Class 10 Chapter Arithmetic Progression

Q1

Find the common difference of an A.P. whose first term is $\frac{1}{2}$ and the 8th term is $\frac{17}{6}$. Also write its 4^{th} term.

$a = \frac{1}{2}$, $t_8 = \frac{17}{6}$

$t_8 = a + 7d$

or $\frac{17}{6} = \frac{1}{2} + 7d$ **1**

$\Rightarrow d = \frac{1}{3}$, $t_4 = a + 3d$

$= \frac{1}{2} + 3.\frac{1}{3} = \frac{3}{2}$ **1**

Q2

Find the sum of the first 23 terms of the A.P : 7, $10\frac{1}{2}$, 14, ...

$a = 7, d = \frac{7}{2}$ ½

$S_{23} = \frac{23}{2}\left[14 + (22)\left(\frac{7}{2}\right)\right]$ (½ Mark for formula)

$= 1046.5$ **1**

Q3

Find the sum of first twelve multiples of 7.

$7+14+21+.........$ It is an A.P.

here $a=7,\ d=7$ **1**

$\therefore\ S_{12}=\frac{12}{2}[14+11\times 7]$

$=6[91]=546$ **1**

Q4

8th term of an A.P. is 37 and its 12th term is 57. Find the A.P.

$a+7d=37$ and $a+11d=57$ **½**

$\Rightarrow 4d=20 \Rightarrow d=5$ **1**

$\therefore\ a+35=37 \Rightarrow a=2$

$\therefore$ A.P. is 2, 7, 12, ------- **½**

Q5

Which term of the A.P. 3, 15, 27, 39,....... is 132 more than its 54^{th} term ?

Here $a=3,\ d=12$

$a_n=a_{54}+132$

$a+(n-1)d=a+53d+132$ **½**

$3+12(n-1)=3+53\times 12+132$

$-9+12n=135+636$ **1**

$12n=780$

$n=780/12=65$ **½**

Q6

Find the sum of the first 50 odd natural numbers.

50 odd natural numbers are 1, 3, 5,....a_n ½

$a_{50} = a + (n-1)d$

$= 1 + 49.2$ ½

$= 99$

$Sn = n/2(a + 99)$

$= 50/2\ (100)$ **1**

$= 2500$

Q7

Find the 12th term of the A.P. $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}$, ...

$a = \sqrt{2}$

$d = 3\sqrt{2} - \sqrt{2} = 2\sqrt{2}$

$a_{12} = a + 11d = \sqrt{2} + 11\left(2\sqrt{2}\right)$ ½

$= \sqrt{2} + 22\sqrt{2}$

$= 23\sqrt{2}$ **1**

12^{th} term $= 23\sqrt{2}$ ½

Q8

Find the 10th term from the end of the A.P. 4, 9, 14, ... 254.

AP in reverse order : 254, ..., 14, 9, 4 ½

$a = 254$

$d = 4 - 9 = -5$ ½

$a_{10} = a + (10-1)d$

$= 254 + 9(-5)$ ½

$= 209$

$\therefore$ 10^{th} term from the end of given AP = 209 ½

Q9

09. If S_n denotes the sum of n terms of an AP whose common difference is d and first term is *a*, find $S_n - 2S_{n-1} + S_{n-2}$.

$T_n = S_n - S_{n-1}$ ½

$T_{n-1} = S_{n-1} - S_{n-2}$ ½

$S_n - 2S_{n-1} + S_{n-2} = S_n - S_{n-1} - S_{n-1} + S_{n-2}$

$= (S_n - S_{n-1}) - (S_{n-1} - S_{n-2})$ ½

$= T_n - T_{n-1} = d$ ½

Q10

Find the sum of the first 25 terms of an AP whose n^{th} term is given by $t_n = 7 - 3n$

$t_n = 7 - 3n$

$a = t_1 = 7 - 3 = 4,\ t_2 = 7 - 6 = 1 \Rightarrow d = -3$ ½

$t_{25} = 7 - 3\ (25) = -68$ ½

$S_{25} = \frac{25}{2}[4 - 68] = -800$ **1**

Q11

How many terms are there in A.P. ?

7, 16, 25, ..., 349 ?

$a = 7,\ d = 16 - 7 = 9,\ a_n = 349$

$a_n = 349 = a + (n-1)d$ **1**

$349 = 7 + (n-1)9$

$\frac{349 - 7 + 9}{9} = n \quad \Rightarrow \quad n = \frac{351}{9} = 39$

$= 39$ terms **1**

Q12

12. Find the number of terms of the series :

$-5+(-8)+(-11)+\ldots\ldots\ldots\ldots+(-230)$

$-5+(-8)+(-11)+\ldots\ldots\ldots+(-230)$

$a=-5$

$d=-3$

$a_n=-230$

$-230=a+(n-1)d$ 1

$-230=-5+(n-1)(-3)$

$-230=-5-3n+3$

$-228=-3n$

$n=76$ 1

Q13

Find the number of all 2 digit numbers divisible by 3.

12, 15, 18,..........99

$a=12$

$d=3$

$a_n=99$ ½

$a_n=a+(n-1)d$ ½

$99=12+(n-1)3$

$99=12+3n-3$

$90=3n$ ½

$n=30$ ½

Q14

Find the value of p, if the numbers x, $2x+p$, $3x+6$ are three consecutive terms of an A.P..

$2x+p-x=3x+6-2x-p$ 1

$\cancel{x}+p=\cancel{x}+6-p \Rightarrow 2p=6 \Rightarrow p=3$ 1

Q15

15. If 6^{th} term of an A.P. is -10 and its 10^{th} term is -26, then find the 15^{th} term of the A.P.

$a+5d=-10$

$a+9d=-26$

$-4d=16 \quad d=-4$ **1**

$a-20=-10 \quad \Rightarrow a=10$

$a_{15}=a+14d=10+14(-4)$

$=10-56=-46$ **1**

Q16

Which term of an AP 21, 18, 15, ... is zero ?

$a=21,\ d=-3$ **1**

$a_n=0 \qquad a_n=0=21+(n-1)(-3)$

$\Rightarrow \quad n=8$ **1**

Q17

If 8th term of an A.P. is 31 and 15^{th} term is 16 more than 11^{th} term, find the A.P.

$a_8=a+7d=31$

$a_{15}=a+14d=16+a+10d$

$4d=16 \Rightarrow d=4$ **1**

$\therefore \quad a+7\times 4=31$

$a=3$ ½

A.P $\Rightarrow$ 3, 7, 11, 15, ... ½

Q18

For what value of p are 2p, p+10 and 3p+2 in A.P. ?

$(p+10)-2p=(3p+2)-(p+10)$ **1**

Or $p=6$ **1**

Q19

19. Calculate how many multiples of 7 are there between 100 and 300.

105, 112, ... 294 $a=105$, $an=294$ **1**

$294=105+(n-1)7$

or $n=28$ **1**

Q20

Find the sum of first 10 terms of the sequence $\{a_n\}$ where $a_n=5-6n$, where n is a natural number.

$a=5-6=-1$ ½

$d=(5-12)-(-1)=-6$ ½

$S_{10}=5[-2+9(-6)]=5[-56]=-280$ **1**

Q21

Which term of the arithmetic progression 3, 10, 17 ...... will be 84 more than its 13th term.

$a=3 \quad d=7$ ½

$a_n=a_{13}+84$ ½

$a+(n-1)d=a+(13-1)d+84$ ½

$3+(n-1)7=3+12\times 7+84$

$n=25$ ½

Q22

If the n^{th} term of an A.P. is $(2n+1)$, find the sum of first n terms of the A.P.

$t_n=2n+1 \Rightarrow t_1=3,\ t_2=5$ ½

$\therefore\ a=3 \quad d=2$ ½

$Sn=\frac{n}{2}\ [2a+(n-1)d]$ ½

$\therefore\ Sn=2n+n^2$ ½

Q23

23 Which term of the AP : 6, 13, 20, 27, ... is 98 more than its 24^{th} term ?

A.P. is 6, 13, 20

$a = 6,\ d = 7$ **1**

$a_{24} = a + 23d$

$= 6 + 23 \times 7$

$= 6 + 161$

$= 167$

$a_{24} + 98 =$

$167 + 98 = 265$

$\therefore\ 265 = 6 + (n-1) \times 7$

$\frac{259}{7} = n - 1$ **1**

$\Rightarrow\ n = 38$

Q24

In an A.P. the first term is -4, the last term is 29 and the sum of all its terms is 150. Find the common difference of the A.P.

$a = -4$

$a_n = 29$ $\quad 29 = a + (n-1)d$

$S_n = 150$ $\quad 29 = -4 + (n-1)d$

$33 = (n-1)d$ ½

$\frac{33}{n-1} = d$

$150 = \frac{n}{2}(a + l)$

$300 = n(-4 + 29)$ **1**

$\frac{300}{25} = n$

$n = \frac{300}{25} = 12$

$d = \frac{33}{12-1} = \frac{33}{11} = 3$ ½

Q25

25. For an A.P. show that $a_p + a_{p+2q} = 2\,a_{p+q}$

$$\left.\begin{aligned} a_p + a_{p+2q} &= a + (p-1)d + a + (p+2q-1)d \\ &= a + pd - d + a + pd + 2qd - d \\ &= 2a + 2pd + 2qd - 2d \\ &= 2[a + (p+q-1)d] \quad - \text{(i)} \end{aligned}\right]$$ **1**

$2a_{p+q} = 2[a + (p+q-1)d] \ -$ (ii) **½**

From (i) and (ii)

$a_p + a_{p+2q} = 2a_{p+q}$ **½**

Q26

Find the 20[th] term from the last term (end) of the AP : 3, 8, 13 ....253.

20[th] term from the end $= l - (n-1)d$. $\quad l = 253,\ a = 3,\ d = 5$ 1

20[th] term from the last term is 158 1

(3 MARKS)

Q27

27. How many terms of the A.P. 9, 17, 25, ... , must be taken to get a sum of 450 ?

$a = 9,\ d = 8,\ s_n = 450$ ½

$s_n = \frac{n}{2}[2a + (n-1)d]$

$450 = \frac{n}{2}[18 + (n-1)(8)]$

$450 = 4n^2 + 5n$ **1**

or $4n^2 + 5n - 450 = 0$

$4n^2 + 45n - 40n - 450 = 0$

or $4n^2 - 40n + 45n - 450 = 0$

$4n(n-10) + 45(n-10) = 0$

or $n = -\frac{45}{4}$ or $n = 10$ **1**

Rejecting $n = -\frac{45}{4}$ as number of terms can not be negative.

$\therefore n = 10$

Ten terms of the given A.P. will make sum as 450. ½

Q28

Determine 'a' so that $2a+1$, a^2+a+1 and $3a^2-3a+3$ are consecutive terms of an A.P.

For terms to be in A.P.

$a^2 + a + 1 - (2a+1) = 3a^2 - 3a + 3 - (a^2 + a + 1)$ **1**

or $a^2 - 3a + 2 = 0$ **1**

$\Rightarrow$ $a = 1$ or $a = 2$ **1**

Q29

Which term of the A.P. 3, 15, 27, 39, ... will be 132 more than its 60th term ?

$a_{60} = 3 + (60-1)12 = 3 + 708 = 711$ ½

132 more than $711 = 843$ ½+½

Let 843 be n^{th} term

$\therefore 843 = 3 + (n-1)\ 12 \Rightarrow n = 71$ **1+½**

Q30

30. Find three numbers in A.P. whose sum is 15 and the product is 80.

Let the numbers be $a-d$, a, $a+d$ **1**

$\therefore \quad 3a = 15 \Rightarrow a = 5$

$5\,(5-d)\,(5+d) = 80$

$25 - d^2 = 16 \Rightarrow d = \pm 3$ **1**

$\therefore$ the numbers are 2, 5, 8 **1**

or 8, 5, 2

Q31

Find the sum of all three digit numbers which leave the same remainder 2 when divided by 5.

The required A.P. is 102, 107, ...997 which leave the same remainder 2 when divided by 5, $a = 102$, $d = 5$, $a_n = 997$ **½**

$\Rightarrow a + (n-1)d = 997$

$102 + (n-1)5 = 997$ **1½**

$900 = 5n, \; n = 900/5 = 180$

$S_n = n/2[a + l] = 180/2 \times [102 + 997] = 90 \times 1099 = 98910$ **1**

Q32

32. Sum of the first n terms of an A.P. is $5n^2-3n$. Find the A.P. and also find its 16^{th} term.

Sum of the n terms of an A.P. is $5n^2-3n$. Find the terms of the A.P. and also find the 16^{th} term

$S_n = 5n^2 - 3n \Rightarrow a_n = S_n - S_{n-1}$ ½

$a_n = 5n^2 - 3n - [5\ (n-1)^2 - 3\ (n-1)]$

$a_n = 5n^2 - 3n - [5(n^2 - 2n + 1)\ - 3n + 3]$

$a_n = 5n^2 - 3n - 5n^2 + 10n - 5 + 3n - 3$ **1**

$a_n = 10\ n - 8$ ½

$a_1 = 2$

$a_2 = 20 - 8 = 12$ ½

$a_3 = 30 - 8 = 22$

$a_{16} = 2 + 150 = 152$ ½

Q33

How many terms of the A.P. 78, 71, 64, ... are needed to give the sum 465 ? Also find the last term of this A.P.

$a = 78,\ d = 71 - 78 = -7$ ½

Let n be the required no. of terms

$S_n = 465$

$\Rightarrow \frac{n}{2}\left[2a + (n-1)d\right] = 465$ ½

$\Rightarrow n[\ 156 + (n-1)(-7)\ ] = 930$

$\Rightarrow 7n^2 - 163n + 930 = 0$

$\Rightarrow 7n^2 - 70n - 93n + 930 = 0$ ½

$\Rightarrow (7n - 93)(n - 10) = 0$

$\Rightarrow n = \frac{93}{7}$ or $n = 10$ ½

Neglect $n = \frac{93}{7}$ as n cannot be a fraction ½

$\therefore$ No. of terms (n) = 10 ½

Q34

34. Find the sum of all two digit odd positive numbers.

11, 13, 15, ..., 99

$a = 11,\ d = 2,\ a_n = a + (n-1)d = 99$ **1**

$\Rightarrow \quad 11 + (n-1)2 = 99 \quad \Rightarrow \quad n = \frac{99 - 11 + 2}{2} = 45$ **1**

$S_{45} = \frac{45}{2}[2(11) + 44(2)]$

$= \frac{45}{2}[22 + 88]$ **1**

$= \frac{45}{2} \times 110 = 45 \times 55 = 2475$

Q35

If the sum of first n terms of an A.P. is $4n^2 - n$, find the 12^{th} term.

$S_n = 4n^2 - n$

$S_1 = 4 - 1 = 3 = a$ ½

$S_2 = 4(2)^2 - 2$

$= 4(4) - 2$

$= 14$ ½

$\Rightarrow \quad 3 + II = 14$

$II = 14 - 3$ ½

$II = 11$

A.P = 3, 11, 19, ... ½

now

$a_{12} = a + 11d$

$= 3 + 11(8)$

$= 3 + 88$

$a_{12} = 91$ **1**

36. The sum of the first n terms of an A.P. is $5n^2-3n$. Find the A.P. and hence find its 12th term.

$S_1=a_1=2$

$S_2=a_1+a_2=20-6=14$ **1**

$2+a_2=14 \quad a_2=12$

2, 12, 22, 32 ______ **½**

$a=2 \quad d=10$ **½**

$a_{12}=a+11d=2+11\ (10)=112$ **1**

Q37

The angles of a triangle are in A.P. The greatest angle is twice the least. Find all angles of the triangle.

Angles in A.P

$a-d,\ a,\ a+d$ **½**

$a+d=2\ (a-d)$ **½**

$a-\not{d}+a+a+\not{d}=180$

$a=60$ **1½**

$60+d=2\ (60-d)$

$2\ d+d=60 \quad 3d=60 \quad d=20$

$40°,\ 60°,\ 80°,$ **½**

Q38

The sum of first three terms of an A.P. is 33. If the product of the first and third term exceeds the second term by 29, find the A.P.

$\frac{3}{2}[2a+2d]=33$

$2a+2d=22$ **1**

$a+d=11 \quad \Rightarrow \quad d=11-a$

$a(a+2d)=a+d+29$

$a^2+2a(11-a)=40 \ \Rightarrow \ a^2+22a-2a^2=40$ **1**

$a^2-22a-40=0$

$a=20,\ 2$

$a=20$ $\qquad a=2$

$d=-9$ $\qquad d=9$ **1**

A.P. = 20, 11, 2,..... or 2, 11, 20,.....

Q. 39. Find the sum of all natural numbers between 250 and 1000 which are exactly divisible by 9.

252, 261, ..., 999 **1**

$999 = 252 + (n-1)9$

$n = 84$ **1**

$S_{84} = 42(252 + 999)$

$= 52542$ **1**

Q40

Which term of the A.P : 3, 15, 27, 39, ... will be 120 more then its 21^{st} term ?

Let $a_n = 120 + a_{21}$ $a = 3,\ d = 12$ 1

$a + (n-1)d = 120 + a + 20d$

$12n - 12 = 120 + 240$ 1

$n = \frac{372}{12} = 31$

$\therefore\ a_{31} = 120 + a_{21}$ 1

Q41

Find the sum of all two-digit odd positive numbers.

All two digit odd positive numbers are 11, 13, 15,............., 99.

Here, $a = 11,\ d = 13 - 11 = 2,\ l = 99$ **1**

Let the number of terms be n

Then $a_n = a + (n-1)d$

$\Rightarrow 99 = 11 + (n-1)2 \quad \Rightarrow 99 = 11 + 2n - 2$ ½

$\Rightarrow 2n = 90 \quad \Rightarrow n = 45$ ½

Now, $S_n = \frac{n}{2}(a + l) = \frac{45}{2}(11 + 99) = 2475$ ½+½

Q.42. If the sum of all the terms of an A.P. 1, 4, 7, 10, ..., x. is 287, find x.

$S_n = 287$

$a = 1,\ d = 3$ **1**

$S_n = \frac{n}{2}[2 + (n - 1)3] = 287$ **1**

or $n = 14$

$\Rightarrow x = 1 + 13 \times 3 = 40$ **1**

Q43

Find the value of the middle most term(s) of the arithmetic progression :

$-11,\ -7,\ -3,\ ....49.$

$a = -11 \quad d = -7 - (-11) = 4 \quad a_n = 49$ ½

$a_n = a + (n-1)d \quad$ i.e. $49 = -11 + (n-1)4$

$n = 16$ **1**

As n is even, there will be two middle terms which are $\left(\frac{16}{2}\right)$th and $\left(\frac{16}{2} + 1\right)$th terms

i.e. 8^{th} and 9^{th} terms ½

$a_8 = -11 + (8-1) \times 4 = 17$ ½

$a_9 = -11 + 8 \times 4 = 21$ ½

Q.44. The sum of first six terms of an arithmetic progression is 42. The ratio of its 10th term to its 30th term is 1 : 3. Find the first and the thirteenth term of the A.P.

Let a be the first term and d be the common diff.

$S_6 = 42$ ½

$\frac{6}{2}(2a + 5d) = 42$ ½

$2a + 5d = 14 \qquad -(1)$

$\frac{a_{10}}{a_{30}} = \frac{1}{3}$ ½

$\frac{a + 9d}{a + 29d} = \frac{1}{3}$

$\frac{2a + 18d}{2a + 58d} = \frac{1}{3}$

$\frac{14 - 5d + 18d}{14 - 5d + 58d} = \frac{1}{3}$ or $\frac{14 + 13d}{14 + 53d} = \frac{1}{3}$ ½

$42 + 39d = 14 + 53d$

$28 = 14d$ ½

$\Rightarrow d = 2$

$2a + 5d = 14 \Rightarrow a = 2$

$a_{13} = 2 + 12 \times 2 = 26$ ½

Q.45 Find the sum of all the two-digit natural numbers which are divisible by 4.

The two digit numbers

divisible by 4 are

12, 16, 20, ... 96, ½

$\therefore\ a = 12\ ;\ d = 4.$

$t_n = a + (n-1)\ d.$ ½

But $t_n = 96$

i.e $96 = 12 + (n-1)\ 4$

$4n = 88 \Rightarrow n = 22$ **1**

$S_n = \frac{n}{2}[2a + (n-1)\ d]$

$= \frac{22}{2}\ [2 \times 12 + (22-1)\ 4]$

$= 11 \times 108 = 1188$ **1**

Q46

The 4^{th} term of an AP is equal to 3 times the first term and the 7^{th} term exceeds twice the 3^{rd} term by 1. Find the first term and the common difference.

AP be a, a + d, a + 2d ......

$a_4 = 3a \quad a_4 = 2a_3 + 1$

$a + 3d = 3a$ i.e. : $d = \frac{2}{3}\ a$ ____ (1)

$a + 6d = 2\ (a + 2d) + 1$

$a - 2d + 1 = 0$ ____ (2) **1**

from (1) and (2)

$a - 2 \times \left(\frac{2}{3}a\right) + 1 = 0$ **1½**

$\Rightarrow \quad 3a - 4a + 3 = 0$

$a = 3$

$d = \frac{2}{3} \times 3 = 2$ ½

Q.47. The sum of the 5^{th} and 7^{th} terms of an AP is 52 and its 10^{th} term is 46. Find the AP.

Let a be the first term and d be the common diff. 1

$T_5 + T_7 = 52$ 1

$T_{10} = 46$

$a + 4d + a + 6d = 52$

$\Rightarrow a + 5d = 26$

$a + 9d = 46$ 1

On solving $a = 1$, $d = 5$

(4 MARKS)

Q48

In November 2009, the number of visitors to a zoo increased daily by 20. If a total of 12300 people visited the zoo in that month, find the number of visitors on 1^{st} Nov. 2009.

Let number of visitors in zoo on 1^{st} November be x Then the daily visitors in November in the zoo are :

$x,\ x + 20, \ldots$ **1**

Total no. of visitors in Nov. $= 12300$

$\therefore S_{30} = 12300$ **1½**

$$S_{30} = \frac{30}{2}[2x + (29)20]$$

$12300 = 15\ (2x + 29 \times 20)$

$= 30x + 8700$ **1**

$x = 120.$

$\therefore$ Visitors on 1^{st} Nov.2009 $= 120.$ **½**

Q.49. For what value of n, the n^{th} terms of the A.P. 63, 65, 67, ... and 3, 10, 17, ... are equal ? Also find that term.

For A.P 63, 65, 67, ...

$a = 63, \ d = 2$

$a_n = 63 + (n-1)2$

$= 61 + 2n \quad (1) \quad$ **1**

For A.P. 3, 10, 17, ...

$b_n = 3 + (n-1)(7)$

$= -4 + 7n \quad (2) \quad$ **1**

From (1) and (2)

$61 + 2n = -4 + 7n$

or $\quad n = 13 \quad$ **1**

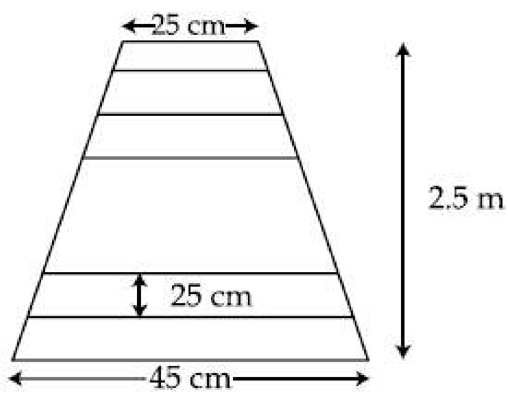
$\therefore \quad 13^{th}$ term of both A.P. will be same

$a_{13} = 61 + 2(13) = 87$

$b_{13} = -4 + 7(13) = 87 \quad$ **1**

Q50

A ladder has rungs 25 cm apart. The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top (see figure). If the top and bottom rungs are 2.5 m apart, what is the length of the wood required for the rungs ?



Gap between two consecutive rungs = 25 cm

Number of rungs = $\frac{250}{25} + 1 = 11$ **1**

(as total distance between top and bottom rung is 2.5 m = 250 cm)
Rungs are decreasing uniformly in length from 45 cm at bottom to 25 cm at the top.

$\therefore$ A.P. will form with **1**
$a = 45,\ a_{11} = 25$

length of the wood required = $S_{11} = \frac{11}{2}(45+25)$
= 385 cm
= 3.8 m **2**