur dioxide in the interval 0.12-0.16 on any of these days.

Sol:

Given that

The frequency distribution table regarding the concentration of sulphurdioxide in the air in parts per million of a certain city for 30 days is as follows

Conc of SO_2	$0\cdot 00-0\cdot 04$	$0\cdot 04-0\cdot 88$	$0\cdot 88-0\cdot 12$	$0\cdot 12-0\cdot 16$	$0\cdot 16-0\cdot 20$	$0\cdot 20-0\cdot 24$
No. of days	4	8	9	2	4	2

Total number of days = 30.

Probability of concentration of SO_2 in the internal $0\cdot 12-0\cdot 16$ is

$$= \frac{Favourable\ outcome}{Total\ outcome} = \frac{2}{30} = 0\cdot 06.$$

12. Companies selected 2400 families at random and survey them to determine a relationship between income level and the number of vehicles in a home. The information gathered is listed in the table below:

Monthly income (in Rs)	Vehicles per family			
	0	1	2	Above2
Less than 7000	10	160	25	0
7000-10000	0	305	27	2
10000-13000	1	535	29	1
13000-16000	2	469	29	25
16000 or more	1	579	82	88

If a family is chosen, find the probability that the family is:

(i) earning Rs 10000- 13000 per month and owning exactly 2 vehicles.

(ii) earning Rs 16000 or more per month and owning exactly 1 vehicle.

(iii) earning less than Rs 7000 per month and does not own any vehicle.

(iv) earning Rs 13000-16000 per month and owning more than 2 vehicle.

(v) owning not more than 1 vehicle

(vi) owning at least one vehicle.

Sol:

(i) The probability that the family is earning Rs 10000-13000 per month and owning exactly 2 vehicles

$$= \frac{Favorable\ outcomes}{Total\ outcome} = \frac{29}{2400}$$

(ii) The probability that the family is earning Rs 16000 or more per month and owning exactly one vehicle $= \frac{Favorable\ outcomes}{Total\ outcome} = \frac{579}{2400}$

(iii) The probability that the family is earning less than Rs 7,000 per month and does not own any vehicle $= \frac{Favorable\ outcomes}{Total\ outcome}$

$$= \frac{10}{2400} = \frac{1}{240}$$

(iv) The probability that thee family is earning Rs 13,000 – 16,000 per month and owning more than 2 vehicle $= \frac{Favorable\ outcomes}{Total\ outcome} = \frac{25}{2400} = \frac{1}{96}$

(v) The probability that the family is owning not more Than 1 vehicle

$$= \frac{Favorable\ outcomes}{Total\ outcome} = \frac{10+0+1+2+1+160+305+535+469+579}{2400}$$

$$= \frac{2062}{2400} = \frac{1031}{1200}$$

(vi) The probability that the family is owning at-least one vehicle

$$= \frac{Favorable\ outcomes}{Total\ outcome}$$

$$= \frac{160+305+535+469+579+25+27+29+29+82+0+2+1+25+88}{2400}$$

$$= \frac{2356}{2400} = \frac{589}{600}$$

13. The following table gives the life time of 400 neon lamps:

Life time (in hours)	300-400	400-500	500-600	600-700	700-800	800-900	900-1000
No. of lamps	14	56	60	86	74	62	48

A bulb is selected at random. Find the probability that the life time of the selected bulb is:

(i) less than 400

(ii) between 300 to 800 hours

(iii) at least 700 hours.

Sol:

Life time	300-400	400-500	500-600	600-700	700-800	800-900	900-1000
(in hours)	14	56	60	86	74	62	48

(i) The probability that the life time of the selected bulb is less than 400

$$= \frac{Favorable\ outcomes}{Total\ outcome}$$

$$= \frac{14}{400} = \frac{7}{200}$$

(ii) The probability that the life time of the selected bulb is between 300 – 800 hours

$$= \frac{Favorable\ outcomes}{Total\ outcome}$$

$$= \frac{14+56+60+86+74}{400}$$

$$= \frac{290}{400} = \frac{29}{40}$$

(iii) The probability that the life time of the selected bulb is at least 700 hours

$$= \frac{Favorable\ outcomes}{Total\ outcome} = \frac{74+62+48}{400} = \frac{184}{400} = \frac{23}{50}$$

14. Given below is the frequency distribution of wages (in Rs) of 30 workers in a certain factory:

Wages (in Rs)	110-130	130-150	150-170	170-190	190-210	210-230	230-250
No. of workers	3	4	5	6	5	4	3

A worker is selected at random. Find the probability that his wages are:

(i) less than Rs 150

(ii) at least Rs 210

(iii) more than or equal to 150 but less than Rs 210.

Sol:

Wages (in Rs)	110-130	130-150	150-170	170-190	190-210	210-230	230-250
No. of workers	3	4	5	6	5	4	3

Total no. of workers = 30.

(i) The probability that his wages are less than Rs 150 $= \frac{Favorable\ outcomes}{Total\ outcome}$

$$= \frac{3+4}{30} = \frac{7}{30}$$

(ii) The probability that his wages are at least Rs 210 $= \frac{Favorable\ outcomes}{Total\ outcome}$

$$= \frac{4+3}{30} = \frac{7}{30}.$$

The probability that his wages are more than or equal to 150 but less than Rs 200 $= \frac{Favorable\ outcomes}{Total\ outcome} = \frac{5+6+5}{30} = \frac{16}{30} = \frac{8}{15}$