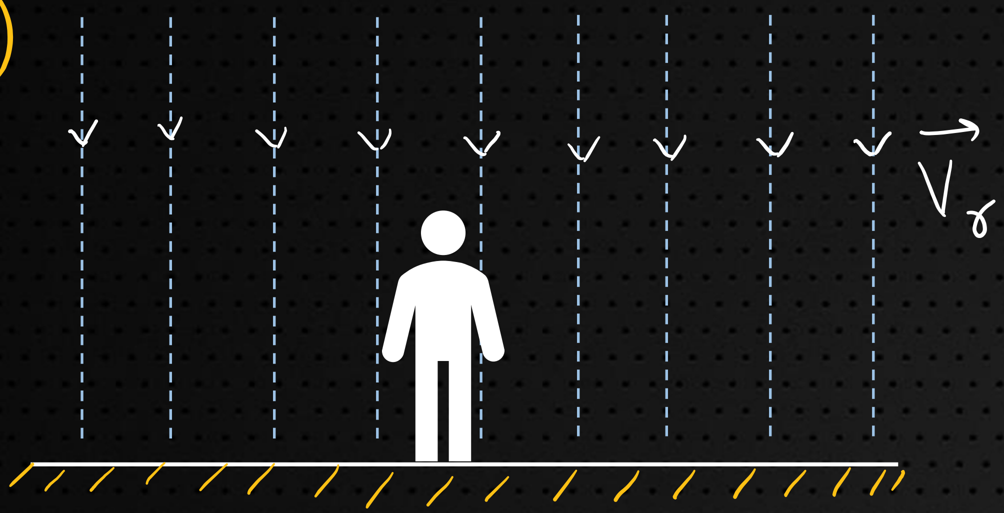
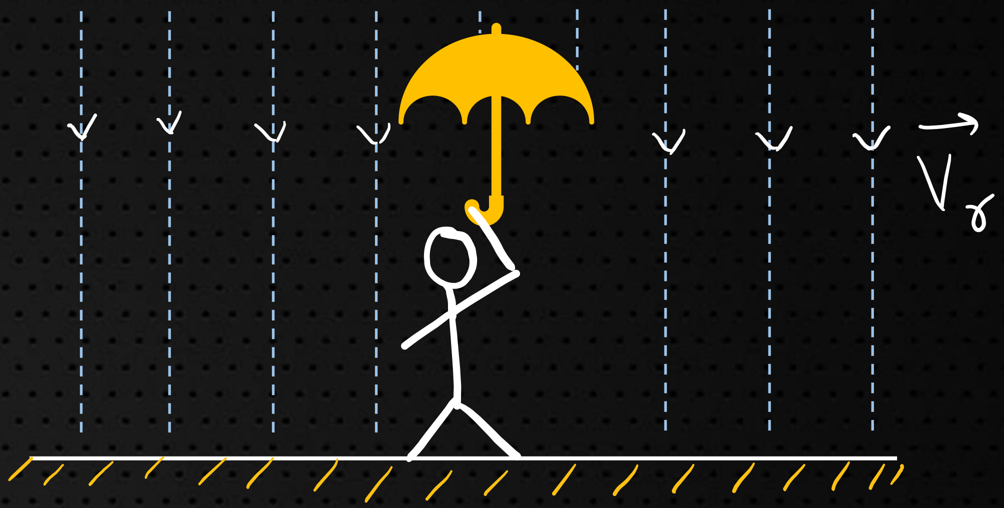
Rain-Man Problems

(A)



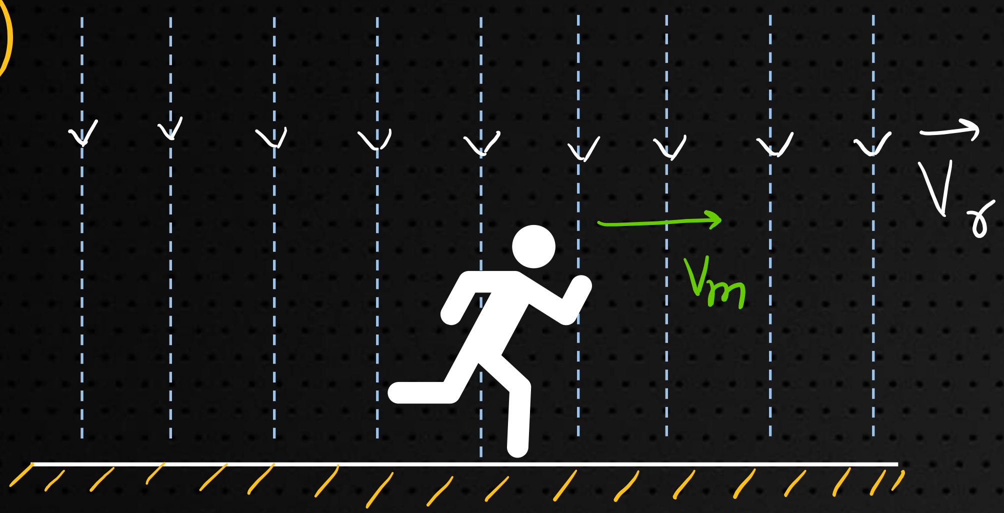
Man at rest



↳ He has to hold umbrella as shown

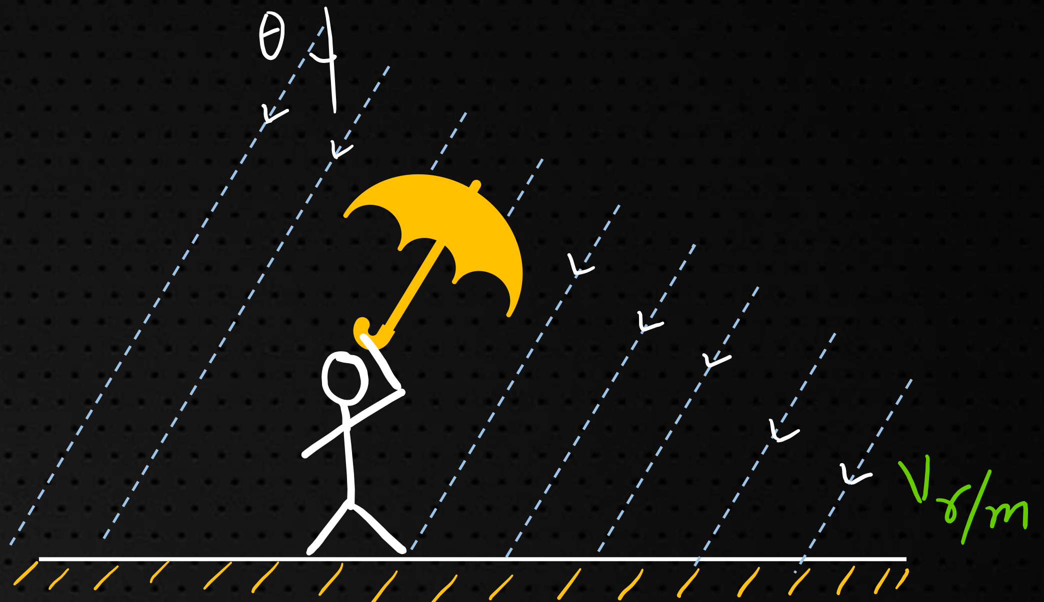
Rain-Man Problems

(A)

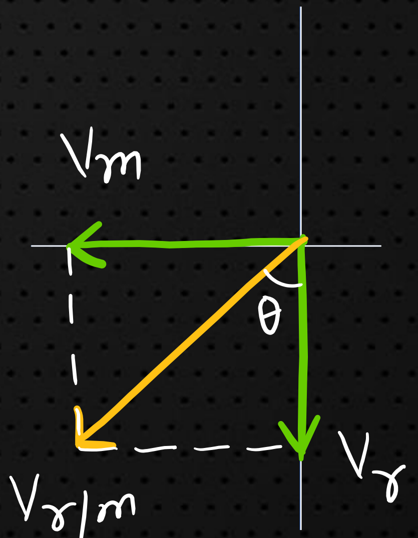


Ground Frame

Man Running



Man's Frame

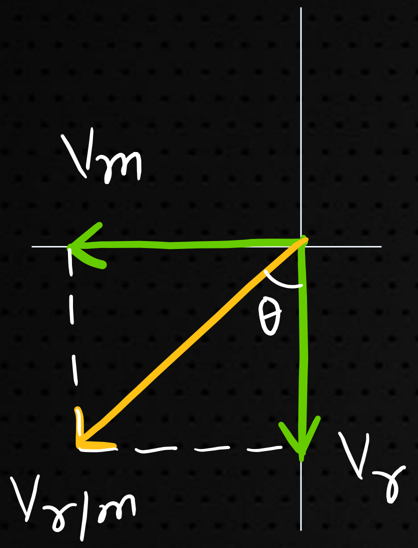


(i) $\tan\theta = V_m / V_r$

(ii) $V_{r/m} = \sqrt{V_m^2 + V_r^2}$

$\vec{V}_{r/m} = \vec{V}_r - \vec{V}_m$

Rain-Man Problems



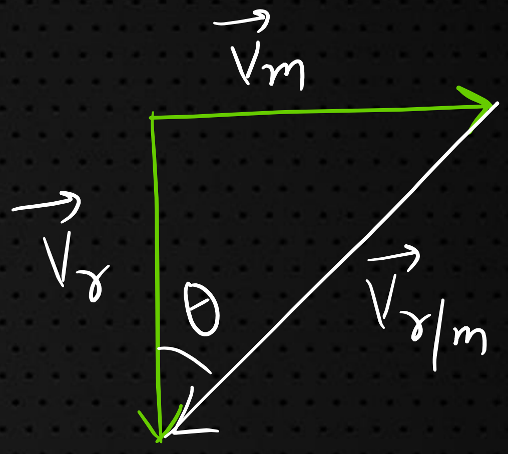
(i) $\tan\theta = V_m / V_r$

(ii) $V_{r/m} = \sqrt{V_m^2 + V_r^2}$

$\vec{V}_{r/m} = \vec{V}_r - \vec{V}_m$

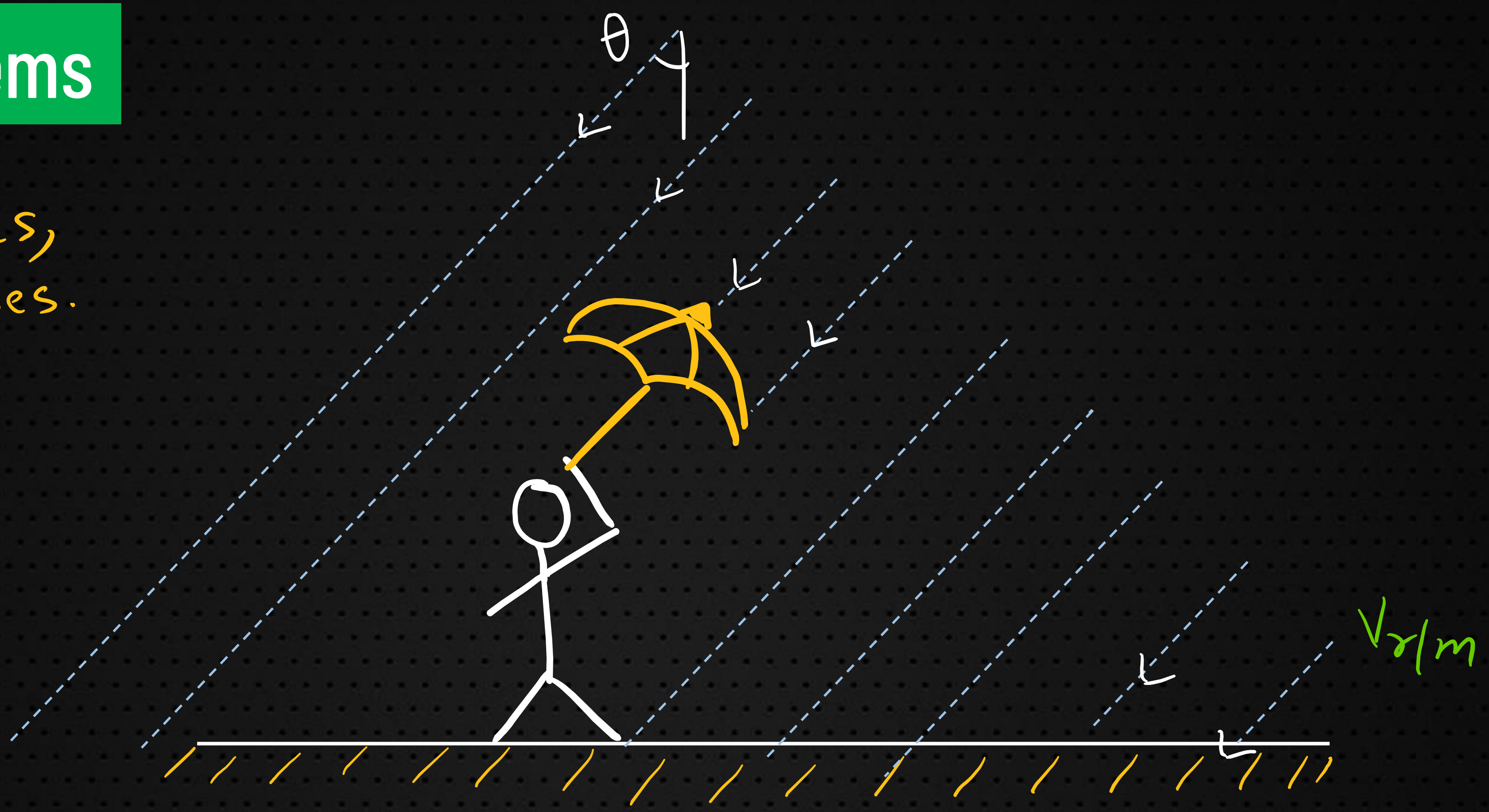
Another way:

$\vec{V}_r = \vec{V}_m + \vec{V}_{r/m}$



Rain-Man Problems

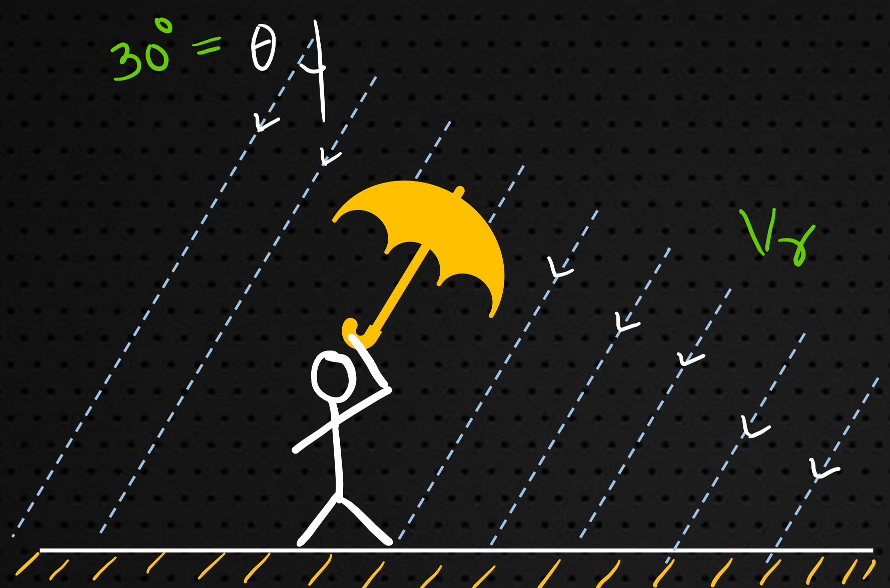
As V_m increases,
θ also increases.



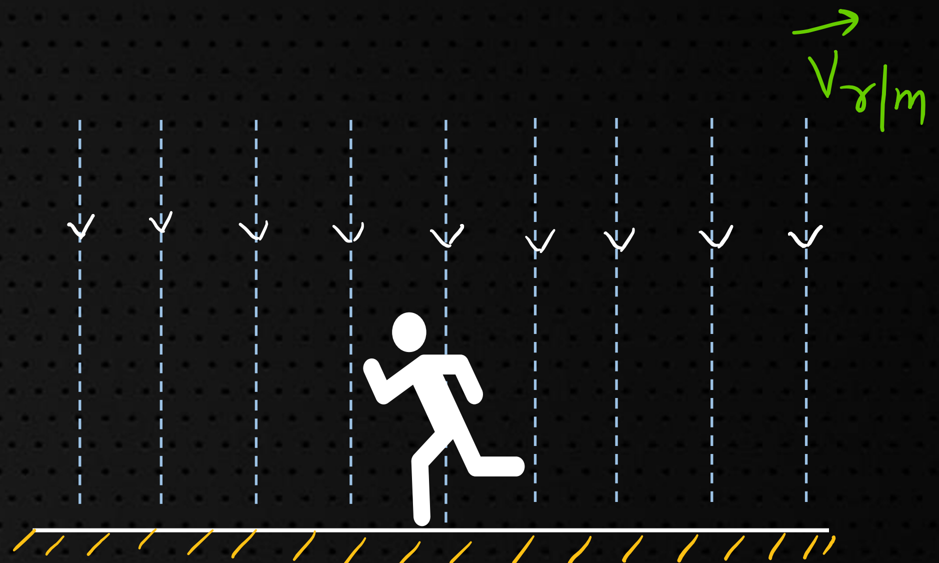
Man's Frame

Q1. **A man standing on a road has to hold his umbrella at 30° with the vertical to keep the rain away. He throws the umbrella and starts running at 10 km/h. He finds that raindrops are hitting his head vertically. Find the speed of raindrops with respect to (a) the road, (b) the moving man.**

Sol^n:



Ground frame



Man's frame

Given: (i) Dir^n of V_r
(ii) Dir^n of $V_{r/m}$
(iii) $V_m = 10$ Km/h and dir^n.

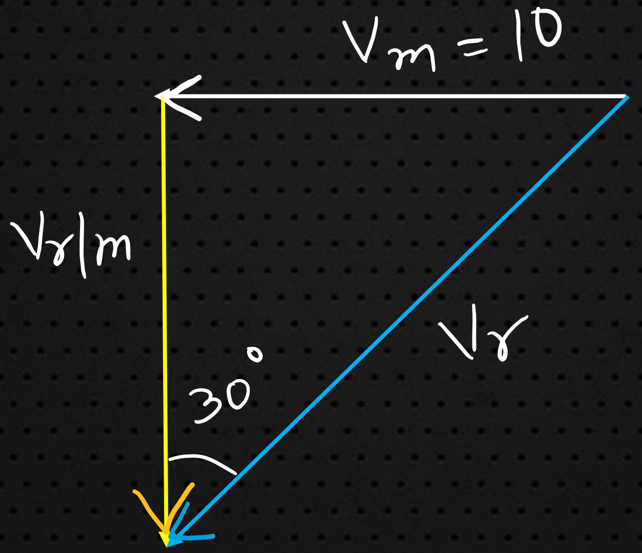
Q1. **A man standing on a road has to hold his umbrella at 30° with the vertical to keep the rain away. He throws the umbrella and starts running at 10 km/h. He finds that raindrops are hitting his head vertically. Find the speed of raindrops with respect to (a) the road, (b) the moving man.**

Sol^n: Given: (i) Dir^n of V_r

(ii) Dir^n of $V_{r/m}$

(iii) $V_m = 10$ Km/h and dir^n.

$\vec{V}_{r/m} = \vec{V}_r - \vec{V}_m \Rightarrow \vec{V}_r = \vec{V}_m + \vec{V}_{r/m}$



(a) $\sin 30° = \dfrac{V_m}{V_r}$

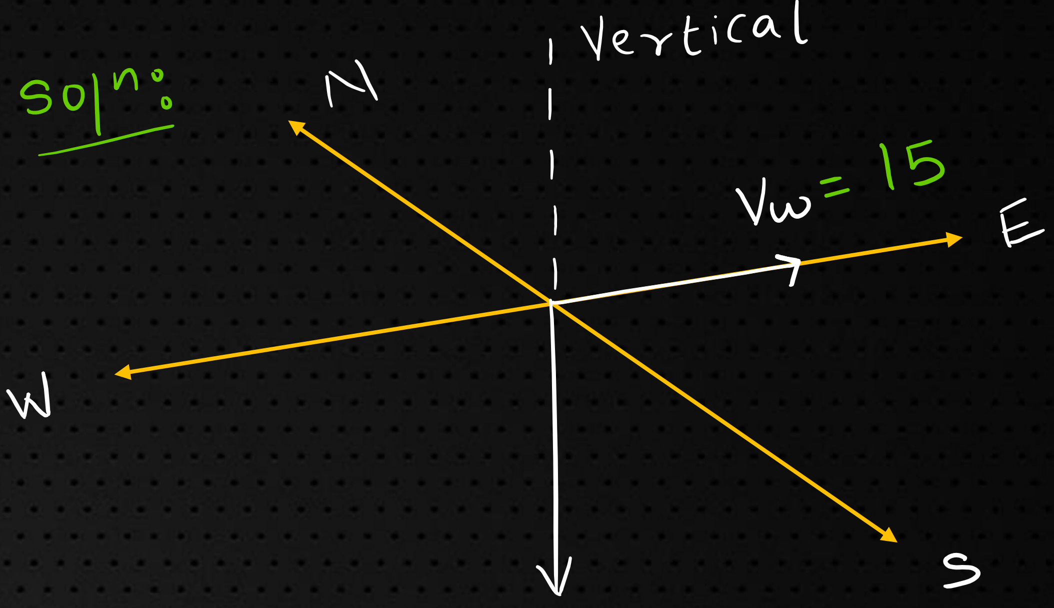
$\Rightarrow V_r = \dfrac{10}{1/2} = 20$ Km/h

(b) $\tan 30° = V_m / V_{r/m}$

$\Rightarrow V_{r/m} = \dfrac{10}{1/\sqrt{3}} = 10\sqrt{3}$ Km/h

Q2. Rain is falling vertically with a speed of 20 ms^{-1} relative to air. A person is running in the rain with a velocity of 5 ms^{-1} and a wind is also blowing with a speed of 15 ms^{-1} (both towards east). Find the angle with the vertical at which the person should hold his umbrella so that he may not get drenched.

Soln:

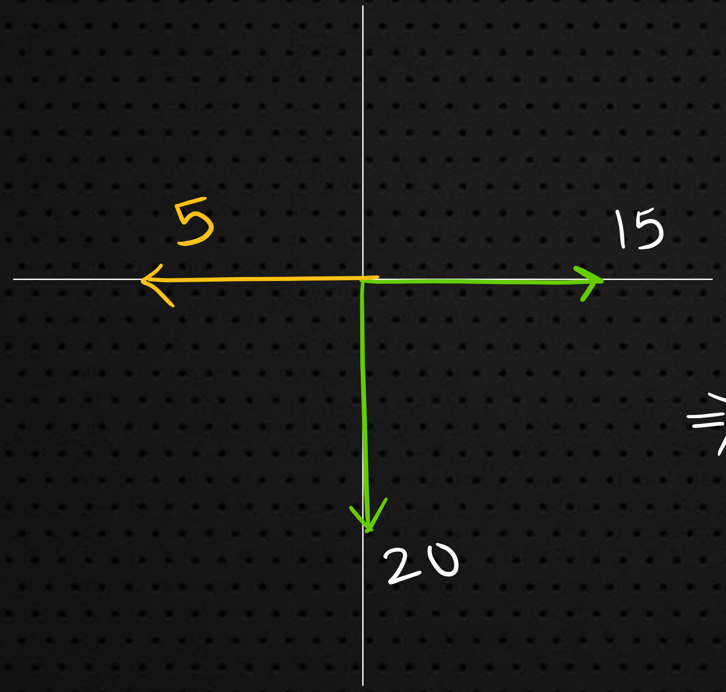


$20 = V_{r/w}$

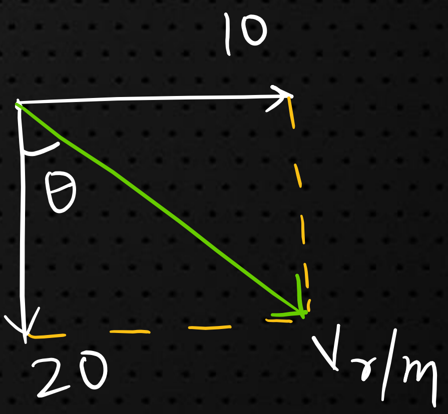
$\vec{V}_r = \vec{V}_w + \vec{V}_{r/w}$

$\vec{V}_{r/m} = \vec{V}_r - \vec{V}_m$

$\Rightarrow \vec{V}_{r/m} = \vec{V}_r + (-\vec{V}_m)$



$\Rightarrow$

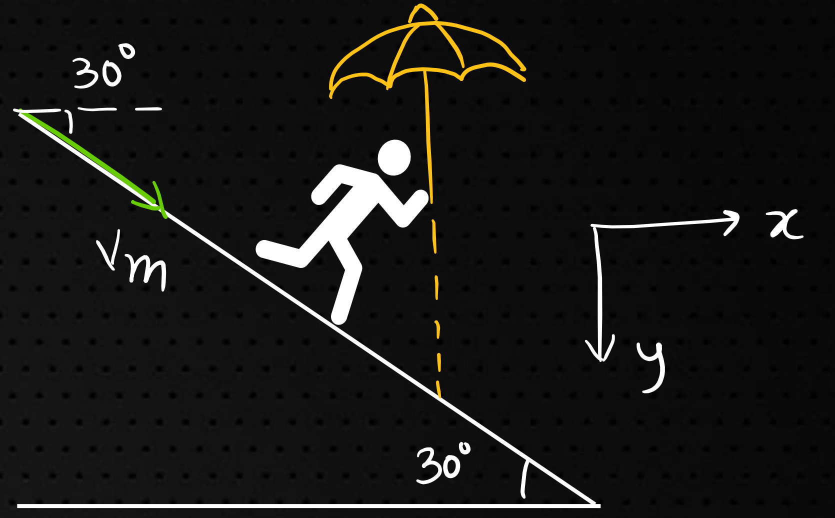


$\tan\theta = \frac{10}{20}$

$\therefore \theta = \tan^{-1}\frac{1}{2}$

Ans

Q3. A man is coming down an incline of angle 30°. When he walks with speed $2\sqrt{3}$ m/s, he has to keep his umbrella vertical to protect himself from rain. The actual speed of rain is 5 m/s. At what angle with vertical should he keep his umbrella when he is at rest so that he does not get drenched?



Sol^n: Given : (i) $V_m = 2\sqrt{3}$ m/s and direction

(ii) $V_{r/m}$ direction (Vertically down)

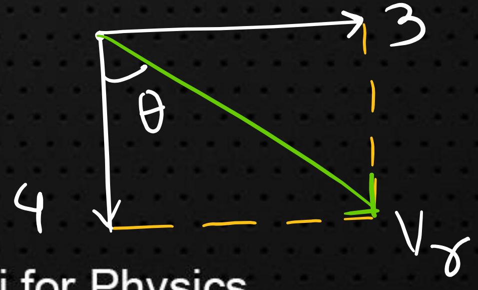
(iii) $V_r = 5$ m/s

To Find : direction of V_r

$$\vec{V}_{r/m} = \vec{V}_r - \vec{V}_m \Rightarrow \vec{V}_r = \vec{V}_m + \vec{V}_{r/m}$$

$$\Rightarrow a\hat{i} + b\hat{j} = (3\hat{i} + \sqrt{3}\hat{j}) + V_{r/m}\hat{j}$$

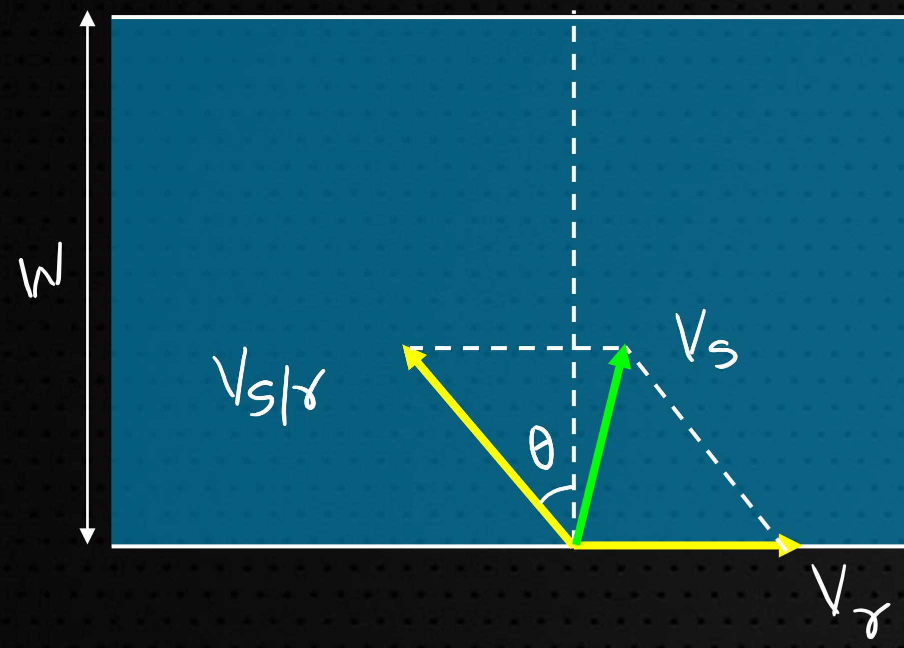
$\therefore \boxed{a = 3}$ also, $\sqrt{a^2 + b^2} = 5 \Rightarrow b = \sqrt{25 - 9} = 4$ m/s



$\tan\theta = 3/4$

$\therefore \boxed{\theta = 37°}$ Ans.

River-Swimmer



River Crossing

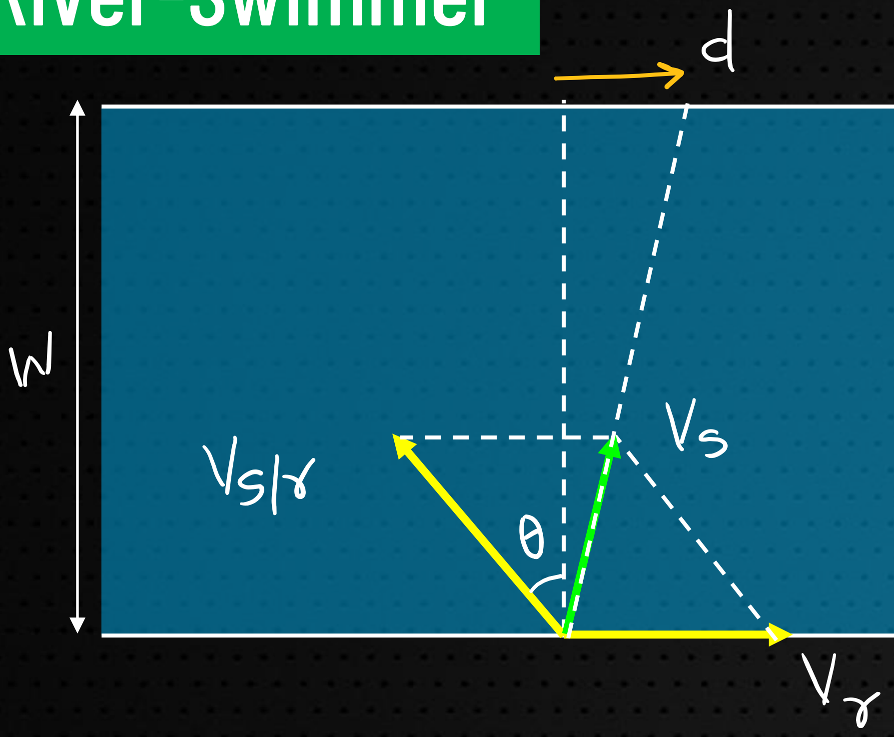
$\vec{V}_r$: velocity of river w.r.t Ground

$\vec{V}_{s/r}$: velocity of swimmer w.r.t river (vel of swimmer in still river)

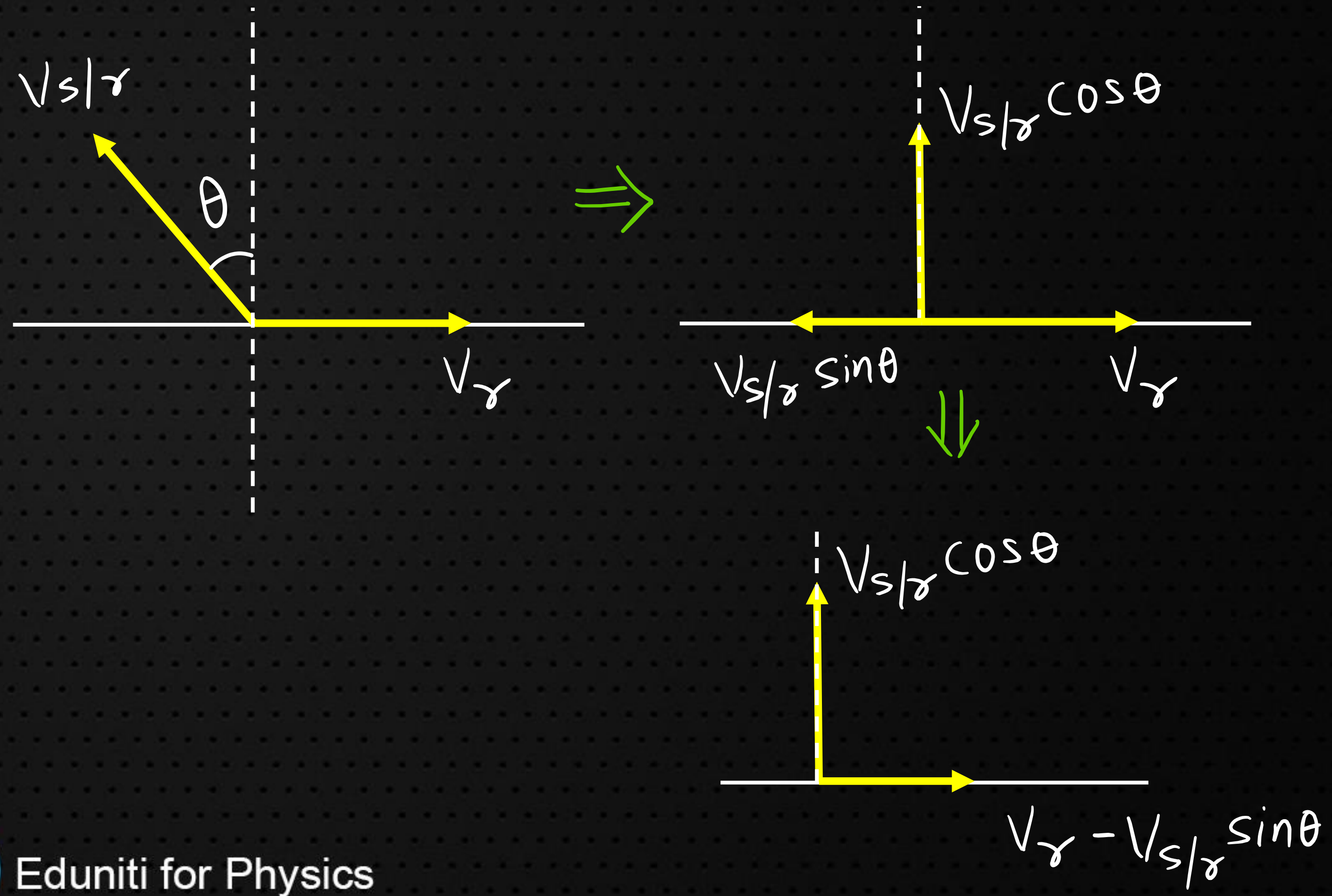
$\vec{V}_s$: velocity of swimmer w.r.t Ground

River-Swimmer

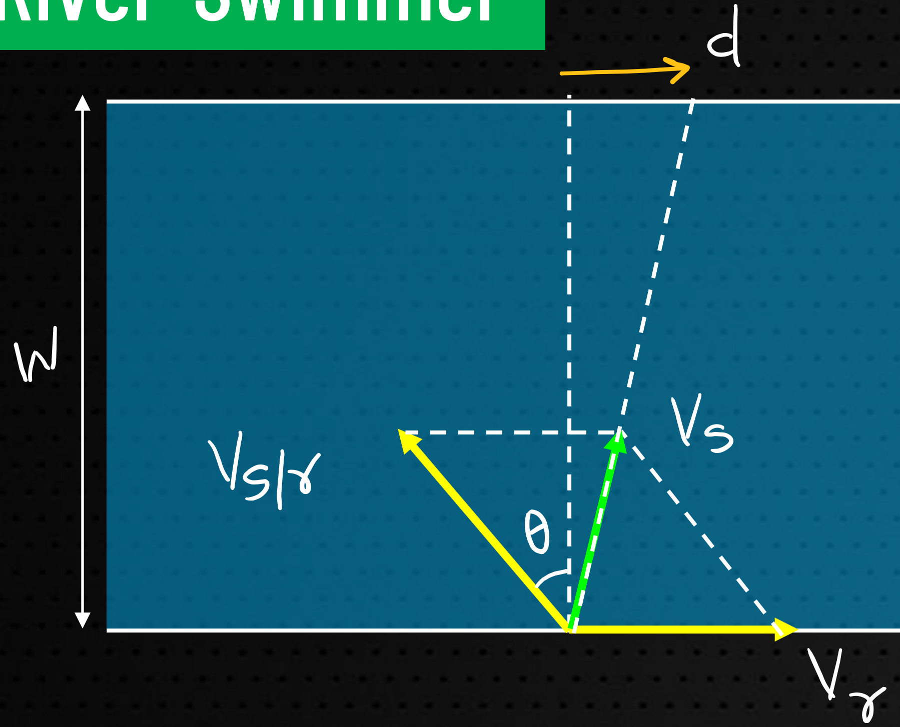
River Crossing



1. Drift (d) : Displacement along x-axis

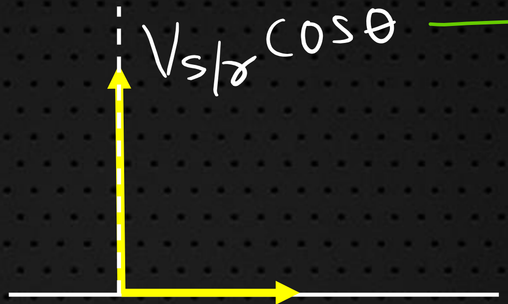


River-Swimmer



River Crossing

1. Drift (d) : Displacement along x-axis



$V_{s/r}\cos\theta \longrightarrow$ Responsible for crossing river $\left(t = \dfrac{W}{V_{s/r}\cos\theta} \right)$

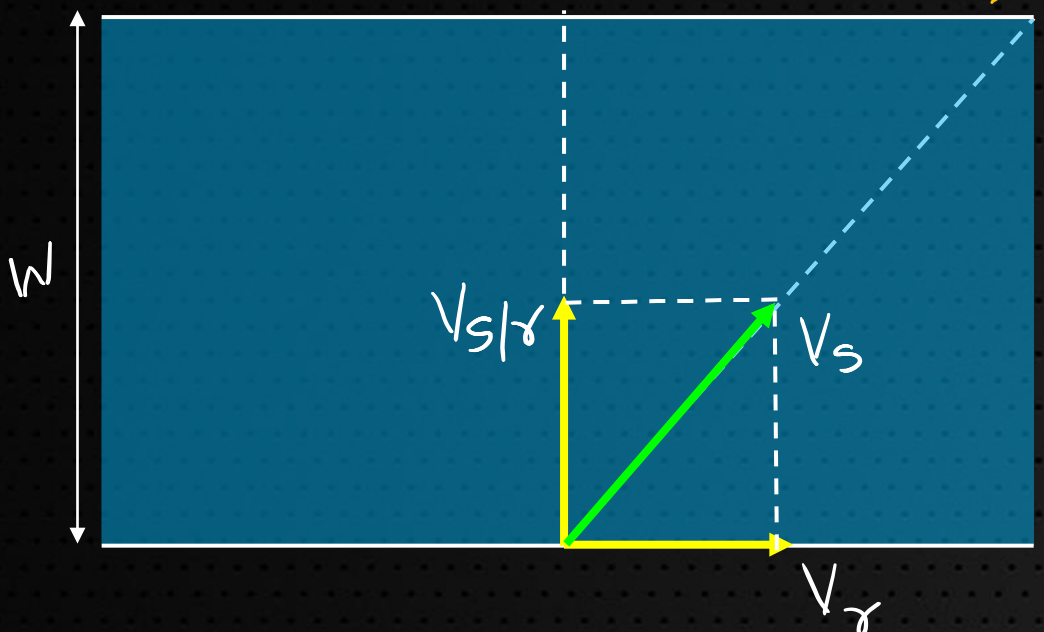
$V_r - V_{s/r}\sin\theta$

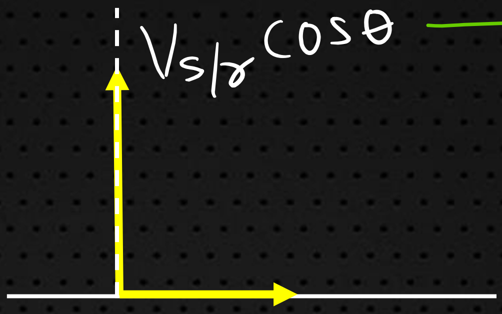
$\hookrightarrow$ Responsible for Drift

$$d = (V_r - V_{s/r}\sin\theta)\cdot t$$

River-Swimmer

River Crossing Case A



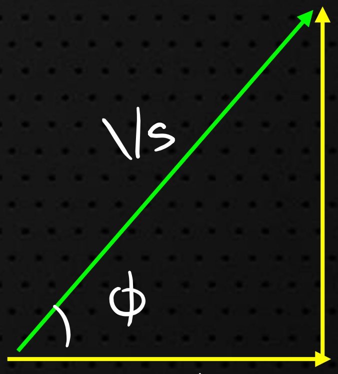


$V_{s/r}\cos\theta \longrightarrow$ Responsible for crossing river $\left(t = \dfrac{W}{V_{s/r}\cos\theta}\right)$

Condition to cross in minimum time:

For t_{min}, $\cos\theta = 1 \Rightarrow \theta = 0^\circ \Rightarrow V_{s/r}$ should be $\perp$ to V_r

$$\boxed{t_{min} = \frac{W}{V_{s/r}}}$$



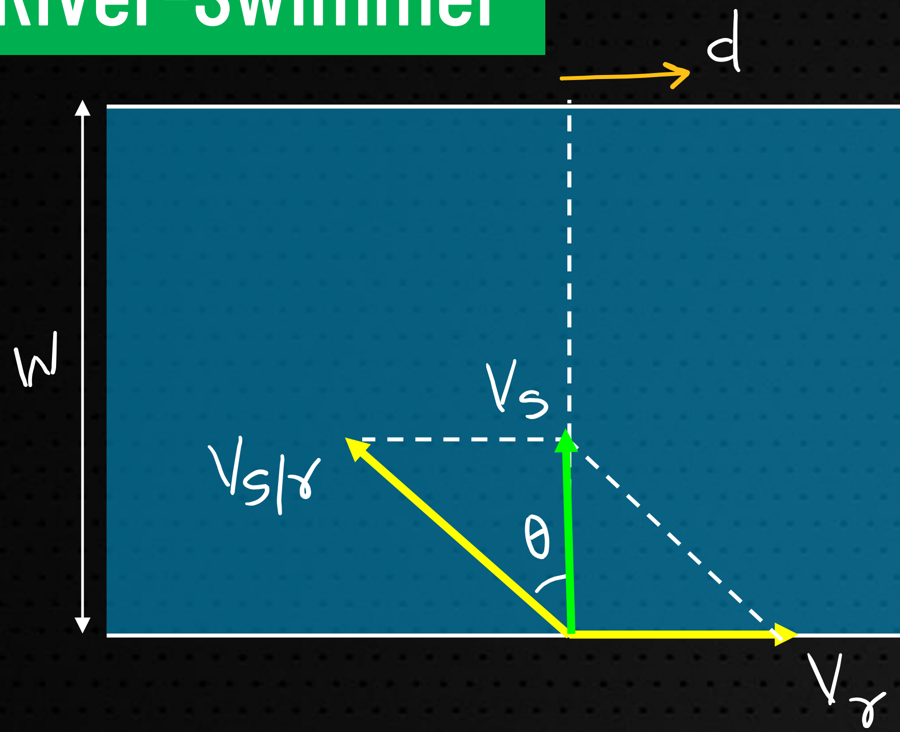
$$\vec{V_s} = \vec{V_{s/r}} + \vec{V_r}$$

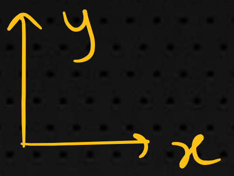
$$V_s = \sqrt{V_{s/r}^2 + V_r^2}$$

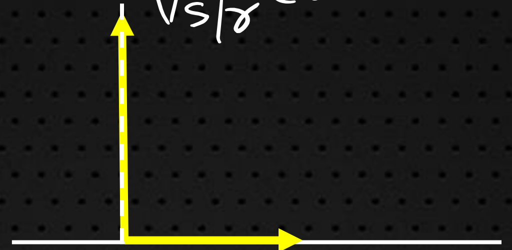
$$\tan\phi = \frac{V_{s/r}}{V_r}$$

River-Swimmer

River Crossing Case B







$V_{s/r}\cos\theta \longrightarrow$ Responsible for crossing river $\left(t = \dfrac{W}{V_{s/r}\cos\theta}\right)$

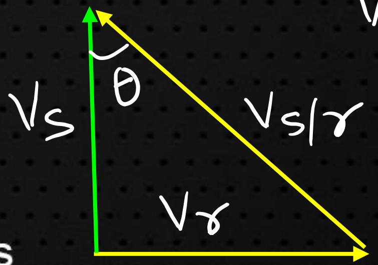
$V_r - V_{s/r}\sin\theta \longrightarrow$ Causes drift

$d = (V_r - V_{s/r}\sin\theta)\cdot t$

Condition for drift to be zero (shortest path)

For $d = 0$, $V_r - V_{s/r}\sin\theta = 0 \Rightarrow V_r = V_{s/r}\sin\theta$

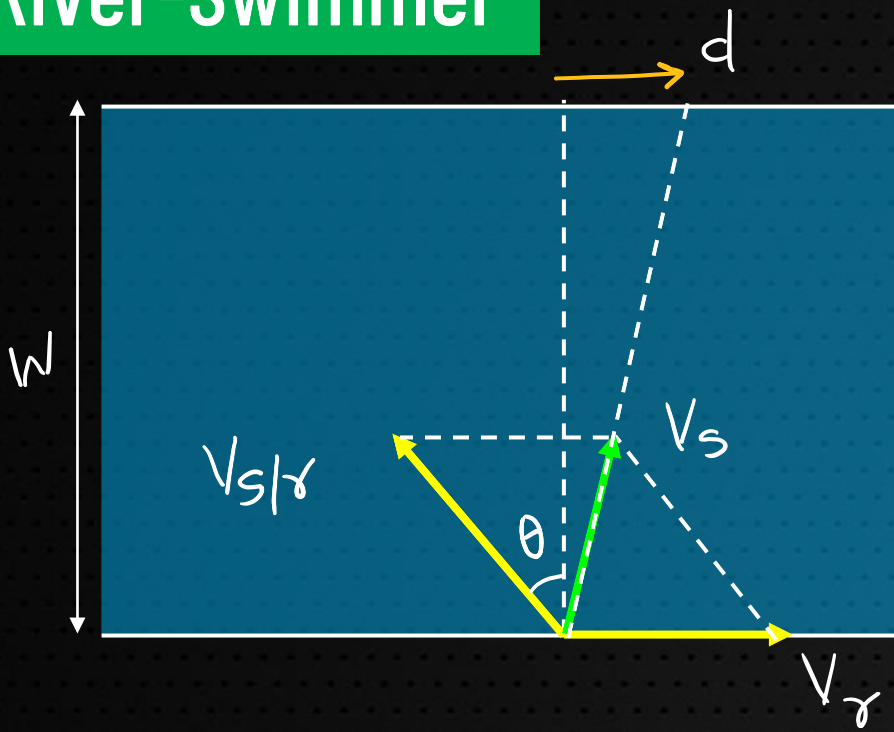
also, $\sin\theta = \dfrac{V_r}{V_{s/r}}$ $\because \sin\theta \leq 1 \Rightarrow \dfrac{V_r}{V_{s/r}} \leq 1 \Rightarrow \boxed{V_r \leq V_{s/r}}$

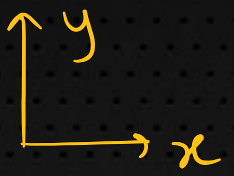


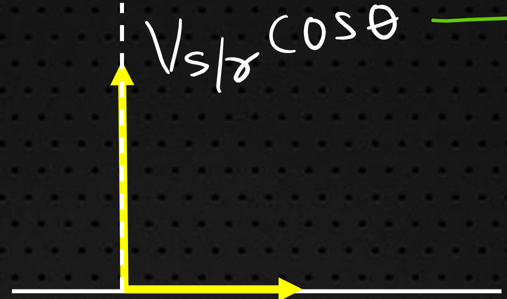
$V_s = \sqrt{V_{s/r}^2 - V_r^2}$

River-Swimmer

River Crossing Case C







$V_{s/r}\cos\theta \longrightarrow$ Responsible for crossing river $\left(t = \dfrac{W}{V_{s/r}\cos\theta}\right)$

$V_r - V_{s/r}\sin\theta \longrightarrow$ causes drift

$d = (V_r - V_{s/r}\sin\theta)\cdot t$

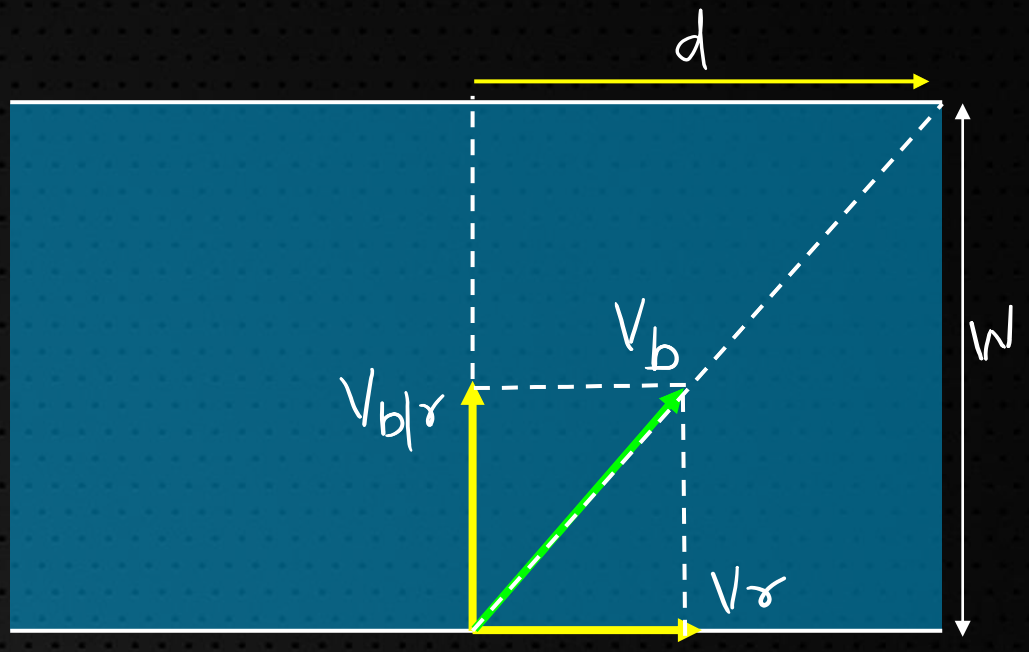
If $V_r > V_{s/r}$ d can never be zero :

so, $d = (V_r - V_{s/r}\sin\theta)\cdot\dfrac{W}{V_{s/r}\cos\theta}$ $\therefore$ Cond^n for d_{min}?

Differentiate w.r.t θ and equate to zero.

We get : $\sin\theta = \dfrac{V_{s/r}}{V_r} \Rightarrow \boxed{\theta = \sin^{-1}\left(\dfrac{V_{s/r}}{V_r}\right)}$

Q1. A river 400 m wide is flowing at a rate of 2·0 m/s. A boat is sailing at a velocity of 10 m/s with respect to the water, in a direction perpendicular to the river. (a) Find the time taken by the boat to reach the opposite bank. (b) How far from the point directly opposite to the starting point does the boat reach the opposite bank ?

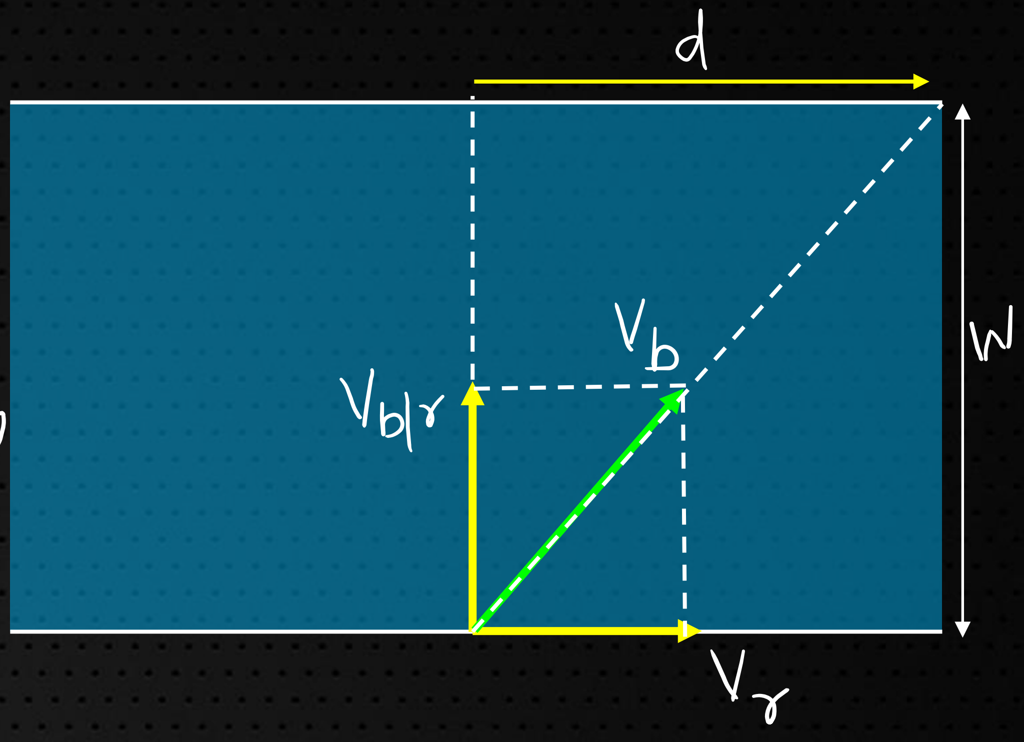


Sol^n: $W = 400m$, $V_r = 2m/s$, $V_{b/r} = 10m/s$

(a) $t = \frac{W}{V_{b/r}} = \frac{400}{10} = \boxed{40\,s}$

(b) $d = V_r \cdot t = 2 \times 40 = \boxed{80\,m}$

Q2. A man wishes to cross a river in a boat. If he crosses the river in minimum time, he takes 10 min with a drift of 120 m. If he crosses the river taking shortest route, he takes 12.5 min Find the velocity of boat with respect to water. (in m/min)



Soln: For t_{min}: $d = V_r \cdot t \Rightarrow V_r = \frac{120}{10} = 12$ m/min

Also, $t_{min} = \frac{W}{V_{b/r}} \Rightarrow W = 10 V_{b/r}$ — (1)

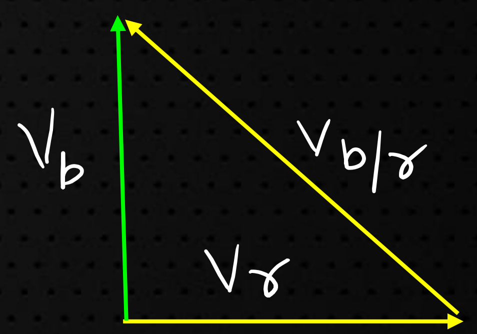
$$V_b \cdot t = W$$

$$\Rightarrow \sqrt{V_{b/r}^2 - V_r^2} \cdot t = 10 V_{b/r}$$

$$\Rightarrow (V_{b/r}^2 - 144) \frac{625}{4} = 100 V_{b/r}$$

$$\therefore \boxed{V_{b/r} = 20 \text{ m/min}}$$

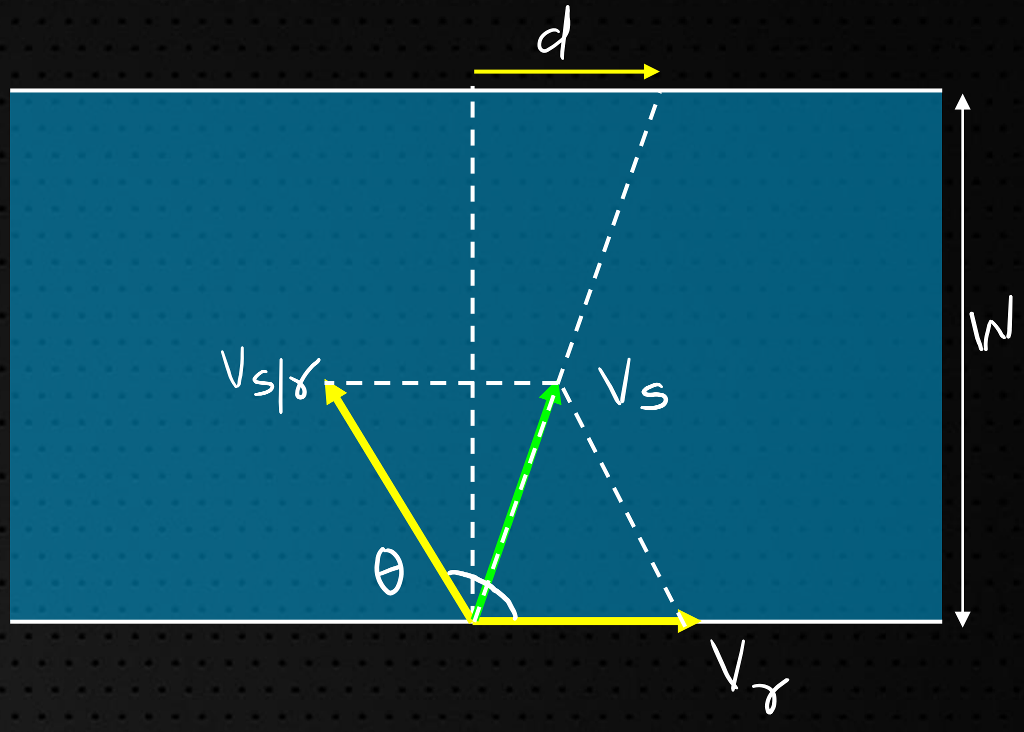
Ans



For shortest Path

Q3. A swimmer wishes to cross a 500 m wide river flowing at 5 km/h. His speed with respect to water is 3 km/h. (a) If he heads in a direction making an angle θ with the flow, find the time he takes to cross the river. (b) Find the shortest possible time to cross the river.

Consider the situation of the previous problem. The man has to reach the other shore at the point directly opposite to his starting point. If he reaches the other shore somewhere else, he has to walk down to this point. Find the minimum distance that he has to walk.



Sol^n: $W = 500\,m$, $V_r = 5\,Km/h$, $V_{s/r} = 3\,Km/h$

(a) $t = \dfrac{W}{V_{s/r}\cos(\theta - 90^\circ)} = \dfrac{500}{\frac{3\times 1000}{60}\sin\theta} = \dfrac{10}{\sin\theta}$ min

(b) For t_{min}, $\theta = 90^\circ \Rightarrow t_{min} = 10$ min

(c) $\because V_r > V_{s/r} \Rightarrow d$ can never be zero.

d_{min} occurs at $\sin(\theta - 90^\circ) = \dfrac{V_{s/r}}{V_r} \Rightarrow \cos\theta = \dfrac{-3}{5}$

Q3. A swimmer wishes to cross a 500 m wide river flowing at 5 km/h. His speed with respect to water is 3 km/h. (a) If he heads in a direction making an angle θ with the flow, find the time he takes to cross the river. (b) Find the shortest possible time to cross the river.

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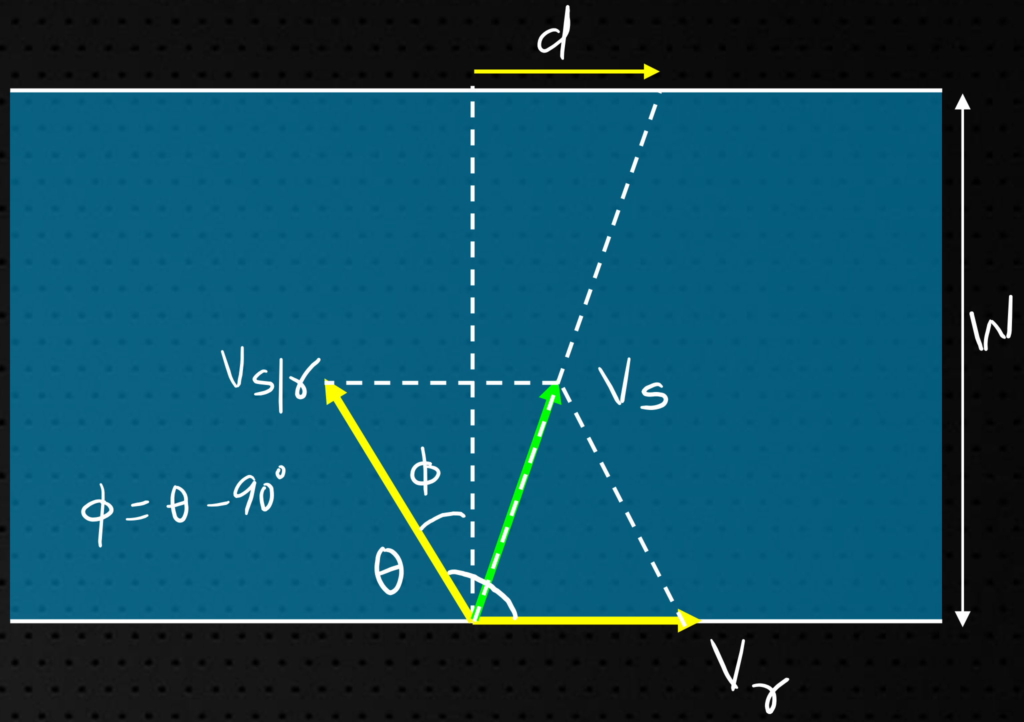

Sol^n: $W = 500m$, $V_r = 5\,Km/h$, $V_{s/r} = 3\,Km/h$

(a) $t = \dfrac{W}{V_{s/r}\cos(\theta - 90^\circ)} = \dfrac{500}{3 \times \frac{1000}{60} \sin\theta} = \dfrac{10}{\sin\theta}$ min

(c) $\because V_r > V_{s/r} \Rightarrow$ d can never be zero.

d_{min} occurs at $\cos\theta = -3/5$

$d_{min} = (V_r + V_{s/r}\cos\theta) \times \dfrac{10}{\sin\theta \times 60} = \left(5 - 3 \times \dfrac{3}{5}\right) \times \dfrac{5}{4 \times 6}$

$= \boxed{2/3\ Km}$