Arithmetic Progression-X Solved Problems

A ladder has rungs 25 cm apart. The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top .If the top and the bottom rungs are two and a half meter apart, what is the length of the wood required for the rungs?

Let the first term, a = 0 and d= common difference =25 cm=0.25m.
When the rungs are measured from top to bottom , then the
0, 0.25, 0.50, 0.75, 1.0. 1.25, 1.50, 1.75, 2.00, 2.25 and 2.50 meters.
⇒The number of rungs is eleven.

Now,

Rung length decreases from 45 at the bottom to 25 at the top, so rung lengths are:
45, 43, 41, 39, 37, 35, 33, 31, 29, 27 and 25 cm.
If the top and the bottom rungs are two and a half meter apart, then
The length of the wood required for the rungs= 11 x 35 = 385 cm.

The sum of first n terms of an AP is given by $S_n = 3n^2 + 5n$ find the n^{th} term of the AP.

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the sum of n terms $S_n = 3n^2 + 5n$
therefore the sum of (n-1) terms

$$S_{n-1} = 3(n-1)^2 + 5(n-1)$$

$$S_{n-1} = 3(n^2 - 2n + 1) + 5n - 5$$

$$S_{n-1} = 3n^2 - n - 2$$

the nth term = sum of n terms - sum of (n-1) terms

$$t_n = S_n - S_{n-1} = 3n^2 + 5n - (3n^2 - n - 2)$$

$$t_n = 6n + 2$$

Hence the nth term is 6n+2.

How many terms of the ap -6,-11/2 , -5,...... are needed to give the sum -25

First term,(a)=-6

Common difference, $(d) = \frac{1}{2}$

We have to find the number of terms(n) in the series such that the summation of the series is -25.

Thus,

$$\frac{n}{2}(2a + (n-1)d) = -25$$

$$\frac{n}{2}\left(-12 + \frac{(n-1)}{2}\right) = -25$$

$$n^2 - 25n + 100 = 0$$

$$(n - 20)(n - 5) = 0$$

$$n = 20, 5$$

Putting the values we observe that only n=5 satisfy the given condition as summation of series till n=20 becomes 25 but we need summation of series to be -25 which is only possible when **n=5.**

Find a, b such that 27, a, b - 6 are in A.P.

If t_1, t_2, t_3, t_4 are in A.P., then, $t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = d$ where d is a constant.

$\therefore (a-27) = b-a$ ....(1)
Also, $b-a = -6-b$
Adding the above equations,
$b-27 = -6-a$ $\qquad a+b = 27-6 = 21$
That is: $21 = a+b$ ....(2) from: (1) - (2) $\Rightarrow$
$a-48 = (b-a)-(a+b)$ $\qquad a-48 = -2a \quad 3a = 48 \quad a = \frac{48}{3}$
$a = 16$ ....(3) $\qquad$ Using (3) in (2), $21 = 16+b \qquad \therefore b = 5$
Thus, $d = b-a = 5-16 = -11$ Thus the A.P is: $27, 16, 5, -6$

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find the sum of all the odd numbers between 50 and 150 divisible by 7

The least number ≥ 50 such that it is divisible by 7 is 56.
The highest number ≤ 50 such that it is divisible by 7 is 147.
Sum of all numbers $\{n : 50 \leq n \geq 50, n \% 7 = 0\}$
$= (56+63+70+\ldots+147)$
$= 7 \times (8+9+10+\ldots+21)$
Let $S_1 = 7 \times (8+9+10+\ldots+21)$
$= 7 \times [(1+2+3+\ldots+21) - (1+2+3+\ldots+7)]$
Since $1+2+3+\ldots+n = \frac{n(n+1)}{2}$,
$S_1 = 7 \times [\frac{21 \times 22}{2} + \frac{7 \times 8}{2}] \quad = 7 \times 7(\frac{3 \times 22}{2} + \frac{8}{2})$
$= 7 \times 7 \times (3 \times 11 + 4)$
$= 49 \times 37 = 1813$
Consider even numbers between 50 and 150 which are divisible by 7:
$56, 70, 84, \ldots, 140$
Let $S_2 = (56+70+84+\ldots+140)$
$= 14 \times (4+5+\ldots+10)$
$= 14 \times [(1+2+3+\ldots+10) - (1+2+3)]$
$= 14 \times [\frac{10 \times 11}{2} + \frac{3 \times 4}{2}] \quad = 14 \times (55+6) = 14 \times 61 = 854$
Hence sum of odd numbers between 50 and 150 is equal to $S_1 - S_2 = 1813 - 854 = 959$

The sum of third and seventh term of an AP is 6 and their product is 8.

Let the Fifth term be p, and the difference between each terms of the arithmetic progression be d.
Then the third, fourth, fifth, sixth and seventh terms of the AP would be :
$p-2d, p-d, p, p+d, p+2d$
Third and seventh terms of the AP are respectively :-
$p-2d, p+2d$
Since sum of third and seventh term is 6,
$(p-2d)+(p+2d) = 6$
$2p = 6$
$p = 3$ www.jsuniltutorial.weebly.com/
Since product of the third and seventh term is 8,
$(p-2d) \times (p+2d) = 8$
$(3-2d) \times (3+2d) = 8$

$9-4d^2=8$

$9-8=4d^2$

$4d^2=1$

$d^2=\frac{1}{4}$

$d=\frac{1}{2}$

First term of the AP would be $2d$ less than the third term.
Hence first term of the AP a would be : $(p-2d)-2d=p-4d$

$=3-(4\times\frac{1}{2})$

$=3-2$

$=1$

Hence, $a=1, d=\frac{1}{2}$

Sum of n terms of an AP is

$S_n=\frac{n}{2}\times[2a+(n-1)d]$

Hence sum of first 16 terms of the AP is:

$=\frac{16}{2}\times[(2\times1)+(16-1)\times(\frac{1}{2})]$

$=8\times[2+\frac{15}{2}]=8\times[\frac{19}{2}]$

$=76$

Sum of first 16 terms of the given arithmetic progression is equal to 76.

If $S_n = n^2p$ and $S_m = m^2p$, (m not equal ton), is an A.P. prove that $S_p = p^3$.

Let first term = a and
Common difference = d
∴According to the question,
$S_n = n^2 p$

$\Rightarrow \frac{n}{2}[2a+(n-1)d]=n^2p$ $\left[\because S_n=\frac{n}{2}[2a+(n-1)d]\right]$

$\Rightarrow [2a+(n-1)d]=2np$...........(i)

and $S_m = m^2 p$

$\Rightarrow \frac{m}{2}[2a+(m-1)d]=m^2p$

$\Rightarrow [2a+(m-1)d]=2mp$...........(ii)

Now, on subtracting (i) and (ii) , we get,

$\Rightarrow 2a+(n-1)d-2a-(m-1)d=2np-2mp$

$\Rightarrow (n-1)d(m-1)d=2(n-m)p$ $\Rightarrow (n-1-m+1)d=2(n-m)p$

$\Rightarrow (n-m)d=2(n-m)p$ $\Rightarrow d=2p$

Put the value of d = 2p in (i), we get,

$\Rightarrow [2a+(n-1)2p]=2np$ $\Rightarrow 2a+(n-1)2p=2np$

$\Rightarrow a+(n-1)p=np$ $\Rightarrow a=np-np+p$ $\Rightarrow a=p$

$S_p = \frac{p}{2}[2a + (p-1)d]$

Putting the value of $a = p$ *and* $d = 2p$, *we get,*

$S_p = \frac{p}{2}[2p + (p-1)2p] \quad = \frac{p}{2} \times 2p + \frac{p}{2} \times (p-1)2p \quad = p^2 + p^2(p-1)$

$= p^2 + p^3 - p^2 \quad \Rightarrow S_p = p^3$

Show that the sum of (m+n)th term and (m-n)th term of an A.P is equal to twice the mth term.

Let first term = a
and common difference = d of an A.P.
$\therefore T_n = a + (n-1)d$
Now, acc. to the question,

$T_{m+n} + T_{m-n} = a + (m+n-1)d + a + (m-n-1)d$

$= 2a + (m+n-1)d + (m-n-1)d \quad = 2a + (m+n-1+m-n-1)d$

$= 2a + (2m-2)d \quad = 2[a + (m-1)d]$

$\Rightarrow T_{m+n} + T_{m-n} = 2T_m$

If the ratio of the sums of n terms of 2 APs is n+1:3n+1, then find the ratio of the 7th terms of the AP.

Let $a_1, and\ a_2$ be the first terms and $d_1, and\ d_2$ be the common differences of two given A.P.s
Then, the sums of their n terms are $S_A and\ S_B$
$S_A = \frac{n}{2}\left[2a_1 + (n-1)d_1\right]$ *and* $S_B = \frac{n}{2}\left[2a_2 + (n-1)d_2\right]$
The ratio of the sums of n terms of 2 A.P.is $\frac{n+1}{3n+1}$
Therefore,
$\frac{S_A}{S_B} = \frac{n+1}{3n+1}$

$\frac{\frac{n}{2}[2a_1+(n-1)d_1]}{\frac{n}{2}\left[2a_2+(n-1)d_2\right]} = \frac{n+1}{3n+1} \qquad \frac{2a_1+(n-1)d_1}{2a_2+(n-1)d_2} = \frac{n+1}{3n+1} \qquad \frac{a_1+\frac{(n-1)}{2}d_1}{a_2+\frac{(n-1)}{2}d_2} = \frac{n+1}{3n+1}$ ......(1)

Since we need to find the 7th term, substitute $\frac{(n-1)}{2} = 6$ in the above equation (1).
Since $\frac{(n-1)}{2} = 6$
$(n-1) = 12$
$n = 12 + 1$
$n = 13$ www.jsuniltutorial.weebly.com/

Consider the equation (1):

$\frac{a_1+\frac{(n-1)}{2}d_1}{a_2+\frac{(n-1)}{2}d_2} = \frac{n+1}{3n+1} \qquad \frac{a_1+6d_1}{a_2+6d_2} = \frac{13+1}{3(13)+1} \qquad \frac{a_1+6d_1}{a_2+6d_2} = \frac{14}{40} \qquad \frac{a_1+6d_1}{a_2+6d_2} = \frac{7}{20}$

In an A.P. if m^{th} term is n and the n^{th} term is m, where $m \neq n$, find the pth term.

We have $am = a + (m - 1)d = n$, ... (1)
and $an = a + (n - 1)d = m$ ... (2)
Solving (1) and (2), we get
$(m - n)d = n - m$, or $d = -1$, ... (3)
and $a = n + m - 1$ ... (4)
Therefore $ap = a + (p - 1)d$
$= n + m - 1 + (p - 1)(-1) = n + m - p$
Hence, the pth term is $n + m - p$.

Determine the sum of the first 30 terms of the sequence whose nth term is given by tn=2n+9/3

$T_n = (2n + 9)/3$
$T_1 = 11/3$ and $T_{30} = 23$
$S_{30} = (n/2)(a + l) = 15 \times [(11/3) + 23] = 15 \times (80/3) = 400$

The sum of the first four terms of an A.P. is 56. The sum of the last four terms is 112. If its first term is 11, then find the number of terms.

Let the A.P. be $a, a + d, a + 2d, a + 3d, \ldots a + (n - 2)d, a + (n - 1)d$.
Sum of first four terms = $a + (a + d) + (a + 2d) + (a + 3d) = 4a + 6d$
Sum of last four terms = $[a + (n - 4)d] + [a + (n - 3)d] + [a + (n - 2)d] + [a + n - 1)d]$
$= 4a + (4n - 10)d$
According to the given condition,
$4a + 6d = 56$ ’ $4(11) + 6d = 56$ [Since $a = 11$ (given)] $6d = 12$ $d = 2$
$4a + (4n - 10)d = 112$ $4(11) + (4n - 10)2 = 112$ $(4n - 10)2 = 68$ $4n - 10 = 34$ $4n = 44$
$n = 11$ Thus, the number of terms of the A.P. is 11.

The sum of the first and the last terms of an AP is 60,the sum of n terms of the AP is720.what is n?

$a + a_n = 60$ $2a + (n-1)d = 60$ ....(1)
$S_n = 720$ $n/2\ (2a + (n-1)d) = 720$ $n/2 \times 60 = 720$ $30n = 720$ $n = 24$

If pth , qth and rth term of an AP are a,b,c respectively , then show that $(a-b)r + (b-c)p + (c-a)q = 0$

Let A be the first term of the A.P. and D be the common difference of the A.P.
Given that,
a = pth term
Therefore,
$a = A + (p - 1)D$
b = qth term
Therefore,
$b = A + (q - 1)D$
c = rth term
Therefore, $c = A + (r - 1)D$

Consider the expression: $a(q - r) + b(r - p) + c(p - q)$
$LHS = \{A + (p - 1)D\}(q - r) + \{A + (q - 1)D\}(r - p) + \{A + (r - 1)D\}(p - q)$
$= A\{(q - r) + (r - p) + (p - q)\} + D\{(p - 1)(q - r) + (q - 1)(r - p) + (r - 1)(p - q)\}$
$= A \times 0 + D\{p(q - r) + q(r - p) + r(p - q) - (q - r) - (r - p) - (p - q)\}$
$= A \times 0 + D \times 0$
$= 0$