

Class 10 Arithmetic progression CBSE Test Paper - 4

Q.1. Find the sum of the first 40 positive integers divisible by 6.

Ans: $a = 6$, $d = 6$ and $n = 40$

$$S_{40} = \frac{40}{2} [2 \times 6 + 39 \times 6] = 20 [12 + 234] = 20 \times 246 = 4920$$

Q.2. Find the sum of the first 15 multiples of 8.

Ans: $a = 8$, $d = 8$ and $n = 15$

$$S_{15} = \frac{15}{2} [2 \times 8 + 14 \times 8] = 7.5 [16 + 112] = 7.5 \times 128 = 960$$

Q.3. Find the sum of the odd numbers between 0 and 50.

Ans: $a = 1$, $d = 2$ and $n = 25$

$$S_{25} = \frac{25}{2} [2 \times 1 + 24 \times 2] = \frac{25}{2} \times 50 = 625$$

Q.4. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs. 200 for the first day, Rs. 250 for the second day, Rs. 300 for the third day, etc., the penalty for each succeeding day being Rs. 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

Ans: $a = 200$, $d = 50$ and $n = 30$

$$S_{30} = \frac{30}{2} [2 \times 200 + 29 \times 50] = 15 [400 + 1450] = 15 \times 1850 = 27750$$

Q. 5. A sum of Rs. 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs. 20 less than its preceding prize, find the value of each of the prizes.

Ans: Let AP be x , $(x - 20)$, $(x - 40)$, $(x - 60)$, $(x - 80)$, $(x - 100)$, $(x - 120)$

$S_n = 700$ and $n = 7$, $l = (x - 120)$

$$S_n = \frac{n}{2} (a + l) \Rightarrow 700 = 3.5 [x + x - 120]$$

$$\Rightarrow \frac{700}{3.5} + 120 = 2x \Rightarrow 200 + 120 = 2x$$

$$\Rightarrow 320 = 2x \Rightarrow x = 160$$

The AP 160 , 140 , 120 , 100 , 80 , 60 , 40

Q. 6. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of class I will plant 1 tree,

a section of class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?

Ans: 3, 6, 9,36

First term, $a = 3$ Common difference, $d = 6 - 3 = 3$ and $n = 12$

$$S_{12} = 6 [2 \times 3 + 11 \times 3] = 6 \times 39 = 234$$

Q.7. A spiral is made up of successive semicircles, with centers alternately at A and B, starting with centre A, of radii 0.5 cm, 1 cm, 1.5 cm, 2 cm, as shown in the figure. What is the total length of such spiral made up of thirteen consecutive semicircles?

Ans: AP will be 0.5π , π , 1.5π

$a = 0.5\pi$, $d = 0.5\pi$ and $n = 13$

$$S_{13} = 6.5 [2 \times 0.5\pi + 12 \times 0.5\pi] = 6.5[\pi + 6\pi] = 45.5\pi = 45.5 \times 3.17 = 144.235$$

Q.8. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how many rows are the 200 logs placed and how many logs are in the top row?

Ans: AP will be: 20, 19, 18...

$a = 20$, $d = 19 - 20 = -1$, $S_n = 200$

$$200 = \frac{n}{2} [2 \times 20 + (n - 1) (-1)] \Rightarrow 400 = n (40 - n + 1) \Rightarrow 400 = 40n - n^2 + n$$

$$\Rightarrow 400 = 41n - n^2 \Rightarrow n^2 - 41n + 400 = 0 \Rightarrow n^2 - 16n - 25n + 400 = 0$$

$$\Rightarrow n (n - 16) - 25 (n - 16) = 0 \Rightarrow (n - 16) (n - 25) = 0$$

$$\text{if } (n - 16) = 0 \Rightarrow n = 16$$

$$\text{if } n - 25 = 0 \Rightarrow n = 25$$

$$a_{16} = 20 + (16 - 1) (-1) = 5$$

$$a_{25} = 20 + (25 - 1) (-1) = 20 - 24 = -4$$

Clearly, the number of logs in 16 row is 5. However, the number of logs in 25 row is negative, which is not possible.

Therefore, 200 logs can be placed in 16 rows and the number of logs in the 16 row is 5.

Q. 9. In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line.

A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

Ans:

To pick up the first potato, the distance covered = 2×5 m

To pick up the second potato, the distance covered = $2 \times (5+3)$
= 2×8 m

To pick up the third potato, the distance covered = $2 \times (5+6)$
= 2×11

and so on.....

The total distance covered

$$= 2 \times 5 + 2 \times 8 + 2 \times 11 + 2 \times 14 \dots\dots\dots$$

$$= 10 + 2(8 + 11 + 14 + \dots\dots\dots)$$

$$= 10 + 2(\text{Sum of nine terms, where 1st term} = 8, \\ \text{Common Difference} = 3)$$

$$= 10 + 2 \times 9/2(2 \times 8 + 8 \times 3)$$

$$= 10 + 9 \times (16 + 24)$$

$$= 10 + 9 \times 40$$

$$= 10 + 360$$

$$= 370$$

Therefore the total distance covered = 370 m.

Q. 10. An arithmetic progression 5, 12, 19, ... has 50 terms. Find its last term. Hence find the sum of its last 15 terms.

Ans: 50^{th} terms = $a_{50} = 5 + 49 \times 7 = 5 + 343 = 348$

$$a_{36} = 5 + 35 \times 7 = 250$$

From last , $a = 348$, $d = -7$

$$\text{Sum of its last 15 terms} = \frac{15}{2} [2 \times 348 + 14 \times -7] = 7.5 [696 - 98] = 7.5 \times 598 = 4485$$